5.5

$m = 5\ kg$

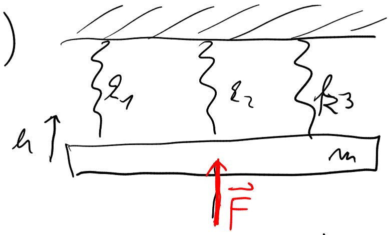
$k_1 = 200\ N/m$

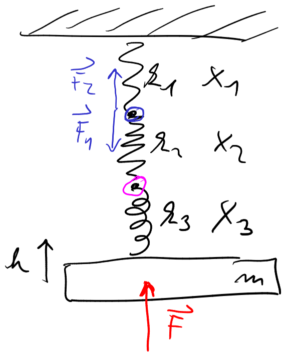
$k_2 = 250\ N/m$

$k_3 = 300\ N/m$

$h = 10\ cm$

$A = ?$

a) 

b) 

$$\Delta W_p + \Delta W_{pr} = A$$

a) $$mgh + \frac{1}{2}k_1h^2 + \frac{1}{2}k_2h^2 + \frac{1}{2}k_3h^2 = A$$

$$A = mgh + \frac{1}{2}(k_1+k_2+k_3)h^2 = mgh + \frac{1}{2}\tilde{k}h^2 \qquad \tilde{k} = k_1+k_2+k_3$$

$$A = 5kg \cdot 9.81\,\frac{m}{s^2} \cdot 0{,}1m + \frac{1}{2}(200+250+300)\frac{N}{m}\cdot(0{,}1m)^2 =$$

$$= 8{,}7\ J$$

b) $$x_1 + x_2 + x_3 = h$$

$$\vec{F_1} + \vec{F_2} = 0 \qquad F_1 = F_2$$

$$k_1x_1 = k_2x_2 \qquad k_2x_2 = k_3x_3$$

$$x_1 = \frac{k_2}{k_1}x_2 \qquad x_3 = \frac{k_2}{k_3}x_2$$

$$\frac{k_2}{k_1}x_2 + x_2 + \frac{k_2}{k_3}x_2 = h$$

$$k_2x_2\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right) = h$$

$$x_2 = \frac{h}{k_2\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}\right)} = \frac{\tilde{k}}{k_2}h$$

$$\frac{1}{\tilde{k}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

$$x_1 = \frac{\tilde{k}}{k_1}h \qquad x_3 = \frac{\tilde{k}}{k_3}h$$

$$A = mgh + \frac{1}{2}k_1\left(\frac{\tilde{k}}{k_1}h\right)^2 + \frac{1}{2}k_2\left(\frac{\tilde{k}}{k_2}h\right)^2 + \frac{1}{2}k_3\left(\frac{\tilde{k}}{k_3}h^2\right) =$$

$$= mgh + \frac{1}{2}\tilde{k}^{\cancel{2}}h^2\cancel{\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)} = mgh + \underbrace{\frac{1}{2}\tilde{k}h^2}$$

$$= 5kg \cdot \frac{9.81m}{s^2} \cdot 0{,}1m + \frac{1}{2}\,\frac{1}{\left(\frac{1}{200}+\frac{1}{250}+\frac{1}{300}\right)}\,N/m \cdot 0{,}1m^2 = 5{,}3\ J$$

3.17

$m_V = 150\ kg$

$m_č = 70\ kg$

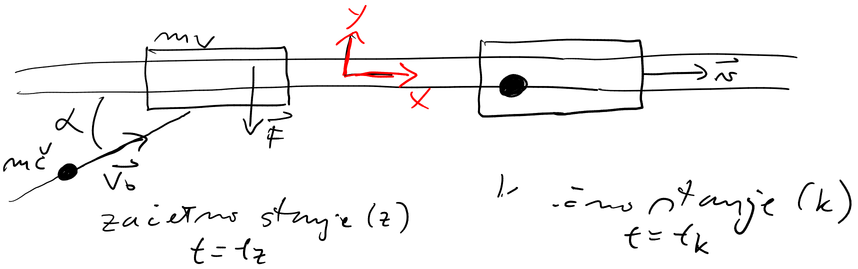
$v_0 = 5\ m/s$

$\alpha = 30°$

---

v

sunek sile



Zakon o gibalni količini:

$$\Delta \vec{G} = \underbrace{\int \vec{F}\, dt} \qquad \vec{G} = m\vec{v}$$

$$\vec{G}_k - \vec{G}_z = \int_{t_z}^{t_k} \vec{F}\, dt$$

$$(m_V + m_č)\vec{v} - (m_č \vec{v}_0 + m_V \cdot \vec{0}) = \int \vec{F}\, dt$$

$$x: \quad (m_V + m_č) v - m_č v_0 \cos\alpha = 0$$

$$y: \quad 0 - m_č v_0 \sin\alpha = \int (-F)\, dt$$

$\vec{F}$ sila tračnice na voziček

$$v = \frac{m_č v_0 \cos\alpha}{m_V + m_č} = \frac{70\,\cancel{kg} \cdot 5\ m/s \cdot \cos 30°}{220\,\cancel{kg}} = \underline{1{,}37\ m/s}$$

$$\boxed{\int F\, dt} = m_č v_0 \sin\alpha = 70\ kg \cdot 5\ m/s \cdot \sin 30° = \underline{175\ Ns}$$

$$\frac{kg\, m}{s} = \frac{kg\, m}{s^2} \cdot s = \underline{\underline{Ns}}$$

3.18

$m_č = 50\,kg$

$v_0 = 2\,m/s$

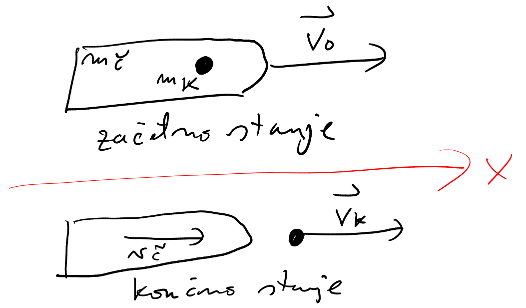
$m_K = 60\,kg$

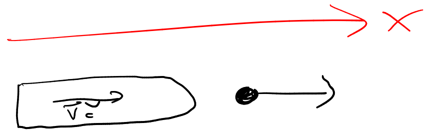
a) $v_K = 4\,m/s$ glede na vodo

$v_č = ?$

b) $v_K = 2.5\,m/s$ glede na čoln

$v_č = ?$





$$\Delta \vec{G} = \int \vec{F}\,dt$$

x: $G_K - G_Z = \int F\,dt$

ni zunanjih sil v smeri x

$$(m_č v_č + m_K v_K) - (m_č + m_K) v_0 = 0$$

$$v_č = \frac{(m_č + m_K) v_0 - m_K v_K}{m_č} = \frac{(50+60)\,\cancel{kg} \cdot 2\,m/s - 60\,\cancel{kg} \cdot 4\,m/s}{50\,\cancel{kg}} = -0{,}4\ m/s$$

x: $G_K - G_Z = 0$

$$(m_č v_č + m_K (v_K + v_č)) - (m_č + m_K) v_0 = 0$$

$$v_č = \frac{(m_č + m_K) v_0 - m_K v_K}{m_č + m_K}$$

$$v_č = v_0 - \frac{m_K}{m_č + m_K} \cdot v_K$$

$$v_č = 2\,m/s - \frac{60}{110} \cdot 2{,}5\,m/s = 0{,}64\ m/s$$

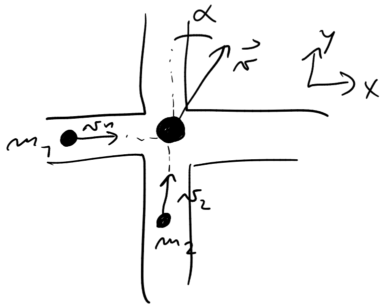
3.22

$v_1 = 40\,\mathrm{km/h}$

$m_2 = k\,m_1 \qquad k = 1{,}2$

$\alpha = 30^\circ$

$v_2 = ?$



$$\Delta \vec G = \int \vec F\, dt$$

$$\vec G_k - \vec G_z = 0$$

$$\vec G_k = \vec G_z$$

$$(m_1 + m_2)\vec v = m_1 \vec v_1 + m_2 \vec v_2$$

$$x: \quad (m_1 + m_2)\, v \sin\alpha = m_1 v_1 + 0$$

$$y: \quad (m_1 + m_2)\, v \cos\alpha = 0 + m_2 v_2$$

$$: \qquad \mathrm{tg}\,\alpha = \frac{m_1 v_1}{m_2 v_2} \qquad\qquad m_2 = k\, m_1$$

$$v_2 = \frac{m_1 v_1}{m_2\, \mathrm{tg}\,\alpha} = \frac{v_1}{k\, \mathrm{tg}\,\alpha} = \frac{40\,\mathrm{km/h}}{1{,}2 \cdot \mathrm{tg}\, 30^\circ} = \underline{57{,}74\,\mathrm{km/h}}$$

3,23

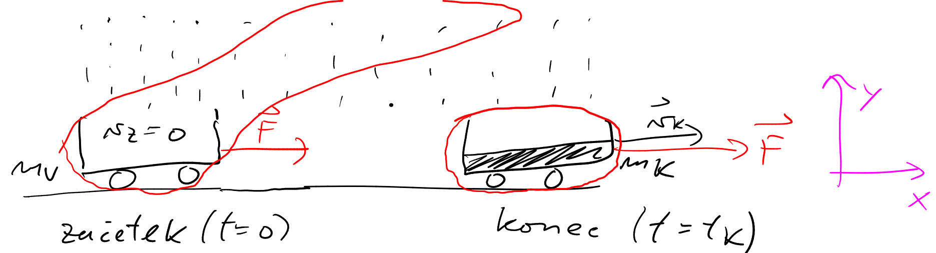
$m_V = 1\,kg$

$\phi_m = 0{,}1\,kg/s$

$F = 0{,}5\,N$

$m_k = 2\,kg$

$v_k = ?$



$$\vec{G}_k - \vec{G}_z = \int \vec{F}\,dt$$

$$x:\quad m_k \cdot v_k - 0 = \int_0^{t_k} F\,dt = F\int_0^{t_k} dt = F t_k$$

$$v_k = \frac{F t_k}{m_k} = \frac{F}{m_k}\,\frac{m_k - m_V}{\phi_m} = \frac{0{,}5\,N \cdot 1\,\cancel{kg}\,s}{2\,\cancel{kg} \cdot 0{,}1\,kg} =$$

$$\phi_m = \frac{\Delta m}{\Delta t} = \frac{m_k - m_V}{t_k}$$

$$t_k = \frac{m_k - m_V}{\phi_m}$$

$$= 2{,}5\,\frac{Ns}{kg} = 2{,}5\,\frac{\cancel{kg}\,m\,\cancel{s}}{s^{\cancel{2}}\,\cancel{kg}} =$$

$$= 2{,}5\ m/s$$

3.30

$\alpha_1 = 25^\circ$

$\alpha_2 = 35^\circ$

$S_1 = 1{,}5\ cm^2$

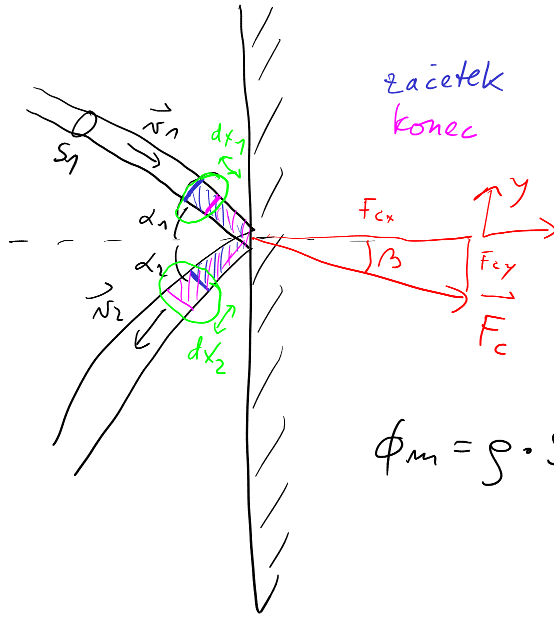
$v_1 = 2\ m/s$

$v_2 = 1{,}3\ m/s$

$\rho = 10^3\ kg/m^3$

$F_c = ?$

$\beta = ?$



$tg\,\beta = \frac{-F_{cy}}{F_{cx}}$

$\phi_m = \rho \cdot S \cdot v$

$$\Delta \vec{G} = \int \vec{F}\,dt$$

$$d\vec{G} = \vec{F}\,dt$$

$$dm \cdot \vec{v}_2 - dm \cdot \vec{v}_1 = -\vec{F}_c \cdot dt$$

$$\vec{F}_c = \frac{dm}{dt}(\vec{v}_1 - \vec{v}_2) = \phi_m(\vec{v}_1 - \vec{v}_2) = \rho S_1 v_1 (\vec{v}_1 - \vec{v}_2)$$

$$F_c = \rho S_1 v_1 \left|\vec{v}_1 - \vec{v}_2\right| = \rho S_1 v_1 \left(\overset{x}{v_1\cos\alpha_1 + v_2\cos\alpha_2},\ \overset{y}{-v_1\sin\alpha_1 + v_2\sin\alpha_2}\right)$$

$$F_c = \rho S_1 v_1 \sqrt{(v_1\cos\alpha_1 + v_2\cos\alpha_2)^2 + (v_2\sin\alpha_2 - v_1\sin\alpha_1)^2} =$$

$$= \rho S_1 v_1 \sqrt{v_1^2\cos^2\alpha_1 + 2v_1v_2\cos\alpha_1\cos\alpha_2 + v_2^2\cos^2\alpha_2 + v_2^2\sin^2\alpha_2 - 2v_1v_2\sin\alpha_1\sin\alpha_2 + v_1^2\sin^2\alpha_1} =$$

$$= \rho S_1 v_1 \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos(\alpha_1+\alpha_2)} =$$

$$= 1000\,\frac{kg}{\cancel{m^3}}\,(1{,}5\cdot 10^{-2}\,\cancel{m})^2 \cdot 2\,\frac{\cancel{m}}{s}\sqrt{\left(2\,\frac{m}{s}\right)^2 + \left(1{,}3\,\frac{m}{s}\right)^2 + 2\cdot 2\cdot 1.3\,\frac{m^2}{s^2}\cos 60^\circ} =$$

$$= \ldots\ \frac{kg}{s}\frac{m}{s} = \underline{0{,}86\ N}$$

$$tg\,\beta = -\frac{F_{cy}}{F_{cx}} = \frac{v_1\sin\alpha_1 - v_2\sin\alpha_2}{v_1\cos\alpha_1 + v_2\cos\alpha_2} = \frac{2\sin 25^\circ - 1{,}3\cdot\sin 35^\circ}{2\cos 25^\circ + 1{,}3\cdot\cos 35^\circ} = \ldots$$

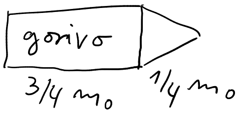
$$\beta = \mathrm{arctg}\ldots = \underline{2^\circ}$$

3.31

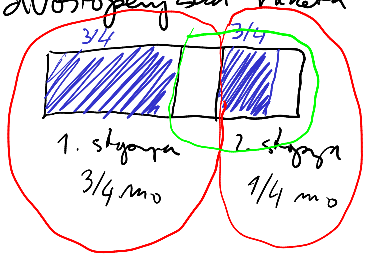
$v_i = 3\ km/s$

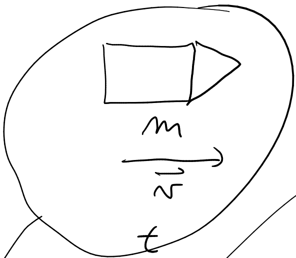
$p = 3/4$

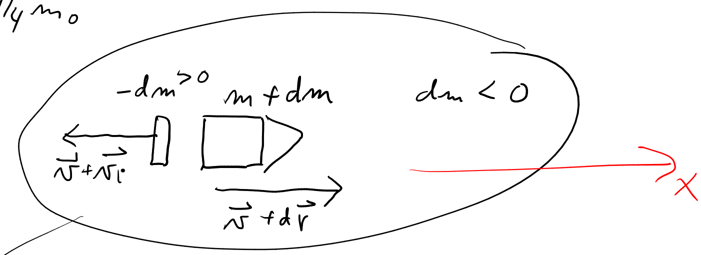
$v_K = ?$



dvostopenjska raketa







$t + dt$

$$d\vec{G} = \vec{F}dt = 0$$

$$\vec{G}(t+dt) - \vec{G}(t) = 0$$

$$\vec{G}(t+dt) = \vec{G}(t)$$

x_i

$$(m+dm)(\vec{v}+d\vec{v}) + (-dm)(\vec{v}+\vec{v}_i) = m\vec{v}$$

$$(m+dm)(v+dv) + (-dm)(v-v_i) = mv$$

$$\cancel{mv} + \underline{m\,dv} + \cancel{dm\,v} + \cancel{dm\,dv} - \cancel{dm\,v} + \underline{dm\,v_i} = \cancel{mv}$$

$$m\,dv + dm\,v_i = 0$$

$$\frac{dm}{m} = -\frac{dv}{v_i} \quad \Big/ \int$$

$$\int_{m_z}^{m_k} \frac{dm}{m} = -\int_{v_z}^{v_k} \frac{dv}{v_i}$$

$$\ln\frac{m_k}{m_z} = -\frac{v_k - v_z}{v_i}$$

$$v_k = v_z - v_i \ln\frac{m_k}{m_z}$$

$$\boxed{v_k = v_z + v_i \ln\frac{m_z}{m_k}} \rightarrow v_k = 0 + 3\frac{km}{s}\cdot\ln\frac{m_0}{\frac{1}{4}m_0}$$

$$v(t) = v_z + v_i \ln\frac{m_z}{m(t)} \qquad v_k = 3\frac{km}{s}\ln 4 =$$

$$= \underline{4{,}16\ \frac{km}{s}}$$

1. stopnja

$$m_z^{(1)} = m_0$$

$$m_k^{(1)} = \frac{1}{4}m_0 + \frac{3}{4}m_0\cdot\frac{1}{4} =$$

$$= \frac{7}{16}m_0$$

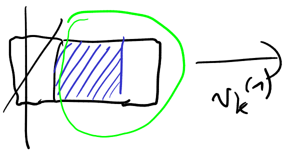
$$v_k^{(1)} = 0 + v_i \ln\frac{m_z^{(1)}}{m_k^{(1)}} =$$

$$= 0 + v_i \ln\frac{m_0}{\frac{7}{16}m_0} =$$

$$= v_i \ln\frac{16}{7}$$

2. stopnja

začetek



$$v_k^{(2)} = v_k^{(1)} + v_i \ln\frac{m_z^{(2)}}{m_k^{(2)}}$$

$$m_z^{(2)} = \frac{1}{4}m_0$$

$$m_k^{(2)} = \frac{1}{4}m_z^{(2)} = \frac{1}{16}m_0$$

$$v_k^{(2)} = v_k^{(1)} + v_i \ln\frac{1/4\,m_0}{1/16\,m_0} =$$

$$= v_k^{(1)} + v_i \ln 4 =$$

$$= v_i \ln\frac{16}{7} + v_i \ln 4 =$$

$$= v_i \ln\frac{64}{7} = \underline{6{,}64}\ \underline{km/s}$$

5.8

$m_1 = 2\,kg$

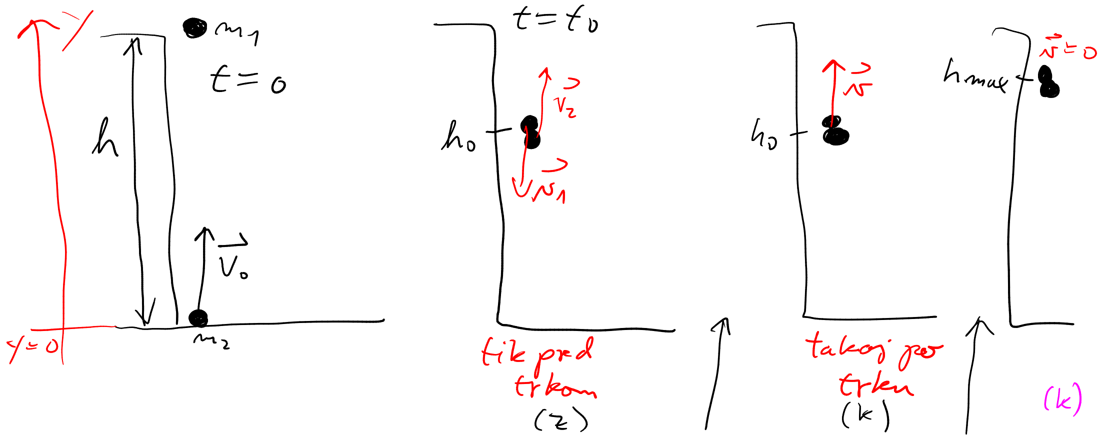
$m_2 = 1\,kg$

$h = 10\,m$

$v_0 = 20\,m/s$

$h_{max} = ?$

$W_{izg} = ?$



1: $y_1 = h - \frac{gt^2}{2}$ $\qquad v_1 = -gt$

2: $y_2 = v_0 t - \frac{gt^2}{2}$ $\qquad v_2 = v_0 - gt$

trk: $y_1 = y_2$

$$h - \cancel{\frac{g t_0^2}{2}} = v_0 t_0 - \cancel{\frac{g t_0^2}{2}}$$

$$t_0 = \frac{h}{v_0}$$

$y_1(t_0)$: $\quad h_0 = h - \frac{g h^2}{2 v_0^2}$

$$v_1 = -g\frac{h}{v_0}$$

$$v_2 = v_0 - g\frac{h}{v_0}$$

$$\Delta \vec{G} = \int \vec{F}\,dt = 0$$

$$\vec{G}_k = \vec{G}_z$$

$g: (m_1+m_2)v = m_1 v_1 + m_2 v_2$

$$v = \frac{m_1 v_1 + m_2 v_2}{m_1+m_2} = \frac{m_2 v_0}{m_1+m_2} - \frac{gh}{v_0}$$

h_{max}:

$$\Delta W = \Delta W_p + \Delta W_k = A' = 0$$

$$\left[\cancel{(m_1+m_2)}\, g (h_{max} - h_0)\right] + \left[0 - \frac{1}{2}\cancel{(m_1+m_2)}\, v^2\right] = 0$$

$$h_{max} = h_0 + \frac{v^2}{2g}$$

$$h_{max} = h - \frac{g h^2}{2 v_0^2} + \frac{1}{2g}\left(\frac{m_2}{m_1+m_2} v_0 - \frac{gh}{v_0}\right)^2 =$$

$$= 10\,m - \frac{9.81\,m \cdot (10\,m)^2}{s^2 \cdot 2 \cdot (20\,m/s)^2} + \frac{1\,s^2}{2 \cdot 9{,}81\,m}\left(\frac{1\,kg}{3\,kg} 20\frac{m}{s} - \frac{9.81\,m \cdot 10\,m}{s^2 \cdot 20\,m/s}\right)^2$$

$$= 8{,}93\,m$$

$$W_{izg} = W_k^{(k)} - W_k^{(z)} = \frac{1}{2}(m_1+m_2)v^2 - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\right) =$$

$$= \frac{1}{2}(m_1+m_2)\left[\frac{m_2}{m_1+m_2} v_0 - \frac{gh}{v_0}\right]^2 - \frac{1}{2} m_1 \left[-\frac{gh}{v_0}\right]^2 - \frac{1}{2} m_2 \left[v_0 - \frac{gh}{v_0}\right]^2 =$$

$$= \frac{1}{2}(m_1+m_2)\left[\left(\frac{m_2 v_0}{m_1+m_2}\right)^2 - \cancel{\frac{2gh}{v_0}\frac{m_2}{m_1+m_2} v_0} + \cancel{\left(\frac{gh}{v_0}\right)^2}\right] - \cancel{\frac{1}{2} m_1 \left(\frac{gh}{v_0}\right)^2} -$$

$$- \frac{1}{2} m_2 v_0^2 + \cancel{m_2 v_0 \frac{gh}{v_0}} - \cancel{\frac{1}{2} m_2 \left(\frac{gh}{v_0}\right)^2} =$$

$$= \frac{1}{2}\frac{m_2^2 v_0^2}{m_1+m_2} - \frac{1}{2} m_2 v_0^2 = \frac{1}{2} m_2 v_0^2 \left(\frac{m_2}{m_1+m_2} - 1\right) =$$

$$= -\frac{1}{2}\frac{m_1 m_2}{m_1+m_2} v_0^2 = -\frac{1}{2}\frac{1\,kg \cdot 2\,kg}{3\,kg} \cdot \left(20\frac{m}{s}\right)^2 = -133\ J$$