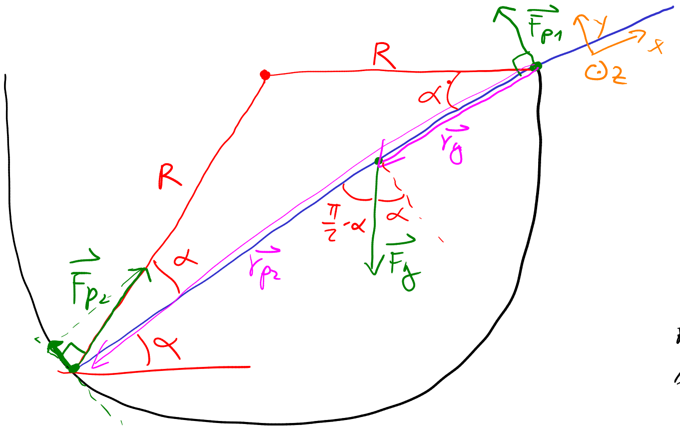
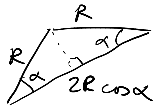
4.12 $\ell = 3R$

$\alpha = ?$





ravnovesje: $\sum \vec F = 0$ $\qquad \vec F_g + \vec F_{p1} + \vec F_{p2} = 0$

$\sum \vec M = 0$ $\qquad \vec r_g \times \vec F_g + \vec r_{p2} \times \vec F_{p2} = 0$

x: $-mg\sin\alpha + F_{p2}\cos\alpha = 0$

y: $-mg\cos\alpha + F_{p2}\sin\alpha + F_{p1} = 0$

z: $\left(2R\cos\alpha - \frac{\ell}{2}\right)\cdot mg \cdot \overset{\sin(\frac{\pi}{2}-\alpha)}{\cancel{\cos\alpha}} - 2R\cancel{\cos\alpha}\cdot F_{p2}\cdot\sin\alpha = 0$

$F_{p2} = mg\,\mathrm{tg}\,\alpha$

$\left(2\cancel{R}\cos\alpha - \frac{3}{2}\cancel{R}\right)\cancel{mg} - 2\cancel{R}\,\cancel{mg}\,\mathrm{tg}\,\alpha\cdot\sin\alpha = 0$

$2\cos\alpha - \frac{3}{2} - 2\frac{\sin^2\alpha}{\cos\alpha} = 0 \quad / \cos\alpha$ $\qquad \sin^2\alpha = 1 - \cos^2\alpha$

$2\cos^2\alpha - \frac{3}{2}\cos\alpha - 2(1-\cos^2\alpha) = 0$

$4\cos^2\alpha - \frac{3}{2}\cos\alpha - 2 = 0$

$\cos\alpha = \dfrac{\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - 4\cdot 4\cdot(-2)}}{2\cdot 4} =$

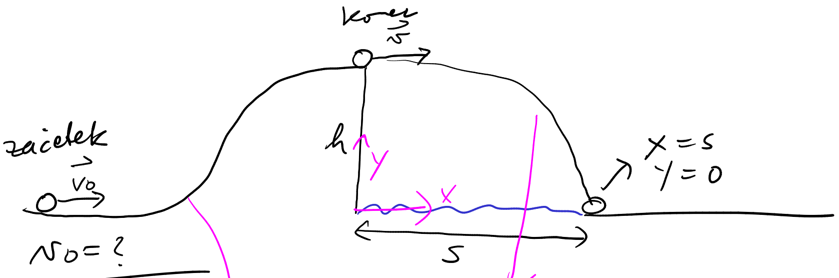
$= \frac{3}{16} \pm \dfrac{\sqrt{\frac{9}{4} + 32}}{8} =$

$= \frac{3}{16} \pm \sqrt{\frac{137}{256}} =$

$= \frac{3}{16} \pm \frac{\sqrt{137}}{16}$ $\qquad$ ~~$\ominus$ $\cos\alpha < 0$ $\alpha > 90°$~~

$\cos\alpha = \dfrac{3 + \sqrt{137}}{16} \longrightarrow \alpha = \underline{23{,}21°}$

5.25



$v_0 = ?$

$h = 0{,}7\,m$
$s = 0{,}5\,m$

vodoravni met:

x: $x = vt = s$

y: $y = h - \frac{gt^2}{2} = 0$

$\hookrightarrow t = \sqrt{\frac{2h}{g}}$

$v = \frac{s}{t} = s\sqrt{\frac{g}{2h}}$

energijski zakon:

$\Delta W = A'$

$\Delta W_k + \Delta W_p = 0$

$\frac{7}{10}\cancel{m} v^2 - \frac{7}{10}\cancel{m} v_0^2 + \cancel{m} g h = 0$

$v_0^2 = v^2 + \frac{10}{7} gh =$

$= \frac{g s^2}{2h} + \frac{10}{7} gh \longrightarrow v_0 = \sqrt{\frac{g s^2}{2h} + \frac{10}{7} gh} =$

$= \sqrt{9{,}81 \frac{m}{s^2}\left(\frac{0{,}5^2 m^2}{2 \cdot 0{,}7 \cancel{m}} + \frac{10}{7} 0{,}7\, m\right)} =$

$= 3{,}4 \quad m/s$

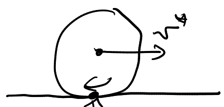
hitrost težišča

$W_k^{konec} = \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2 =$

$= \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{2}{5} m R^2\right)\omega^2 =$

$= \frac{1}{2} m v^2 + \frac{1}{5} m v^2 =$

$= \frac{7}{10} m v^2$



$v = 0 = v^* - R\omega$

$\leftarrow v = R\omega$

5.26

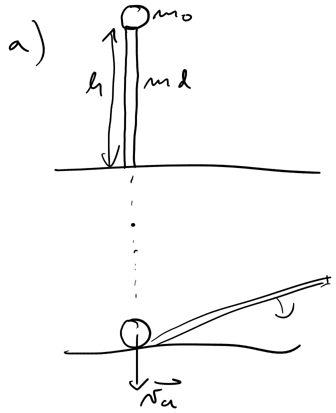
$m_o = 50\,kg$

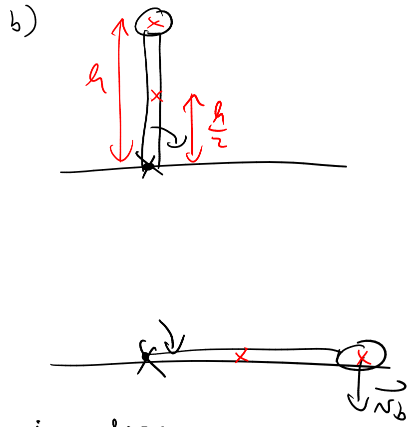
$m_d = 200\,kg$

$h = 5\,m$

$v_a = ?$

$v_b = ?$





opica

a) $\Delta W_p + \Delta W_k = 0$

$$-mgh + \frac{1}{2} m v_a^2 = 0$$

$$v_a = \sqrt{2gh} = \sqrt{2 \cdot 10 \frac{m}{s^2}\, 5m} = 10\ m/s$$

opica + drog

b) $\Delta W_k + \Delta W_p = 0$

$$\left(\frac{1}{2} J \omega^2 - 0\right) + \left(-m_o g h - m_d g \frac{h}{2}\right) = 0$$

$$J = m_o h^2 + \frac{1}{3} m_d h^2$$

$$\left(\frac{1}{2} m_o + \frac{1}{6} m_d\right)(\underbrace{h\omega}_{v_b})^2 - \left(m_o + \frac{m_d}{2}\right) g h = 0$$

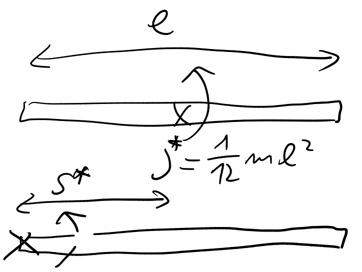
$$v_b = \sqrt{\frac{m_o + \frac{m_d}{2}}{\frac{1}{2} m_o + \frac{1}{6} m_d}\, g h} =$$

$$= \sqrt{\frac{6 m_o + 3 m_d}{3 m_o + m_d}\, g h} =$$

$$= \sqrt{\frac{900}{350}\, g h} =$$

$$= \sqrt{\frac{9}{3{,}5}\, 10 \frac{m}{s^2} \cdot 5m} =$$

$$= 11{,}32\ m/s$$



Steinerjev izrek: $J = J^* + m {s^*}^2 =$

$$= \frac{1}{12} m l^2 + m\left(\frac{l}{2}\right)^2 =$$

$$= \left(\frac{1}{12} + \frac{1}{4}\right) m l^2 = \frac{1}{3} m l^2$$

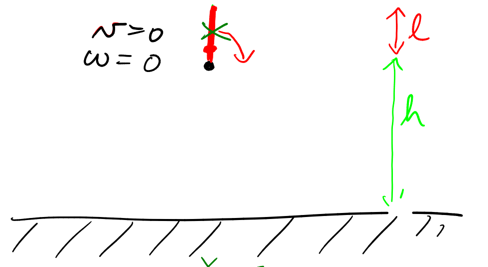
5.28

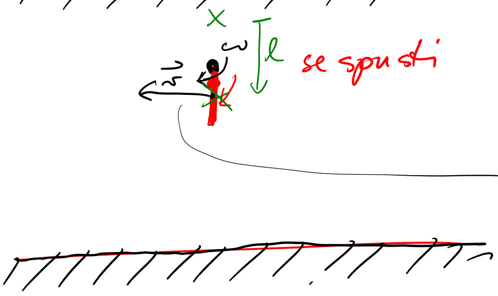
$m = 70\ \mathrm{kg}$

$l = 2\ \mathrm{m}$

$h = ?$

$s = ?$





$$\Delta W_k + \Delta W_p = 0$$

$$\frac{1}{2} J\omega^2 - mgl = 0$$

$$\frac{1}{2}\left(\frac{1}{3} \cancel{m} l^2\right)\omega^2 - \cancel{m} g l = 0$$

$$\omega = \sqrt{\frac{6g}{l}}$$ → kotna hitrost okrog droga = kotna hitrost okoli težišča

$$v = \omega \frac{l}{2} = \sqrt{\frac{3}{2} g l}$$

vodoravni met:

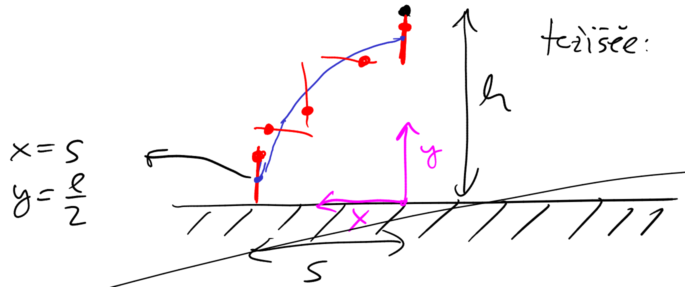
težišče:

x: $x = vt = s$

y: $y = h - \frac{l}{2} - \frac{gt^2}{2} = \frac{l}{2}$

$$h = l + \frac{gt^2}{2}$$

$$s = vt$$



vrtenje okoli težišča: $\varphi = 2\pi N = \omega t$

$N = 1, 2, \dots$

$$t = \frac{2\pi N}{\omega}$$

$$h = l + \frac{\cancel{g}}{2} \frac{4\pi^2 N^2 \cdot l}{6\cancel{g}} = l\left(1 + \frac{\pi^2}{3} N^2\right) \doteq \begin{cases} N=1; & \sim 8\ \mathrm{m} \\ \cancel{N_2 = 2;} & \cancel{\sim 26\ \mathrm{m}} \end{cases}$$

$$s = vt = \sqrt{\frac{3}{2} \cancel{g} l} \cdot \frac{2\pi \sqrt{l}}{\sqrt{6\cancel{g}}} = \pi l = \underline{\underline{6{,}3\ \mathrm{m}}}$$

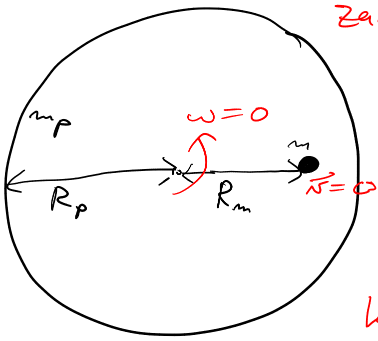
$N = 1$

3.45

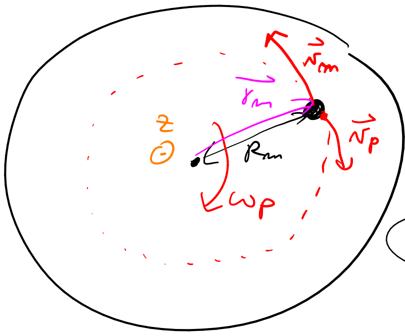
$R_p = 2\,m$
$m_p = 200\,kg$
$R_m = 1{,}5\,m$
$m = 70\,kg$
$v = 4\,m/s$ glede na ploščo
$N = 1$

$t = ?$

začetek



konec



$$|\vec{v}_m - \vec{v}_p| = v$$
$$\downarrow$$
$$v_m + R_m \cdot \omega_p = v$$
$$v_m = v - R_m \omega_p$$

zakon o vrtilni količini:

$$\Delta \vec{\Gamma} = \int \vec{M}\, dt \qquad \underset{\text{vrtilna količina}}{\vec{\Gamma}} = \vec{r} \times \underset{\text{gibalna količina}}{\vec{G}} \quad ; \quad \vec{\Gamma} = J\vec{\omega}$$

$\downarrow$ ni trenja

$$\Delta \vec{\Gamma} = 0$$
$$\vec{\Gamma}^{konec} - \underbrace{\vec{\Gamma}^{začetek}}_{=0} = 0$$
$$\vec{\Gamma}^{konec} = 0$$
$$\vec{\Gamma}^{konec}_{plošča} + \vec{\Gamma}^{konec}_{mož} = 0$$
$$J_p \vec{\omega}_p + \vec{r}_m \times \vec{G}_m = 0 \qquad \vec{G}_m = m \cdot \vec{v}_m$$

z: $-J_p \omega_p + R_m \cdot m v_m = 0$

$$-J_p \omega_p + R_m m (v - R_m \omega_p) = 0$$
$$(-J_p - m R_m^2)\omega_p + m R_m v = 0$$
$$\omega_p = \frac{m R_m v}{J_p + m R_m^2} \qquad J_p = \frac{1}{2} m_p R_p^2$$
$$p = \frac{m R_m v}{\frac{1}{2} m_p R_p^2 + m R_m^2}$$

$$\varphi_p = 2\pi N = \omega_p t$$
$$t = \frac{2\pi N}{\omega_p} = 2\pi N \frac{\frac{1}{2} m_p R_p^2 + m R_m^2}{m R_m \cdot v}$$
$$t = 2 \cdot \pi \cdot 1 \frac{\frac{1}{2} 200\,kg \cdot 4\,m^2 + 70\,kg \cdot 1{,}5^2\,m^2}{70\,kg \cdot 1{,}5\,m \cdot 4\,m/s} = \underline{8{,}34 \quad s}$$

5.24

$J_1 = 2{,}6\ kg\,m^2$

$J_2 = 1{,}4\ kg\,m^2$

$\nu_1 = 1\ Hz$

$A' = ?$

$\Delta \vec{\Gamma} = \cancel{\int \vec{M} dt}$

$\Delta \vec{\Gamma} = 0$

$\vec{\Gamma}^{konec} = \vec{\Gamma}^{začátek}$

$J_2 \omega_2 = J_1 \omega_1$

$\omega_2 = \frac{J_1}{J_2}\omega_1$

$\Delta W = A'$

$\frac{1}{2} J_2 \omega_2^2 - \frac{1}{2} J_1 \omega_1^2 = A'$

$\frac{1}{2} J_2 \left(\frac{J_1}{J_2}\omega_1\right)^2 - \frac{1}{2} J_1 \omega_1^2 = A'$

$A' = \frac{1}{2} J_1 \omega_1^2 \left(\frac{J_1}{J_2} - 1\right)$

$\omega_1 = 2\pi \nu_1$

$A' = 2\pi^2 J_1 \nu_1^2 \left(\frac{J_1}{J_2} - 1\right)$

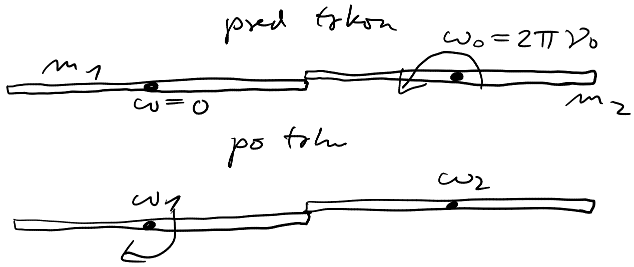
$A' = 2\pi^2\, 2{,}6\ \underline{kg\,m^2 \frac{1}{s^2}} \left(\frac{2{,}6}{1{,}4} - 1\right)$

$$\underline{\underline{A' = 44\ J}}$$

5.34 $l = 1\,m$

$\nu_0 = 1\,Hz$

$\omega_1, \omega_2 = ?$



prožen trk: $\Delta W = 0$

$$W^{pred} = W^{po}$$

$$\frac{1}{2} J_2 \omega_0^2 = \frac{1}{2} J_2 \omega_2^2 + \frac{1}{2} J_1 \omega_1^2$$

$$m_2 \omega_0^2 = m_2 \omega_2^2 + m_1 \omega_1^2$$

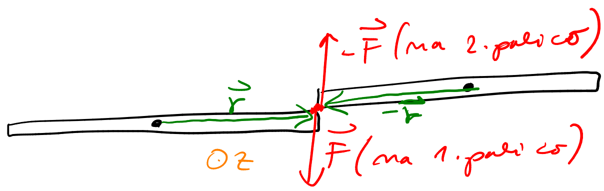
$$J_1 = \frac{1}{12} m_1 l^2$$

$$J_2 = \frac{1}{12} m_2 l^2$$

v. količina:

1. palica: $\Delta \vec{\Gamma}_1 = \int \vec{M}_1 \, dt$ 2. palica: $\Delta \vec{\Gamma}_2 = \int \vec{M}_2 \, dt$

med trkom:



$$\Delta \vec{\Gamma}_1 = \int (\vec{r} \times \vec{F}) dt \qquad \Delta \vec{\Gamma}_2 = \int (-\vec{r}) \times (-\vec{F}) dt = \int \vec{r} \times \vec{F} dt$$

$$\Delta \vec{\Gamma}_1 = \Delta \vec{\Gamma}_2$$

z: $J_1 \omega_1 - 0 = J_2 \omega_2 - J_2 \omega_0 \rightarrow \omega_2 = \omega_0 + \frac{J_1}{J_2} \omega_1$

$$\frac{J_1}{J_2} = \frac{m_1}{m_2}$$

$$m_2 \omega_0^2 = m_2 \omega_2^2 + m_1 \omega_1^2$$

$$m_2 \omega_0^2 = m_2 \left(\omega_0 + \frac{J_1}{J_2} \omega_1\right)^2 + m_1 \omega_1^2$$

$$\cancel{m_2 \omega_0^2} = \cancel{m_2 \omega_0^2} + 2 \cancel{m_2} \frac{m_1}{\cancel{m_2}} \cancel{\omega_1} \omega_0 + \cancel{m_2} \frac{\cancel{m_1^2}}{\cancel{m_2^2}} \cancel{\omega_1^2} + \cancel{m_1} \cancel{\omega_1^2}$$

$$0 = 2\omega_0 + \frac{m_1}{m_2} \omega_1 + \omega_1$$

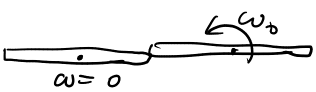
$$\omega_1 = -\frac{2\omega_0}{1 + \frac{m_1}{m_2}} \qquad \omega_1 = -\frac{2 m_2}{m_1 + m_2} \omega_0$$

$$\omega_2 = \omega_0 + \frac{m_1}{m_2} \omega_1 = \omega_0 - \frac{2 m_1}{m_1 + m_2} \omega_0 =$$

$$= \frac{m_1 + m_2 - 2m_1}{m_1 + m_2} \omega_0$$

$$\omega_2 = \frac{m_2 - m_1}{m_1 + m_2} \omega_0$$

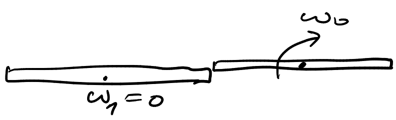
pred trkom



$m_1 >> m_2$ $\omega_1 \sim 0$

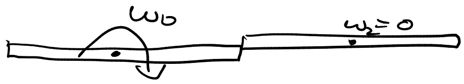
$\omega_2 \sim -\omega_0$

po trku:



$m_1 = m_2$ $\qquad \omega_1 = -\omega_0$ $\qquad$ po trku

$\omega_2 = 0$



$m_1 \ll m_2$ $\qquad \omega_1 \sim -2\omega_0$ $\qquad$ po trku i

$\omega_2 \sim \omega_0$