5.5

$m = 5\,kg$

$k_1 = 200\,N/m$

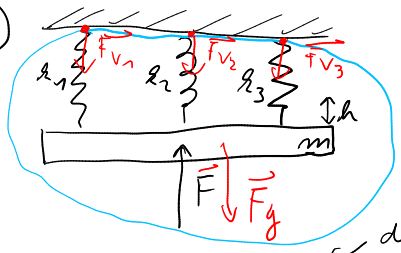
$k_2 = 250\,N/m$

$k_3 = 300\,N/m$

$h = 10\,cm$

$A = ?$

a)



$\Delta W_p + \Delta W_{pr} = A$ ← delos sile $\vec{F}$

$A = mgh + \frac{1}{2}k_1h^2 + \frac{1}{2}k_2h^2 + \frac{1}{2}k_3h^2$

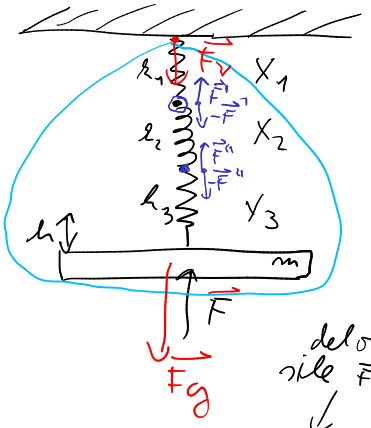
$A = mgh + \frac{1}{2}\tilde{k}h^2$

$\tilde{k} = k_1 + k_2 + k_3 = 750\,\frac{N}{m}$

$A = 5kg \cdot 9{,}81\,\frac{m}{s^2} \cdot 0{,}1\,m + \frac{1}{2} 750\,\frac{N}{m} \cdot (0{,}1\,m)^2 =$

$= 8{,}66\,J$

b)



$\Delta W_p + \Delta W_{pr} = A$ ← delos sile $\vec{F}$

$x_1 + x_2 + x_3 = h$

$A = mgh + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 + \frac{1}{2}k_3x_3^2$

$\vec{F}' + (-\vec{F}') = 0$ | $\vec{F}'' + (-\vec{F}'') = 0$

$|\vec{F}'| = |-\vec{F}'|$ | $|\vec{F}''| = |-\vec{F}''|$

$k_2x_2 = k_1x_1$ | $k_3x_3 = k_2x_2$

$x_1 = \frac{k_2}{k_1}x_2$ | $x_3 = \frac{k_2}{k_3}x_2$

$\frac{k_2}{k_1}x_2 + \frac{k_2}{k_2}x_2 + \frac{k_2}{k_3}x_2 = h$

$k_2\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)x_2 = h$

$x_2 = \frac{h}{k_2\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)} = \frac{\tilde{k}}{k_2}h$ ($\frac{1}{\tilde{k}}$)

$x_1 = \frac{\tilde{k}}{k_1}h$ | $x_3 = \frac{\tilde{k}}{k_3}h$

$\frac{1}{\tilde{k}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$

$A = mgh + \frac{1}{2}k_1\frac{\tilde{k}^2h^2}{k_1^2} + \frac{1}{2}k_2\frac{\tilde{k}^2h^2}{k_2^2} + \frac{1}{2}k_3\frac{\tilde{k}^2h^2}{k_3^2} =$

$= mgh + \frac{1}{2}\tilde{k}^2h^2\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right) =$

$= mgh + \frac{1}{2}\tilde{k}h^2 =$

$= 5kg \cdot 9{,}81\,\frac{m}{s^2} \cdot 0{,}1\,m + \frac{1}{2}\,\frac{1\,\frac{N}{m}}{\frac{1}{200} + \frac{1}{250} + \frac{1}{300}} \cdot (0{,}1\,m)^2 =$

$= 5{,}31\,J$

3.17 $m_V = 150\,kg$

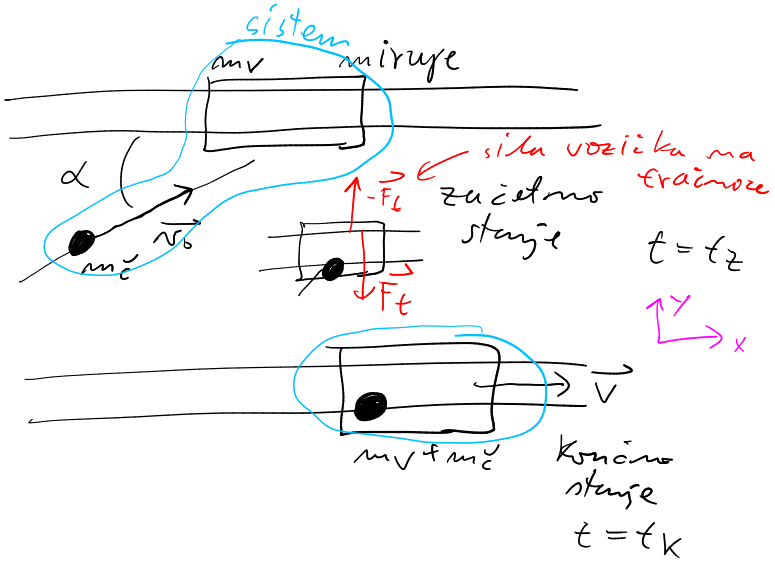
$m_č = 70\,kg$

$v_0 = 5\,m/s$

$\alpha = 30°$

$v = ?$

smer sile = ?



Zakon o gibalni količini

$$\Delta \vec{G} = \int_{t_z}^{t_k} \vec{F}\,dt \quad \leftarrow \text{zunanje sile}$$

$$\vec{G}_k - \vec{G}_t = \int_{t_z}^{t_k} \vec{F}_t\,dt \qquad \vec{G} = m\vec{v}$$

$$(m_V + m_č)\vec{v} - m_č\vec{v}_0 = \int \vec{F}_t\,dt$$

x: $(m_V + m_č)v - m_č v_0 \cos\alpha = 0$

y: $0 - m_č v_0 \sin\alpha = \int (F_t)\,dt$

$$v = \frac{m_č \cdot v_0 \cos\alpha}{m_V + m_č} = \frac{70 \cdot 5\frac{m}{s}\cos 30°}{220} = 1{,}38\ m/s$$

$$\int (-\vec{F}_t)\,dt \Rightarrow y: \int F_t\,dt = m_č v_0 \sin\alpha =$$

$$= 70\,kg \cdot 5\,m/s \cdot \sin 30° =$$

$$= 175\,\left(kg\,\frac{m}{s} = N \cdot \frac{s}{s}\right) =$$

$$= 175\,Ns$$

3.18 $m_č = 50\ kg$

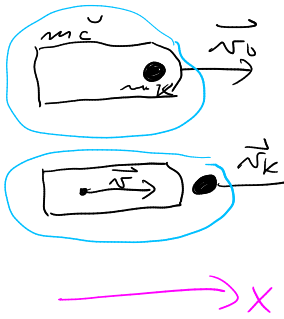
$v_0 = 2\ m/s$

$m_K = 60\ kg$

a) $v_K = 4\ m/s$ glede na vodo

b) $v_K = 2{,}5\ m/s$ glede na čoln

$v = ?$



Začetno stanje

Končno stanje (takoj za tem, ko se kopalec odrine od čolna)

x:

$$\Delta G = \int F\,dt = 0$$

$$G_k - G_z = 0$$

$$G_k = G_z$$

a)
$$m_č v + m_K v_K = (m_č + m_K) v_0$$

$$v = \frac{(m_č + m_K) v_0 - m_K v_K}{m_č} =$$

$$= \frac{110\ \cancel{kg} \cdot 2\ m/s - 60\ \cancel{kg} \cdot 4\ m/s}{50\ \cancel{kg}} = -0{,}4\ m/s$$

b)
$$m_č v + m_K (v + v_K) = (m_č + m_K) v_0$$

$$v = \frac{(m_č + m_K) v_0 - m_K v_K}{m_č + m_K} =$$

$$= v_0 - \frac{m_K}{m_č + m_K} v_K =$$

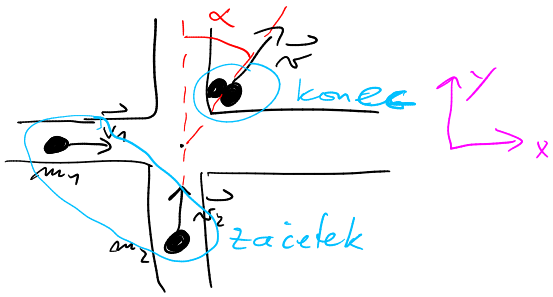
$$= 2\ m/s - \frac{60}{110} \cdot 2{,}5\ m/s = 0{,}6\ m/s$$

3.22 $v_1 = 40\ km/h$

$m_2 = k\, m_1 \quad k = 1{,}2$

$\alpha = 30°$

$v_2 = ?$



$$\Delta \vec{G} = \int \vec{F}\, dt = 0$$

$$\vec{G}_k = \vec{G}_z$$

x: $(m_1 + m_2)\, v \sin\alpha = m_1 v_1$

y: $(m_1 + m_2)\, v \cos\alpha = m_2 v_2$

: $\quad tg\,\alpha = \frac{m_1}{m_2}\frac{v_1}{v_2} = \frac{v_1}{k v_2}$

$$v_2 = \frac{v_1}{k\, tg\,\alpha} = \frac{40\ km/h}{1{,}2 \cdot tg\,30°} = 57{,}7\ km/h$$

3.23

$m_v = 1\ kg$

$\phi_m = 0{,}1\ kg/s$

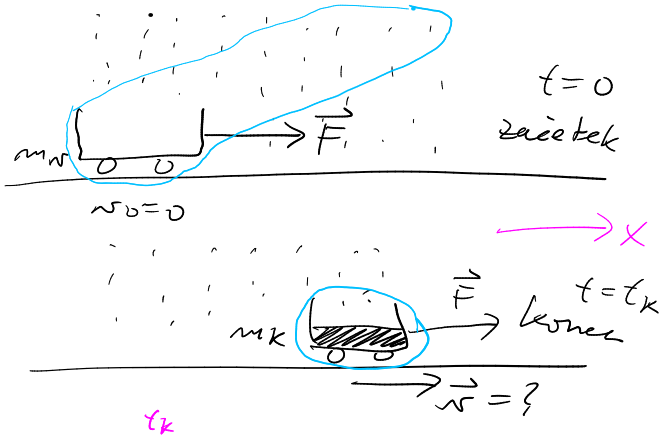
$F = 0{,}5\ N$

$m_k = 2\ kg$

$v_k = ?$

$v_0 = 0$

$\phi_m = \frac{dm}{dt}$



$$x: \quad G_{kx} - G_{zx} = \int F_x\,dt = \int_0^{t_k} F\,dt = F t_k$$

$$m_k v_k - 0 = F t_k$$

$$v_k = \frac{F t_k}{m_k} = \frac{F}{\phi_m}\,\frac{m_k - m_v}{m_k} = \frac{0{,}5\,N\,s}{0{,}1\,kg}\cdot\frac{1\,\cancel{kg}}{2\,\cancel{kg}} = 2{,}5\,\frac{m}{s}$$

$$\phi_m = \frac{m_k - m_v}{t_k} \rightarrow t_k = \frac{m_k - m_v}{\phi_m}$$

3.30

$\alpha_1 = 25°$

$\alpha_2 = 35°$

$S_1 = 1{,}5\,cm^2$

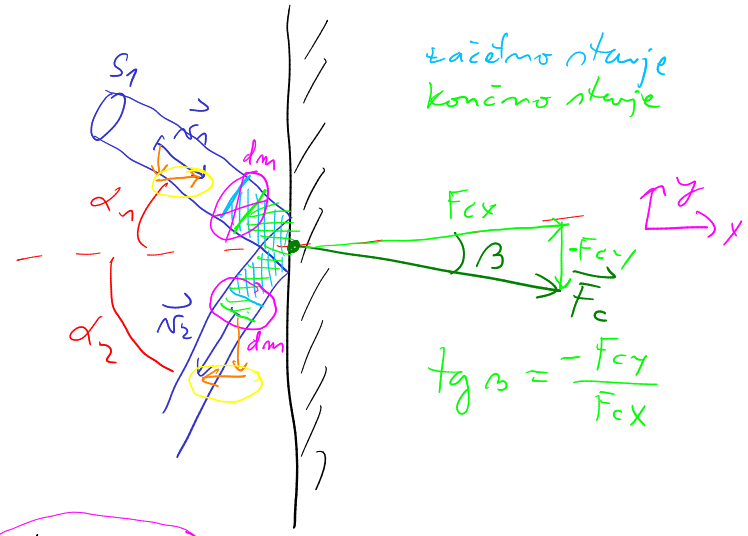
$v_1 = 2\,m/s$

$v_2 = 1{,}3\,m/s$

$F_c = ?$

$\beta = ?$

$\rho = 1000\,kg/m^3$



$$\Delta\vec{G} = \int \vec{F}\,dt$$

$$d\vec{G} = \vec{F}\,dt$$

$$d\vec{G} = -\vec{F}_c\,dt$$

$$dm\,\vec{v}_2 - dm\,\vec{v}_1 = -\vec{F}_c\,dt \quad / : dt \qquad \frac{dm}{dt} = \phi_m$$

$$\phi_m(\vec{v}_2 - \vec{v}_1) = -\vec{F}_c \qquad \phi_m = \rho S v$$

$$\vec{F}_c = \phi_m(\vec{v}_1 - \vec{v}_2)$$

$$\vec{F}_c = \rho S_1 v_1 (\vec{v}_1 - \vec{v}_2)$$

$$\vec{F}_c = \rho S_1 v_1 (v_1\cos\alpha_1 + v_2\cos\alpha_2 \,/\, -v_1\sin\alpha_1 + v_2\sin\alpha_2)$$

$$F_c = \rho S_1 v_1 \sqrt{(v_1\cos\alpha_1 + v_2\cos\alpha_2)^2 + (v_2\sin\alpha - v_1\sin\alpha_1)^2} =$$

$$= \rho S_1 v_1 \sqrt{v_1^2\cos^2\alpha_1 + 2v_1v_2\cos\alpha_1\cos\alpha_2 + v_2^2\cos^2\alpha_2 + v_2^2\sin^2\alpha_2 + 2v_1v_2\sin\alpha_1\sin\alpha_2 + v_1^2\sin^2\alpha_1} =$$

$$= \rho S_1 v_1 \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\alpha_1 + \alpha_2)} =$$

$$= 1000\,\frac{kg}{m^3}\cdot(1{,}5\cdot10^{-2}\,m)^2\cdot 2\,\frac{m}{s}\sqrt{(2\,\tfrac{m}{s})^2 + (1{,}3\,\tfrac{m}{s})^2 + 2\cdot2\,\tfrac{m}{s}\cdot1{,}3\,\tfrac{m}{s}\cos 60°} =$$

$$= \underline{0{,}86\,N}$$

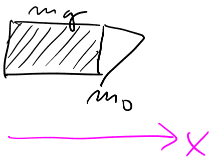
$$tg\,\beta = -\frac{F_{cy}}{F_{cx}} \rightarrow \beta = arctg\left(\frac{v_1\sin\alpha_1 - v_2\sin\alpha_2}{v_1\cos\alpha_1 + v_2\cos\alpha_2}\right) =$$

$$= arctg\,\frac{2\sin 25° - 1{,}3\sin 35°}{2\cos 25° + 1{,}3\cos 35°} = \underline{2°}$$

3.31 a) $v_i = 3\ km/s$

$m_g = 3/4\, m_0$

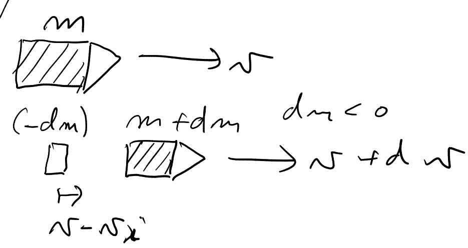
$v_k = ?$



$dG = F\,dt = 0$

začetno stanje ob času t

končno -||- : -||- $t+dt$



$$dG = G(t+dt) - G(t) =$$

$$= \left[(m+dm)(v+dv) + (-dm)(v-v_i)\right] - \left[mv\right] =$$

$$= \cancel{mv} + m\,dv + \cancel{dm\,v} + dm\,dv - \cancel{dm\,v} + dm\,v_i - \cancel{mv} =$$

($dm\,dv$) zanemarimo

$$= m\,dv + dm\,v_i = 0$$

$$\frac{dm}{m} = -\frac{dv}{v_i} \quad \Big/ \int$$

$$\int_{m_z}^{m_k} \frac{dm}{m} = -\frac{1}{v_i}\int_{v_z}^{v_k} dv$$

$$\ln\frac{m_k}{m_z} = -\frac{v_k - v_z}{v_i}$$

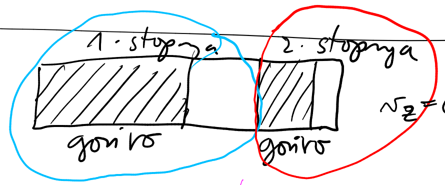
$$v_k = v_z - v_i \ln\frac{m_k}{m_z}$$

$$\boxed{v_k = v_z + v_i \ln\frac{m_z}{m_k}}$$

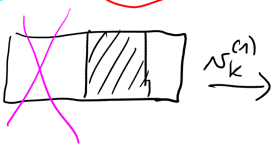
$$v_k = 0 + v_i \ln\frac{m_0}{\frac{1}{4}m_0} =$$

$$= 3\,\frac{km}{s}\cdot \ln 4 = \underline{4{,}2\ km/s}$$

b)

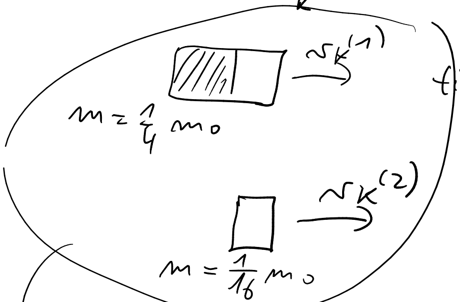


$v_z = 0$ $\quad m_0$



$$m = \frac{1}{4}m_0 + \frac{3}{4}m_0\cdot\frac{1}{4} = \frac{7}{16}m_0$$

$$v_k^{(1)} = 0 + v_i \ln\frac{m_0}{\frac{7}{16}m_0} = v_i \ln\frac{16}{7}$$



tik preden se vklopi motor 2. stopnje (raketa je odklopila motor 1. stopnje)

$$v_k^{(2)} = v_k^{(1)} + v_i \ln\frac{1/4\, m_0}{1/16\, m_0} = v_i \ln\frac{16}{7} + v_i \ln 4 =$$

$$= v_i \ln\frac{64}{7} = 3\,km/s \cdot \ln\frac{64}{7} = \underline{6{,}6\ km/s}$$

5.8 $m_1 = 2\,kg$

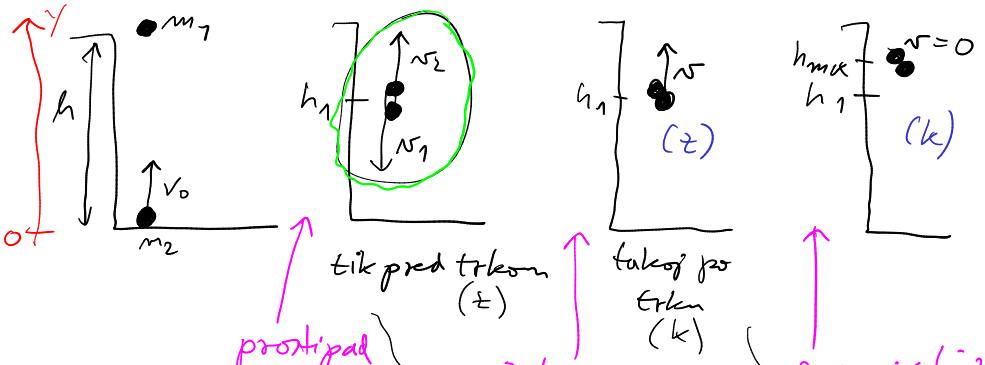
$h = 10\,m$

$m_2 = 1\,kg$

$v_0 = 20\,m/s$

$h_{max} = ?$

$W_{izg} = ?$



protipad — zakon o gibalni količini — energijski zakon

$y_1 = h - \frac{gt^2}{2}$

$y_2 = v_0 t - \frac{gt^2}{2}$

$y_1 = y_2 \longrightarrow h = v_0 t_1$

$t_1 = \frac{h}{v_0}$ čas trka

$v_1 = -gt_1 = -g\frac{h}{v_0}$

$v_2 = v_0 - gt_1 = v_0 - \frac{gh}{v_0}$

$h_1 = y_1 = h - \frac{gh^2}{2v_0^2}$

$G_z = G_k$

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$

$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} =$

$v = \frac{m_2 v_0}{m_1 + m_2} - g\frac{h}{v_0}$

$\Delta W_k + \Delta W_p = 0$

$\left[0 - \frac{1}{2}(m_1 + m_2)v^2\right] + \left[mgh_{max} - mgh_1\right] = 0$

$h_{max} = h_1 + \frac{v^2}{2g}$

$h_{max} = h - \frac{gh^2}{2v_0^2} + \frac{1}{2g}\left(\frac{m_2 v_0}{m_1 + m_2} - g\frac{h}{v_0}\right)^2 =$

$= 10\,m - \frac{9.8\,\frac{m}{s^2} \cdot (10\,m)^2}{s^2 \cdot 2 \cdot (20\,\frac{m}{s})^2} + \frac{1}{2 \cdot 9{,}81\,\frac{m}{s^2}}\left(\frac{1}{3} \cdot 20\,\frac{m}{s} - \frac{9.81\,m \cdot 10\,m}{s^2 \cdot 20\,\frac{m}{s}}\right)^2 =$

$= 8{,}93\,m$

$W_{izg} = W_k(\text{pred trkom}) - W_k(\text{po trku}) =$

$= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{1}{2}(m_1 + m_2)v^2 =$

$= \frac{1}{2}m_1\left(-\frac{gh}{v_0}\right)^2 + \frac{1}{2}m_2\left(v_0 - \frac{gh}{v_0}\right)^2 - \frac{1}{2}(m_1 + m_2)\left(\frac{m_2 v_0}{m_1 + m_2} - \frac{gh}{v_0}\right)^2 =$

$= \frac{1}{2}m_1\left(\frac{gh}{v_0}\right)^2 + \frac{1}{2}m_2 v_0^2 - m_2 v_0 \frac{gh}{v_0} + \frac{1}{2}m_2\left(\frac{gh}{v_0}\right)^2 -$

$- \frac{1}{2}\frac{m_2^2 \cdot v_0^2}{m_1 + m_2} + \frac{1}{2}m_2 v_0 \frac{gh}{v_0} \cdot 2 - \frac{1}{2}(m_1 + m_2)\left(\frac{gh}{v_0}\right)^2 =$

$= \frac{1}{2}m_2 v_0^2\left(1 - \frac{m_2}{m_1 + m_2}\right) = \frac{1}{2}m_2 v_0^2 \frac{m_1}{m_1 + m_2} =$

$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}v_0^2 = \frac{1}{2}\frac{1\,kg \cdot 2\,kg}{3\,kg} \cdot (20\,\frac{m}{s})^2 = 133\,J$