

2.20

$$\omega_0 = 0$$

$$\omega = k\varphi$$

$$k = 0.08 \text{ s}^{-3}$$

$$N = 1$$

$$r = 30 \text{ cm}$$

$$\omega_1 = ?$$

$$\alpha_1 = ?$$

$$\varphi_1 = N \cdot 2\pi$$

$$\omega(t) = \int_0^t k t' dt' = r \frac{t'^2}{2} \Big|_0^t = k \frac{t^2}{2}$$

$$\varphi(t) = \int_0^t \frac{k t'^2}{2} dt' = \frac{k}{2} \frac{t'^3}{3} \Big|_0^t = k \frac{t^3}{6}$$

$$\omega_1 = \omega_1 \cdot r$$

$$\alpha_1 = |\vec{\alpha}_1| = \sqrt{\alpha_R^2 + \alpha_T^2}$$

$$\alpha_R = r \omega_1^2$$

$$\alpha_T = r \alpha_1$$

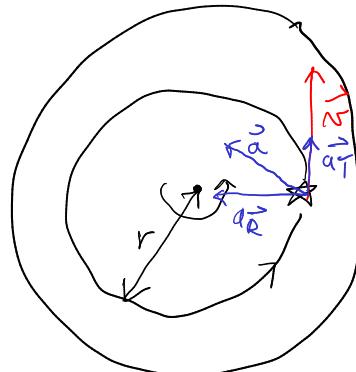
$$\rightarrow \omega_1 = \frac{k}{2} t_1^2 = \frac{k}{2} \left(\frac{12\pi N}{2} \right)^{2/3}$$

$$\rightarrow \alpha_1 = k t_1 = k \left(\frac{12\pi N}{2} \right)^{1/3}$$

$$\alpha_1 = \sqrt{(r \omega_1^2)^2 + (r \alpha_1)^2} = r \sqrt{\omega_1^4 + \alpha_1^2} =$$

$$= r \sqrt{\left(\frac{k}{2} \left(\frac{12\pi N}{2} \right)^{2/3} \right)^4 + \left(k \left(\frac{12\pi N}{2} \right)^{1/3} \right)^2} = r k^{2/3} \sqrt{\frac{(12\pi N)^8/3}{16} + (12\pi N)^2/3} =$$

$$= 1.77 \text{ m/s}^2$$



$$\vec{a} = \vec{\alpha}_R + \vec{\alpha}_T$$

$$\begin{aligned} \varphi(t) &= \int_0^t \omega(t') dt' \\ \omega(t) &= \varphi'(t) \\ \alpha(t) &= \omega'(t) = \varphi''(t) \end{aligned}$$

$$\begin{aligned} \varphi(t) &= \int_0^t \omega(t') dt' \\ \omega(t) &= \int_0^t \alpha(t') dt' \end{aligned}$$

$$\varphi_1 = N \cdot 2\pi = \frac{k t_1^3}{6}$$

občasne vrednosti
plote 1. obrat

$$t_1 = \left(\frac{12\pi N}{k} \right)^{1/3}$$

$$\omega_1 = r \frac{k}{2} \left(\frac{12\pi N}{2} \right)^{2/3} = 0.73 \text{ m/s}$$

2.22

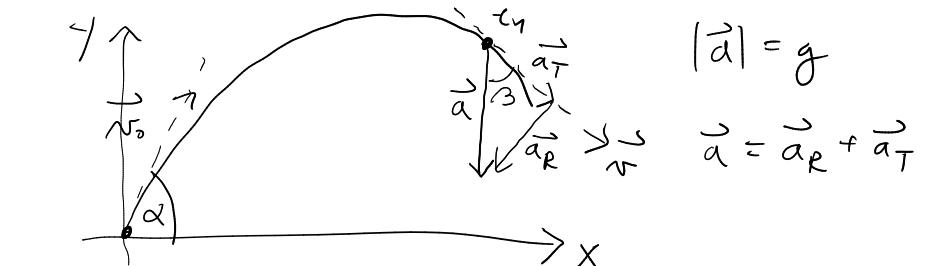
$$\alpha = 60^\circ$$

$$v_0 = 20 \text{ m/s}$$

$$t_1 = 3 \text{ s}$$

$$a_R = ?$$

$$a_T = ?$$



$$|a| = g$$

$$\vec{a} = \vec{a}_R + \vec{a}_T$$

$$\vec{v}(t_1) = ?$$

$$\vec{v} = (v_x, v_y)$$

$$\vec{a} = (0, -g)$$

$$v_x = v_0 \cos \alpha$$

$$v_y = v_0 \sin \alpha - gt$$

$$\vec{v}(t_1) = (v_0 \cos \alpha, v_0 \sin \alpha - gt_1)$$

$$|\vec{v}(t_1) \times \vec{a}| = v(t_1) \underbrace{a \sin \beta}_{a_R}$$

$$\vec{v}(t_1) \cdot \vec{a} = v(t_1) \cdot \underbrace{a \cos \beta}_{a_T}$$

$$a_R = \frac{|\vec{v}(t_1) \times \vec{a}|}{v(t_1)} \dots$$

$$a_T = \frac{\vec{v}(t_1) \cdot \vec{a}}{v(t_1)} = \frac{-g(v_0 \sin \alpha - gt_1)}{\sqrt{(v_0 \cos \alpha)^2 + (v_0 \sin \alpha - gt_1)^2}} =$$

$$= 7.56 \text{ m/s}^2$$

$$a^2 = a_R^2 + a_T^2$$

$$a_R = \sqrt{a^2 - a_T^2} = \sqrt{g^2 - a_T^2} = 6.25 \text{ m/s}^2$$

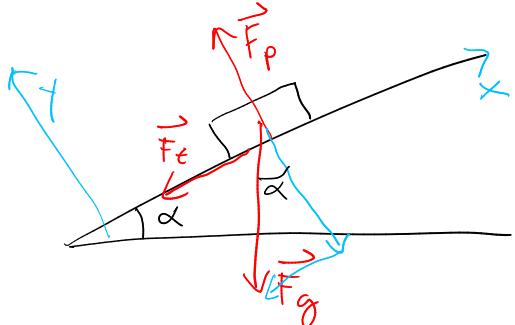
3. 1

$$\alpha = 10^\circ$$

$$v_0 = 14 \text{ m/s}$$

$$s = 20 \text{ m}$$

$$k_t = 0, 1$$

 v 

$$\sum \vec{F} = m \vec{a}$$

2. Newtonov zákon

$$\vec{F}_g + \vec{F}_p + \vec{F}_t = m \vec{a}$$

$$x: -mg \sin \alpha + 0 - F_t = m a$$

$$y: -mg \cos \alpha + F_p + 0 = 0 \rightarrow F_p = mg \cos \alpha$$

$$F_t = k_t \cdot F_p \rightarrow$$

$$F_t = k_t mg \cos \alpha$$

$$-mg \sin \alpha - k_t mg \cos \alpha = ma$$

$$a = -g (\sin \alpha + k_t \cos \alpha)$$

$$v^2 = v_0^2 + 2as$$

$$v = \sqrt{v_0^2 - 2g(\sin \alpha + k_t \cos \alpha) s} = 9.45 \text{ m/s}$$

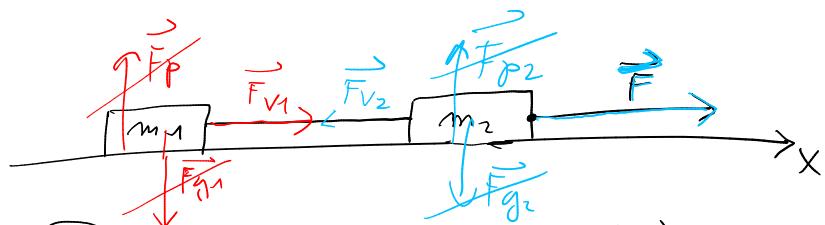
3.5

$$m_1 = 1.5 \text{ kg}$$

$$m_2 = 2 \text{ kg}$$

$$F_V = 60 \text{ N}$$

$$F_{\max} = ?$$



(1.)

$$\vec{F}_{V1} = m_1 \vec{a}_1$$

$$|\vec{F}_{V1}| = |\vec{F}_{V2}| = F_N$$

(2.)

$$\vec{F} + \vec{F}_{V2} = m_2 \vec{a}_2$$

3. Newton's 2. law

$$|\vec{a}_1| = |\vec{a}_2| = a$$

$$X: F_N = m_1 a \rightarrow a = \frac{F_N}{m_1}$$

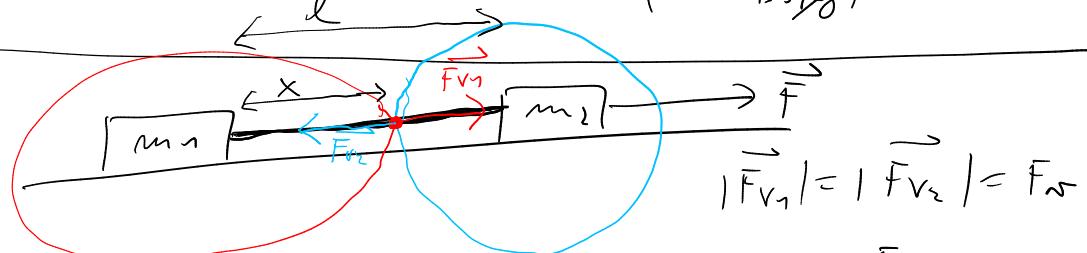
$$F - F_N = m_2 a$$

$$F - F_N = \frac{m_2}{m_1} F_N$$

$$F_N \left(1 + \frac{m_2}{m_1} \right) = F$$

$$F = 60 \text{ N} \left(1 + \frac{2}{1.5} \right) = 140 \text{ N}$$

$$m_V = 0.1 \text{ kg}$$



$$F_N = (m_1 + m_V \frac{x}{l}) a \rightarrow a = \frac{F_N}{m_1 + m_V \frac{x}{l}}$$

$$F - F_N = (m_2 + m_V \frac{l-x}{l}) a$$

$$F - F_N = \frac{m_2 + m_V \frac{l-x}{l}}{m_1 + m_V \frac{x}{l}} F_N$$

$$F = F_N \left(1 + \frac{m_2 + m_V \frac{l-x}{l}}{m_1 + m_V \frac{x}{l}} \right)$$

 $F_N(x) = \max.$
 $\text{pri } x=l$

$$F = F_N \frac{m_1 + m_2 + m_V}{m_1 + m_V \frac{x}{l}}$$

$$F_N = F \frac{\left(m_1 + m_V \frac{x}{l} \right)}{m_1 + m_2 + m_V} = F_N(x)$$

$$x = l$$

$$F = F_N \frac{m_1 + m_2 + m_V}{m_1 + m_V}$$

$$= 60 \frac{3.6}{1.6} \text{ N} =$$

$$= \underline{135 \text{ N}}$$

3.7

$$m_1 = 1 \text{ kg}$$

$$m_2 = 2 \text{ kg}$$

$$\alpha = 30^\circ$$

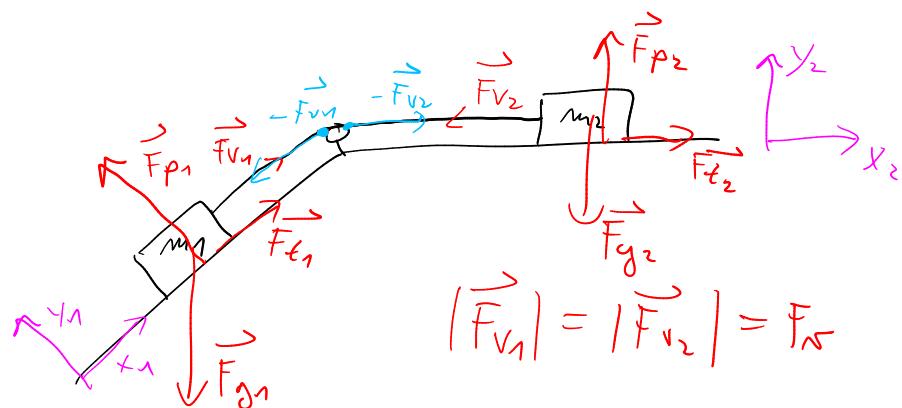
$$k_t = 0.1$$

$$\rho = 10\%$$

$$a = ?$$

$$F_N = ?$$

$$\Delta a = ?$$



$$1.) \vec{F_{g1}} + \vec{F_{p1}} + \vec{F_{v1}} + \vec{F_{t1}} = m_1 \vec{a}_1$$

$$2.) \vec{F_{g2}} + \vec{F_{p2}} + \vec{F_{v2}} + \vec{F_{t2}} = m_2 \vec{a}_2$$

$$1.) x_1: -m_1 g \sin \alpha + F_v + \vec{F_{t1}} = m_1 a$$

$$y_1: -m_1 g \cos \alpha + F_{p1} = 0 \quad F_{t1} = F_{p1} k_t$$

$$2.) x_2: -F_v + \vec{F_{t2}} = m_2 a$$

$$y_2: -m_2 g + F_{p2} = 0 \quad \vec{F_{t2}} = F_{p2} k_t$$

$$F_{t1} = m_1 g \cos \alpha k_t$$

$$F_{t2} = m_2 g k_t$$

$$1.) -m_1 g \sin \alpha + F_N + m_1 g \cos \alpha k_t = m_1 a$$

$$2.) -F_N + m_2 g k_t = m_2 a$$

$$\sum: -m_1 g \sin \alpha + m_1 g \cos \alpha k_t + m_2 g k_t = (m_1 + m_2) a$$

$$F_N = m_2 (g k_t - a) = 3.36 \text{ N}$$

$$a = g \frac{-m_1 \sin \alpha + k_t (m_1 \cos \alpha + m_2)}{m_1 + m_2}$$

$$a = -0.7 \text{ m/s}^2$$

$$k_t \rightarrow k_t (1 + \rho) = k_t (1 + 10\%) = k_t \cdot 1.1 = 0.11$$

$$\rightarrow \Delta a = g \frac{\Delta k_t (m_1 \cos \alpha + m_2)}{m_1 + m_2} \quad \Delta k_t = \rho k_t$$

$$\Delta a = g \frac{\rho k_t (m_1 \cos \alpha + m_2)}{m_1 + m_2} = 0.094 \text{ m/s}^2$$

3.6

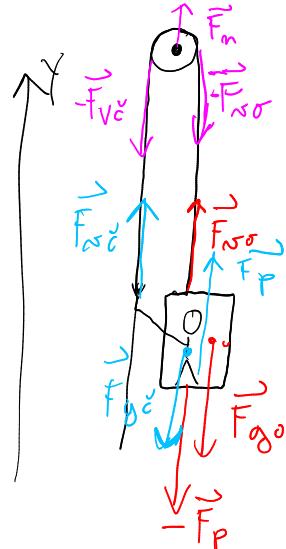
$$m_o = 30 \text{ kg}$$

$$m_c = 90 \text{ kg}$$

$$F_p = 500 \text{ N}$$

$$a = ?$$

$$F_m = ?$$

 $\sigma:$

$$\vec{F}_{N\sigma} + \vec{F}_{g\sigma} - \vec{F}_p = m_o \vec{a}_\sigma$$

 $c:$

$$\vec{F}_{Nc} + \vec{F}_{gc} + \vec{F}_p = m_c \vec{a}_c$$

$$|\vec{a}_\sigma| = |\vec{a}_c| = a$$

$$|\vec{F}_{Nc}| = |\vec{F}_{N\sigma}| = F_N$$

$$y: \begin{aligned} F_N - m_o g - F_p &= m_o a \\ F_N - m_c g + F_p &= m_c a \end{aligned}$$

$$-: \begin{aligned} (-m_o + m_c)g - 2F_p &= (m_o - m_c)a \\ a &= -g + \frac{2F_p}{m_c - m_o} = \\ &= -10 \frac{\text{m}}{\text{s}^2} + \frac{2 \cdot 500 \text{ N}}{60 \text{ kg}} = \\ &= \frac{40}{6} \text{ m/s}^2 = 6.7 \text{ m/s}^2 \end{aligned}$$

$$\vec{F}_m - \vec{F}_{Vc} - \vec{F}_{N\sigma} = 0$$

$$F_m = 2F_V$$

$$2F_V = (m_o + m_c)(a + g) =$$

$$F_m = (m_o + m_c)(a + g) = \\ = 120 \text{ kg} \cdot \left(\frac{40}{6} + 10\right) \text{ m/s}^2 = 1000 \text{ N}$$