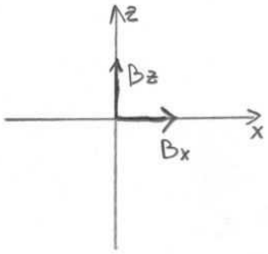
Ob času $t=-\infty$ imamo elektron v izhodišču v osnovnem stanju.
Vklopimo polje in opazujemo, kaj se dogaja z njim



$$|\psi|{-\infty}\rangle = |\downarrow\rangle \qquad H = \frac{g\mu_B}{\hbar}\vec{S}\cdot\vec{B}$$

$$B_x \ll B_z \qquad H_0 = \frac{g\mu_B}{\hbar} B_z S_z$$

Najprej izračunajmo za nemoten sistem

$$H_0|\uparrow\rangle = \frac{g\mu_B}{\hbar} B_z \frac{\hbar}{2}|\uparrow\rangle = \hbar\omega_z|\uparrow\rangle \qquad \frac{g\mu_B B_z}{2} = \hbar\omega_z$$

$$H_0|\downarrow\rangle = -\frac{g\mu_B}{\hbar} B_z \frac{\hbar}{2}|\downarrow\rangle = -\hbar\omega_z|\downarrow\rangle$$

$$|\uparrow,t\rangle = |\uparrow\rangle e^{-i\omega_z t} \qquad |\downarrow,t\rangle = |\downarrow\rangle e^{i\omega_z t}$$

Sedaj pa upoštevajmo naše podatke

$$C_\uparrow(-\infty) = 0 \qquad C_\downarrow(-\infty) = 1$$

$$|\psi,t\rangle = C_\uparrow(t)|\uparrow,t\rangle + C_\downarrow(t)|\downarrow,t\rangle$$

$$C_\uparrow(0) = C_\uparrow(-\infty) - \frac{i}{\hbar}\int_{-\infty}^{0}\langle\uparrow,t|V(t)|\downarrow,t\rangle dt = -\frac{i}{\hbar}\int_{-\infty}^{0}\langle\uparrow,t|V(t)|\downarrow,t\rangle dt$$

$$C_\downarrow(0) = C_\downarrow(-\infty) - \frac{i}{\hbar}\int_{-\infty}^{0}\langle\downarrow,t|V(t)|\downarrow,t\rangle dt = \underline{\underline{1}}$$

Če hočemo izračunati verjetnost, da izmerimo vrednost S_z ob $t=0$ $\frac{\hbar}{2}$, moramo izračunati $C_\uparrow(0)$.

$$S_+ = S_x + iS_y \qquad S_- = S_x - iS_y \qquad S_x = \frac{1}{2}(S_+ + S_-)$$

$$V(t)|\downarrow,t\rangle = \frac{g\mu_B}{\hbar} B_x e^{\frac{t}{\tau}}\frac{1}{2}(S_+ + S_-)|\downarrow,t\rangle = \frac{g\mu_B}{\hbar}\frac{B_x e^{\frac{t}{\tau}}}{2}\hbar|\uparrow\rangle e^{i\omega_z t} =$$

$$\frac{g\mu_B B_x}{2} = \hbar\omega_x \qquad = \hbar\omega_x e^{\frac{t}{\tau}}|\uparrow\rangle e^{i\omega_z t}$$

$$\langle\uparrow,t|V(t)|\downarrow,t\rangle = \hbar\omega_x e^{\frac{t}{\tau}} e^{2i\omega_z t}$$

$$C_\uparrow(0) = -\frac{i}{\hbar}\int_{-\infty}^{0}\hbar\omega_x e^{\frac{t}{\tau}} e^{2i\omega_z t} dt = -i\omega_x\int_{-\infty}^{0} e^{\frac{t}{\tau}} e^{2i\omega_z t} dt = \frac{-i\omega_x}{\frac{1}{\tau}+2i\omega_z}$$

$$P_{S_z=\frac{\hbar}{2}}(0) = |C_\uparrow(0)|^2 = \left|\frac{-i\omega_x}{\frac{1}{\tau}+2i\omega_z}\right|^2 = \frac{\omega_x^2}{\left(\frac{1}{\tau}\right)^2+4\omega_z^2}$$

Sedaj nas pa še zanima kako je ob $t=0$ obrnjen elektronov spin, v limitah $\tau \to 0$ in $\tau \to \infty$.

$$|\psi,\tau\rangle = |\downarrow\rangle e^{i\omega_z\tau} + \frac{-i\omega_x}{\frac{1}{\tau}+2i\omega_z}|\uparrow\rangle e^{-i\omega_z\tau}$$

$$|\psi\rangle = \cos\frac{\vartheta}{2}|\uparrow\rangle + \sin\frac{\vartheta}{2}e^{i\varphi}|\downarrow\rangle$$

① $\tau \to 0$

$$\lim_{\tau\to 0}\frac{-i\omega_x}{\frac{1}{\tau}+2i\omega_z} = 0 \qquad \cos\frac{\vartheta}{2}=0 \qquad \vartheta=\pi$$

$$\sin\frac{\vartheta}{2}=1 \qquad \varphi=0$$

② $\tau \to \infty$ Valovno funkcijo lahko pomnožimo z -1 in se ne spremeni:

$$\lim_{\tau\to\infty}\frac{-i\omega_x}{\frac{1}{\tau}+2i\omega_z} = \frac{\omega_x}{2\omega_z} \qquad \cos\frac{\vartheta}{2} = \frac{\omega_x}{2\omega_z} \qquad \vartheta = 2\cdot\arccos\left(\frac{\omega_x}{2\omega_z}\right) \xrightarrow{\text{razvijemo po Taylorju}}$$

$$\sin\frac{\vartheta}{2}=1 \qquad \varphi=\pi \qquad \to \vartheta = \pi - \frac{\omega_x}{\omega_z}$$

Morda se zdi, da pri točki 2 sinus in kosinus nakazujeta drugačno rešitev; toda ne smemo pozabiti, da smo uporabili samo prvi red perturbacije, ter da je $\omega_x \ll \omega_z$ in zato $\frac{\omega_x}{\omega_z} \sim 0$

Sedaj pa še poglejmo, pri katerem τ bo prišlo do prehoda med stanji. Ker je prehajanje zvezno, poglejmo kakšen τ je na sredini.

$$-\frac{i\omega_x}{\frac{1}{\tau}+2i\omega_z} = \frac{1}{2}\frac{\omega_x}{2\omega_z} \quad \to \quad \tau = \frac{i}{2\omega_z}$$

Ker τ ne more biti imaginaren, ga zanemarimo, in dobimo:

$$\tau = \frac{1}{2\omega_z}$$