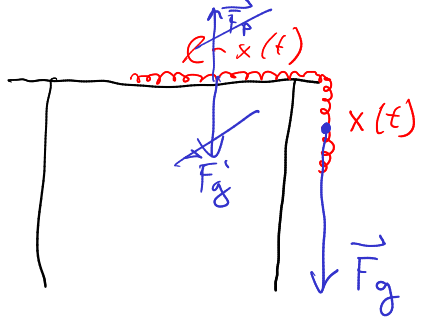
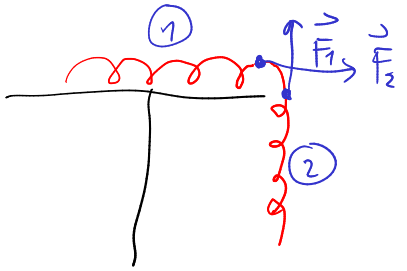
3.9




$\ell$, $m$

$x(t) = ?$ $\quad x(0) = x_0$

$F_g = m(t) \cdot g = m \frac{x(t)}{\ell} g$




$|\vec{F}_1| = |\vec{F}_2| = F$

① $F_2 = m \frac{\ell - x(t)}{\ell} a$

② $m \frac{x(t)}{\ell} g - F_1 = \frac{m x(t)}{\ell} a$

$\Sigma$: $\frac{m x(t)}{\ell} g = m a = m x''(t)$

$$x''(t) = \frac{g}{\ell} x(t)$$

navadna linearna diferencialna enačba drugega reda

$x''(t) = x(t)$

$e^t$

$e^{-t}$

$$x(t) = A f_1(t) + B f_2(t)$$

$$x(t) = A e^{\sqrt{\frac{g}{\ell}} t} + B e^{-\sqrt{\frac{g}{\ell}} t}$$

2 začetna pogoja: $x(0) = x_0$

$x'(0) = 0$

$x(0) = A + B = x_0$

$x'(0) = \left( A \sqrt{\frac{g}{\ell}} e^{\sqrt{\frac{g}{\ell}} t} + B \left( -\sqrt{\frac{g}{\ell}} \right) e^{-\sqrt{\frac{g}{\ell}} t} \right)_{t=0} =$

$= \sqrt{\frac{g}{\ell}} (A - B) = 0$

$\hookrightarrow A = B$

$A = B = \frac{x_0}{2}$

$$x(t) = \frac{x_0}{2} \left( e^{\sqrt{\frac{g}{\ell}} t} + e^{-\sqrt{\frac{g}{\ell}} t} \right)$$

$$x(t) = x_0 \cosh \sqrt{\frac{g}{\ell}} t$$

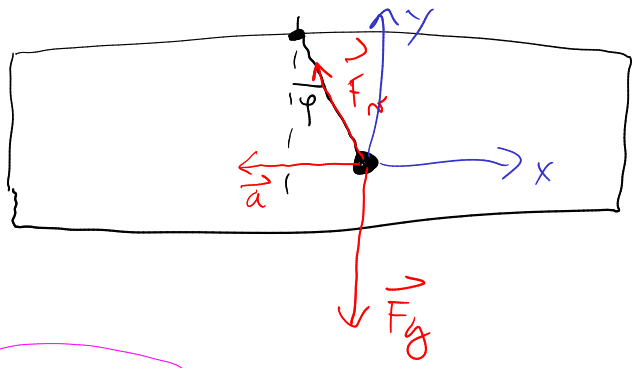
3.51

$a = 0.7\ m/s^2$

$l = 1\ m$

---

$\varphi = ?$




1.)

$\sum \vec{F} = m\vec{a}$

$\vec{F}_g + \vec{F}_v = m\vec{a}$

x: $0 - F_v \sin\varphi = -ma \rightarrow F_v \sin\varphi = ma$

y: $-mg + F_v \cos\varphi = 0 \rightarrow F_v \cos\varphi = mg$

$\therefore \quad tg\,\varphi = \frac{a}{g}$

$\varphi = \arctg \frac{a}{g} = 4^\circ$

2.) s sistemskimi silami

$\sum \vec{F} + \vec{F}_s = 0$

$\sum \vec{F} - m\vec{a} = 0$

3.52

$m_D = 2\ kg$

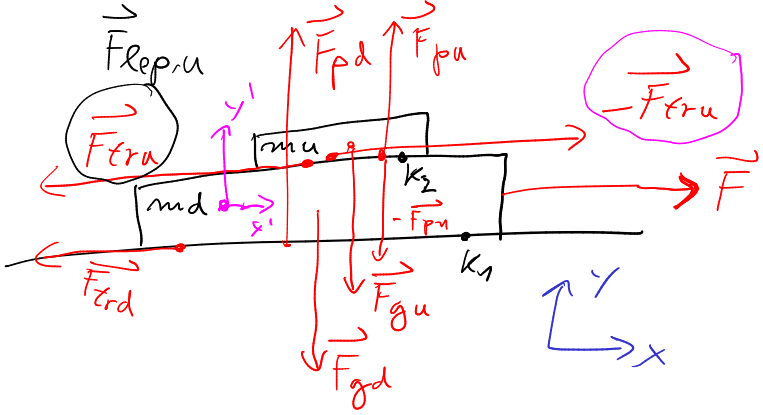
$m_u = 1\ kg$

$k_1 = 0.4$

$k_2 = 0.3$

---

$F_{min} = ?$




1.) sistem deska + utež:

$$\vec{F}_{pd} + \vec{F}_{gd} + \vec{F}_{gu} + \vec{F} + \vec{F}_{trd} = (m_D + m_u)\vec{a}$$

$x$: $-F_{trd} = (m_D + m_u)a$

$y$: $-m_D g - m_u g + F_{pd} = 0$

$F_{trd} = F_{pd} \cdot k_1$

$F_{trd} = (m_D + m_u) g k_1$

$F - (m_D + m_u) g k_1 = (m_D + m_u) a$

$$a = -g k_1 + \frac{F_{min}}{m_D + m_u}$$

2.) v pospešenem koordinatnem sistemu

drugi Newtonov zakon za utež:

$$\vec{F}_{pu} + \vec{F}_{gu} - \vec{F}_{tru} \underbrace{- m_u \vec{a}}_{\text{sistemska sila}} = 0$$

$x'$: $F_{tru} - m_u a = 0$

$y'$: $F_{pu} - m_u g = 0$

$F_{tru} = F_{pu} \cdot k_2$

$F_{tru} = m_u g k_2$

$m_u g k_2 - m_u a = 0$

$a = g k_2$

$F_{eep,u} \leq F_{pu} \cdot k_2$

↳ pospešek, pri katerem utež zdrsne, ker sila lepenja ne more več obdržati na deski

$g k_2 = -g k_1 + \frac{F_{min}}{m_D + m_u}$

$$F_{min} = (m_D + m_u) \cdot g (k_1 + k_2) = 3\ kg \cdot 10 \frac{m}{s^2} \cdot 0.7 = \underline{21\ N}$$

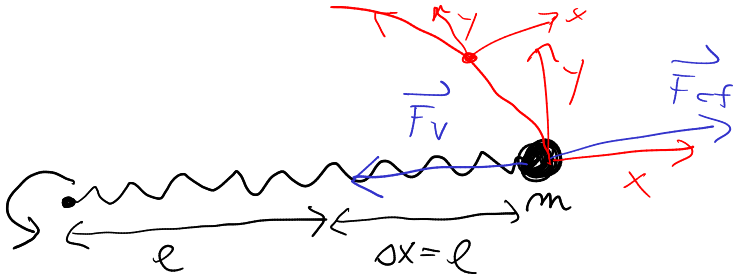
3.56

$k = 20 \frac{N}{cm}$

$m = 100 g$

$\Delta x = l$

$\nu$




$\sum \vec{F} + \vec{F}_s = 0$

$\vec{F}_v + \vec{F}_{cf} = 0$

$\vec{F}_v = F_{cf}$

$k \Delta x = m R \omega^2$

$k l = m (2l) (2\pi\nu)^2$

$2\pi\nu = \sqrt{\frac{k}{2m}}$

$\nu = \frac{1}{2\pi} \sqrt{\frac{k}{2m}} = \frac{1}{2\pi} \sqrt{\frac{20 N}{0.01 m \cdot 2 \cdot 0.1 kg}} =$

$= 15,9 \frac{1}{s} = 15,9 \; Hz$

$N = \frac{kg\, m}{s^2}$

brez sistemskih sil

$\sum \vec{F} = m \vec{a}$

$\vec{F}_v = m \vec{a}_R$

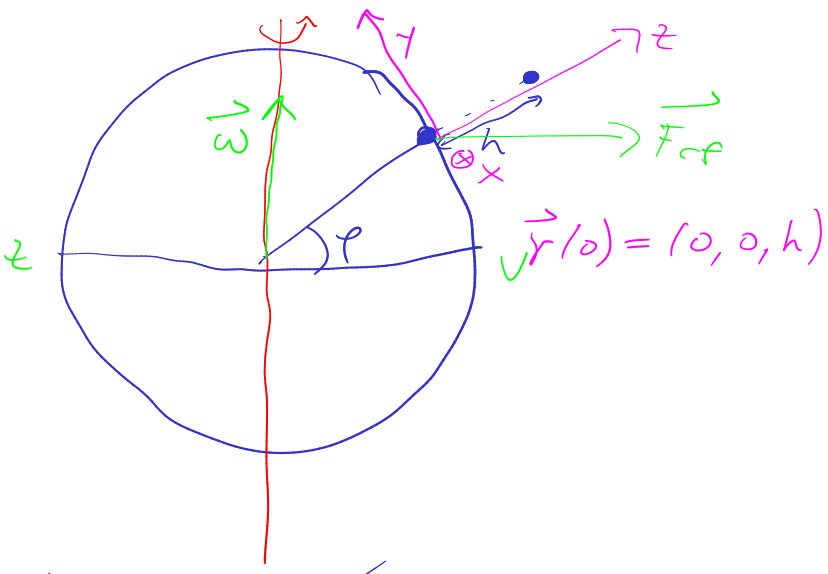
$a_R = R \omega^2$

$F_v = m R \omega^2$

3.66 $\varphi = 30°$

$h = 125\ m$

$\Delta x = ?$




$$\sum \vec{F} - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m\vec{\omega} \times \vec{v} = m\vec{a}$$

$\vec{F}_g$ — centrifugalna sila — Coriolisova sila

zanemarimo

1) prosti pad:

$$z(t) = h - \frac{g t^2}{2}$$

$$v_z(t) = z'(t) = -gt \qquad \vec{v} = \begin{pmatrix} 0 \\ 0 \\ -gt \end{pmatrix}$$

2.) popravek zaradi Coriolisove sile:

$$\vec{F}_c = -2m\vec{\omega} \times \vec{v} \qquad \vec{\omega} = \begin{pmatrix} 0 \\ \omega\cos\varphi \\ \omega\sin\varphi \end{pmatrix}$$

$$\vec{F}_c = -2m \begin{pmatrix} \omega\cos\varphi \cdot (-gt) \\ \cdots \\ \cdots \end{pmatrix}$$

$$F_{cx} = 2m\omega\cos\varphi\, g t = m a_x$$

$$a_x = 2\omega\cos\varphi\, g t$$

$$v_x = \int_0^t a_x\, dt' = \omega\cos\varphi\, g \frac{t^2}{2}$$

$$\boxed{x(t) = \int_0^t v_x\, dt' = \omega\cos\varphi\, g \frac{t^3}{3}}$$

$$z(t) = h - \frac{g t^2}{2}$$

$$z(t) = 0 = h - \frac{g t^2}{2}$$

$$t = \sqrt{\frac{2h}{g}}$$

↑ čas, ko pade kamen na tla

$$\Delta x = \frac{\omega\cos\varphi\, g}{3} \left(\frac{2h}{g}\right)^{3/2}$$

$$\Delta x = \frac{1}{3}\omega\cos\varphi \sqrt{\frac{8h^3}{g}}$$

$$\Delta x = \frac{1}{3} \frac{2\pi}{24 \cdot 3600\,s} \cos 30° \sqrt{\frac{8 \cdot (125\,m)^3}{9.81\,m}} =$$

$$= 2{,}6\ cm$$

5.2

$v_0 = 5\,m/s$

$h_1 = 1\,m$

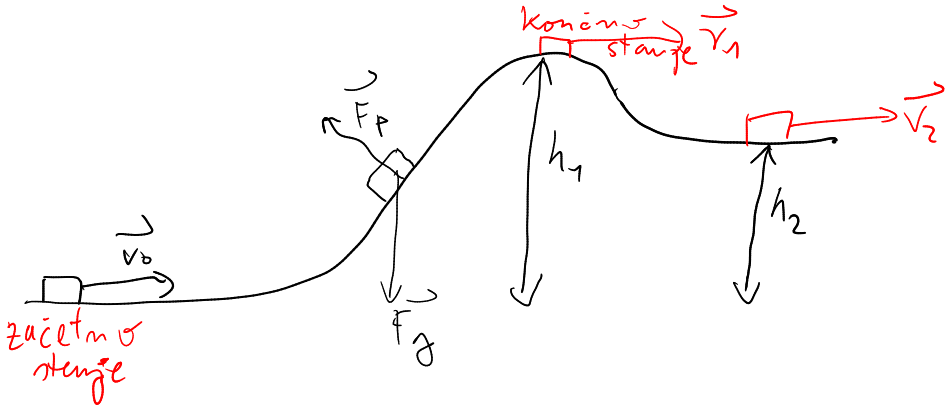
$h_2 = 0.6\,m$

---

$v_1 = ?$

$v_2 = ?$

$h_{max} = ?$




$$\Delta W = A'$$

$$\Delta W_k + \Delta W_p + \Delta W_{pr} = A'$$

$\vec{F}_p \cdot d\vec{s} = 0$

$$W_k = \frac{1}{2} m v^2$$

$$W_p = mgh$$

$$W_{pr} = \frac{1}{2} k \Delta x^2$$

$$A = F s$$

$$A = \vec{F} \cdot \vec{s}$$

$$A = \int \vec{F} \cdot d\vec{s}$$

$$\Delta W_k + \Delta W_p = 0$$

$$\left( \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 \right) + mgh_1 = 0$$

$$v_1 = \sqrt{v_0^2 - 2gh_1} =$$

$$\sqrt{(5\,m/s)^2 - 2 \cdot 10\,\frac{m}{s^2} 1\,m} = 2.32\,\frac{m}{s}$$

$$v_2 = \sqrt{v_0^2 - 2gh_2} = \sqrt{(5\,m/s)^2 - 2 \cdot 10\,\frac{m}{s^2} 0.6\,m} = 3,64\,\frac{m}{s}$$

$$v_1 = 0 = \sqrt{v_0^2 - 2gh_{max}} \rightarrow h_{max} = \frac{v_0^2}{2g} = \frac{(5\,m/s)^2}{2 \cdot 9,89\,m/s^2} =$$

$$= 1,28\,m$$