Naloga iz Fizike Trdne Snovi

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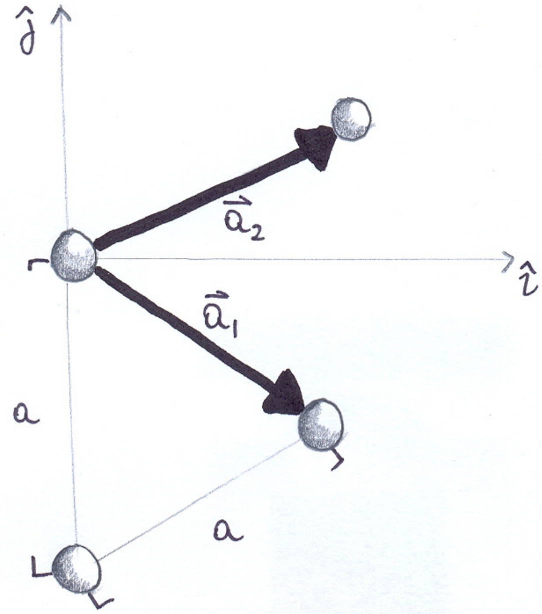
Poišči recipročni mreži za ⓐ trikotno mrežo in ⓑ ploskovno centrirano kubično mrežo (FCC)

ⓐ



Bazni vektorji

$$\vec{a}_1 = \frac{a}{2}\left[\sqrt{3}, -1, 0\right]$$

$$\vec{a}_2 = \frac{a}{2}\left[\sqrt{3}, 1, 0\right]$$

$$\vec{a}_3 = c\left[0, 0, 1\right]$$

Prostornina osnovne celice: $V_0 = (\vec{a}_1, \vec{a}_2, \vec{a}_3) =$

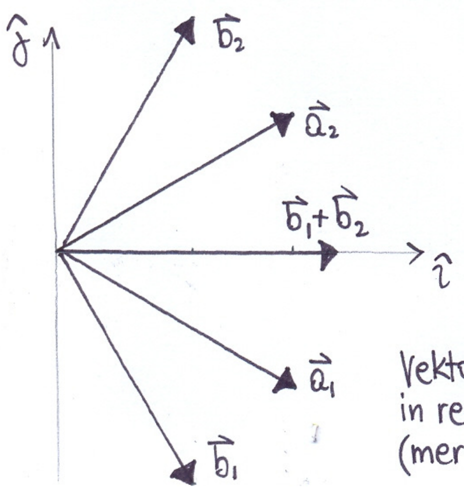
$$= \frac{a}{2}\cdot\frac{a}{2}\cdot c \begin{vmatrix} \sqrt{3} & -1 & 0 \\ \sqrt{3} & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \frac{a^2c}{4}\left(\sqrt{3} - (-\sqrt{3})\right) = \frac{\sqrt{3}}{2}a^2c$$

Vektorji recipročne mreže

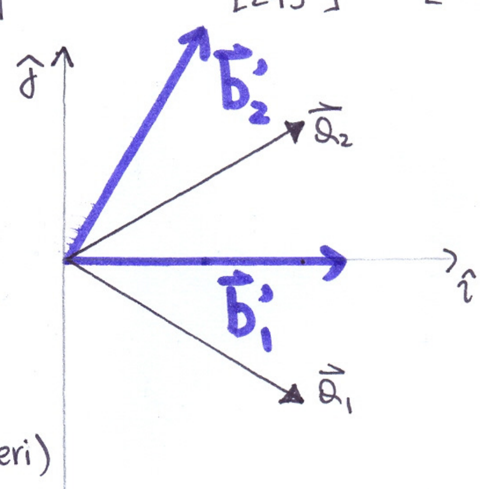
$$\vec{b}_1 = \frac{2\pi}{(\vec{a}_1, \vec{a}_2, \vec{a}_3)}\cdot(\vec{a}_2 \times \vec{a}_3) = \frac{4\pi}{\sqrt{3}a^2c}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \sqrt{3} & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}\frac{a}{2}\cdot c = \frac{2\pi}{a\sqrt{3}}\begin{bmatrix} 1 \\ -\sqrt{3} \\ 0 \end{bmatrix}$$

$$\vec{b}_2 = \frac{2\pi}{V_0}(\vec{a}_3 \times \vec{a}_1) = \frac{4\pi}{\sqrt{3}a^2c}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ \sqrt{3} & -1 & 0 \end{vmatrix}\cdot\frac{a}{2}c = \frac{2\pi}{a\sqrt{3}}\begin{bmatrix} 1 \\ \sqrt{3} \\ 0 \end{bmatrix}$$

$$\vec{b}_3 = \frac{2\pi}{V_0}(\vec{a}_1 \times \vec{a}_2) = \frac{4\pi}{\sqrt{3}a^2c}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \sqrt{3} & -1 & 0 \\ \sqrt{3} & 1 & 0 \end{vmatrix}\frac{a^2}{4} = \frac{\pi}{c\sqrt{3}}\begin{bmatrix} 0 \\ 0 \\ 2\sqrt{3} \end{bmatrix} = \frac{2\pi}{c}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



Vektorji Bravaisove in recipročne mreže (merilo ni točno, zanimajo nas smeri)



(b) Bazni vektorji

$$\vec{a}_1 = \frac{a}{2}[0,\ 1,\ 1]$$
$$\vec{a}_2 = \frac{a}{2}[1,\ 0,\ 1]$$
$$\vec{a}_3 = \frac{a}{2}[1,\ 1,\ 0]$$

Prostornina osnovne celice

$$V_0 = (\vec{a}_1, \vec{a}_2, \vec{a}_3) = \frac{a^3}{8}\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \frac{a^3}{8}\cdot 2 = \frac{a^3}{4}$$

$$\vec{b}_1 = \frac{2\pi}{V_0}(\vec{a}_2 \times \vec{a}_3) = \frac{8\pi}{a^3}\frac{a^2}{4}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \frac{2\pi}{a}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\vec{b}_2 = \frac{2\pi}{V_0}(\vec{a}_3 \times \vec{a}_1) = \frac{2\pi}{a}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \frac{2\pi}{a}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{b}_3 = \frac{2\pi}{V_0}(\vec{a}_1 \times \vec{a}_2) = \frac{2\pi}{a}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \frac{2\pi}{a}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Povzetek

(a) Recipročna mreža trikotne mreže je trikotna mreža. Če je stranica Bravaisove mreže enaka a, je stranica recipročne mreže enaka $2 \cdot \frac{2\pi}{a\sqrt{3}} = \frac{4\pi}{a\sqrt{3}}$. Recipročna mreža je zasukana za kot $30°$ glede na Bravaisov

(b) Recipročna mreža ploskovno centrirane kubične mreže s stranico a telesno centrirana kubična mreža s stranico $2 \cdot \frac{2\pi}{a} = \frac{4\pi}{a}$.