# DEGENERIRANA PERTURBACIJA 7

Matjaž Ivančič

maj, 2008

**Naloga**

Obravnavaj lastna stanja dveh delcev s spinom 1 in Hamiltonianom

$$H = J\vec{S}_1 \cdot \vec{S}_2 + \frac{g\mu_b}{\hbar}\vec{S}_1 \cdot \vec{B} \tag{1}$$

perturbativno v limiti, ko je sklopitev med spini $J$ majhna.

**Rešitev**

Problem se lotimo s teorijo perturbacij, kjer Hamiltonian (1) predelamo v osnovni del $H_0$ in perturbacijski $H_1$

$$H = H_0 + H_1 , \tag{2}$$

kjer je

$$H_0 = \frac{g\mu_b}{\hbar}\vec{S}_1 \cdot \vec{B} = \gamma \vec{S}_{1z} , \tag{3}$$

kjer smo privzeli, da je magnetno polje usmerjeno v z smer

in

$$H_1 = J\vec{S}_1 \cdot \vec{S}_2. \tag{4}$$

Iz dveh delcev s spinom 1 lahko pridelamo naslednje produktne funkcije, kjer bomo skrajšali zapis na $|s_{1z}, s_{2z}\rangle$

$$|1,1\rangle, |1,0\rangle, |1,-1\rangle,$$

$$|0,1\rangle, |0,0\rangle, |0,-1\rangle,$$

$$|-1,1\rangle, |-1,0\rangle, |-1,-1\rangle,$$

kjer vidimo, da imamo tri trikrat degenerirana stanja.

Ker nas zanimajo popravki zaradi perturbacije, izračunajmo prvo neperturbirana lastna stanja tako, da delujemo z operatorjem $H_0$ na vsa stanja.

$$H_0|1,1\rangle = E_0|1,1\rangle = \gamma\hbar|1,1\rangle,$$

$$H_0|1,0\rangle = \gamma\hbar|1,0\rangle,$$

$$H_0|1,-1\rangle = \gamma\hbar|1,-1\rangle,$$

$$H_0|0,1\rangle = 0,$$

$$H_0|0,0\rangle = 0,$$

$$H_0|0,-1\rangle = 0,$$

$$H_0|-1,1\rangle = -\gamma\hbar|-1,1\rangle,$$

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Zaradi lažjega računanja lahko enačbo (4) preoblikujemo v pripravnejšo obliko tako, da namesto produkta $\vec{S}_1 \cdot \vec{S}_2$ vstavimo izraz $1/2\left(S^2 - {S_1}^2 - {S_2}^2\right)$, kjer je $\vec{S} = \vec{S}_1 + \vec{S}_2$. V kolikor upoštevamo še relacijo ${S_i}^2 = \hbar^2\vec{S}_i\left(\vec{S}_i + 1\right)$ in vstavimo vrednosti spinov dobimo, da je perturbacija

$$H_1 = J\frac{1}{2}(S^2 - 4\hbar^2)\,. \tag{5}$$

Ker bomo s perturbacijo delovali na celoten spin si poglejmo, še razvoj produktnih stanj v stanja z dobrim celotnim spinom, torej iz $|s_{1z}, s_{2z}\rangle$ v $|s, s_z\rangle$

$$|1,1\rangle = |2,2\rangle$$

$$|1,0\rangle = \sqrt{\frac{1}{2}}|2,1\rangle + \sqrt{\frac{1}{2}}|1,1\rangle$$

$$|1,-1\rangle = \sqrt{\frac{1}{6}}|2,0\rangle + \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle$$

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$$|-1,-1\rangle = |2,-2\rangle$$

Sedaj ko imamo perturbacijo in funkcije lahko izračunamo prvi red popravka lastnih stanj.

Pri računanju popravkov degeneriranih stanj moramo izračunati

$$\det\left(V_{ij} - {E_n}^{(1)}\delta_{ij}\right) = 0\,, \tag{6}$$

kjer je $V_{ij} = \left\langle {n_i}^{(0)}\middle|H_1\middle|{n_j}^{(0)}\right\rangle$.

Tako pri prvem tripletu dobimo sledečo determinanto, kjer smo pustili v zapisu še produktne funkcije,

$$\begin{vmatrix} \left\langle 1,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,1\right\rangle - E_1{}^{(1)} & \left\langle 1,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,0\right\rangle & \left\langle 1,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,-1\right\rangle \\ \left\langle 1,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,1\right\rangle & \left\langle 1,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,0\right\rangle - E_2{}^{(1)} & \left\langle 1,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,-1\right\rangle \\ \left\langle 1,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,1\right\rangle & \left\langle 1,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,0\right\rangle & \left\langle 1,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,-1\right\rangle - E_3{}^{(1)} \end{vmatrix}. \tag{7}$$

Zamenjajmo produktna stanja s stanji z dobrim celotnim spinom, ki smo prej razvili, saj operator deluje na stanja s celotnim spinom

$$\begin{vmatrix} \left\langle 2,2\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|2,2\right\rangle - E_1{}^{(1)} & \left\langle 2,2\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle & \left\langle 2,2\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle \\ \left\langle\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|2,2\right\rangle & \left\langle\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle - E_2{}^{(1)} & \left\langle\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle \\ \left\langle\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|2,2\right\rangle & \left\langle\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle & \left\langle\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle - E_3{}^{(1)} \end{vmatrix} \tag{8}$$

Zaradi ortogonalnosti vektorjev vidimo, da so vsi izvendiagonalni elementi enaki 0 tako, da dobimo popravke $E_1{}^{(1)} = J\hbar^2$, $E_2{}^{(1)} = 0$, $E_3{}^{(1)} = -7/3\,J\hbar^2$

Pri drugem tripletu postopamo enako.

$$\begin{vmatrix} \left\langle 0,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,1\right\rangle - E_4{}^{(1)} & \left\langle 0,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,0\right\rangle & \left\langle 0,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,-1\right\rangle \\ \left\langle 0,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,1\right\rangle & \left\langle 0,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,0\right\rangle - E_5{}^{(1)} & \left\langle 0,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,-1\right\rangle \\ \left\langle 0,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,1\right\rangle & \left\langle 0,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,0\right\rangle & \left\langle 0,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,-1\right\rangle - E_6{}^{(1)} \end{vmatrix}. \tag{9}$$

$$\begin{vmatrix} \left\langle\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle - E_4{}^{(1)} & \left\langle\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle & \left\langle\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\right\rangle \\ \left\langle\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle & \left\langle\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle - E_5{}^{(1)} & \left\langle\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\right\rangle \\ \left\langle\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle & \left\langle\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle & \left\langle\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\right\rangle - E_6{}^{(1)} \end{vmatrix}. \tag{10}$$

Zaradi ortogonalnosti vektorjev ponovno vidimo, da so vsi izvendiagonalni elementi enaki 0 tako, da dobimo popravke $E_4{}^{(1)} = 0$, $E_5{}^{(1)} = J\hbar^2$, $E_6{}^{(1)} = 0$.

Pri tretjem tripletu ponovno postopamo enako.

$$\begin{vmatrix} \left\langle -1,1 \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| -1,1 \right\rangle - E_7{}^{(1)} & \left\langle -1,1 \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| -1,0 \right\rangle & \left\langle 1-,1 \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| -1,-1 \right\rangle \\ \left\langle -1,0 \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| -1,1 \right\rangle & \left\langle -1,0 \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| -1,0 \right\rangle - E_8{}^{(1)} & \left\langle -1,0 \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| -1,-1 \right\rangle \\ \left\langle -1,-1 \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| -1,1 \right\rangle & \left\langle -1,-1 \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| -1,0 \right\rangle & \left\langle -1,-1 \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| -1,-1 \right\rangle - E_9{}^{(1)} \end{vmatrix}.$$

$$\begin{vmatrix} \left\langle \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \right\rangle - E_7{}^{(1)} & \left\langle \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \right\rangle & \left\langle \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| |2,-2\rangle \right\rangle \\ \left\langle \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \right\rangle & \left\langle \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \right\rangle - E_8{}^{(1)} & \left\langle \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| |2,-2\rangle \right\rangle \\ \left\langle |2,-2\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \right\rangle & \left\langle |2,-2\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \right\rangle & \left\langle |2,-2\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| |2,-2\rangle \right\rangle - E_9{}^{(1)} \end{vmatrix}.$$

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Tako pri prvem tripletu dobimo sledečo determinanto, kjer smo pustili v zapisu še produktne funkcije,

$$\begin{vmatrix} \left\langle 1,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,1\right\rangle - {E_1}^{(1)} & \left\langle 1,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,0\right\rangle & \left\langle 1,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,-1\right\rangle \\ \left\langle 1,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,1\right\rangle & \left\langle 1,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,0\right\rangle - {E_2}^{(1)} & \left\langle 1,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,-1\right\rangle \\ \left\langle 1,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,1\right\rangle & \left\langle 1,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,0\right\rangle & \left\langle 1,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|1,-1\right\rangle - {E_3}^{(1)} \end{vmatrix}. \tag{7}$$

Zamenjajmo produktna stanja s stanji z dobrim celotnim spinom, ki smo prej razvili, saj operator deluje na stanja s celotnim spinom

$$\begin{vmatrix} \left\langle 2,2\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|2,2\right\rangle - {E_1}^{(1)} & \left\langle 2,2\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle & \left\langle 2,2\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle \\ \left\langle\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|2,2\right\rangle & \left\langle\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle - {E_2}^{(1)} & \left\langle\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle \\ \left\langle\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|2,2\right\rangle & \left\langle\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle+\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle & \left\langle\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{6}}|2,0\rangle+\sqrt{\frac{1}{2}}|1,0\rangle+\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle - {E_3}^{(1)} \end{vmatrix} \tag{8}$$

Zaradi ortogonalnosti vektorjev vidimo, da so vsi izvendiagonalni elementi enaki 0 tako, da dobimo popravke ${E_1}^{(1)} = J\hbar^2$, ${E_2}^{(1)} = 0$, ${E_3}^{(1)} = -J\hbar^2$

Pri drugem tripletu postopamo enako.

$$\begin{vmatrix} \left\langle 0,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,1\right\rangle - {E_4}^{(1)} & \left\langle 0,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,0\right\rangle & \left\langle 0,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,-1\right\rangle \\ \left\langle 0,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,1\right\rangle & \left\langle 0,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,0\right\rangle - {E_5}^{(1)} & \left\langle 0,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,-1\right\rangle \\ \left\langle 0,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,1\right\rangle & \left\langle 0,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,0\right\rangle & \left\langle 0,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|0,-1\right\rangle - {E_6}^{(1)} \end{vmatrix}. \tag{9}$$

$$\begin{vmatrix} \left\langle\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle - {E_4}^{(1)} & \left\langle\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle & \left\langle\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\right\rangle \\ \left\langle\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle & \left\langle\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle - {E_5}^{(1)} & \left\langle\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\right\rangle \\ \left\langle\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,1\rangle-\sqrt{\frac{1}{2}}|1,1\rangle\right\rangle & \left\langle\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{2}{3}}|2,2\rangle-\sqrt{\frac{1}{3}}|0,0\rangle\right\rangle & \left\langle\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|\sqrt{\frac{1}{2}}|2,-1\rangle+\sqrt{\frac{1}{2}}|1,-1\rangle\right\rangle - {E_6}^{(1)} \end{vmatrix}. \tag{10}$$

Zaradi ortogonalnosti vektorjev ponovno vidimo, da so vsi izvendiagonalni elementi enaki 0 tako, da dobimo popravke ${E_4}^{(1)} = 0$, ${E_5}^{(1)} = J\hbar^2$, ${E_6}^{(1)} = 0$.

Pri tretjem tripletu ponovno postopamo enako.

$$\begin{vmatrix} \left\langle -1,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|-1,1\right\rangle - E_7{}^{(1)} & \left\langle -1,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|-1,0\right\rangle & \left\langle 1-,1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|-1,-1\right\rangle \\ \left\langle -1,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|-1,1\right\rangle & \left\langle -1,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|-1,0\right\rangle - E_8{}^{(1)} & \left\langle -1,0\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|-1,-1\right\rangle \\ \left\langle -1,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|-1,1\right\rangle & \left\langle -1,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|-1,0\right\rangle & \left\langle -1,-1\middle|J\frac{1}{2}(S^2-4\hbar^2)\middle|-1,-1\right\rangle - E_9{}^{(1)} \end{vmatrix}.$$

$$\begin{vmatrix} \left\langle \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \right\rangle - E_7{}^{(1)} & \left\langle \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \right\rangle & \left\langle \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| |2,-2\rangle \right\rangle \\ \left\langle \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \right\rangle & \left\langle \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \right\rangle - E_8{}^{(1)} & \left\langle \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| |2,-2\rangle \right\rangle \\ \left\langle |2,-2\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{6}}|2,0\rangle - \sqrt{\frac{1}{2}}|1,0\rangle + \sqrt{\frac{1}{3}}|0,0\rangle \right\rangle & \left\langle |2,-2\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| \sqrt{\frac{1}{2}}|2,-1\rangle - \sqrt{\frac{1}{2}}|1,-1\rangle \right\rangle & \left\langle |2,-2\rangle \middle| J\frac{1}{2}(S^2-4\hbar^2) \middle| |2,-2\rangle \right\rangle - E_9{}^{(1)} \end{vmatrix}.$$

Zaradi ortogonalnosti vektorjev ponovno vidimo, da so vsi izvendiagonalni elementi enaki 0 tako, da dobimo popravke $E_7{}^{(1)} = -J\hbar^2$, $E_8{}^{(1)} = 0$, $E_9{}^{(1)} = J\hbar^2$.