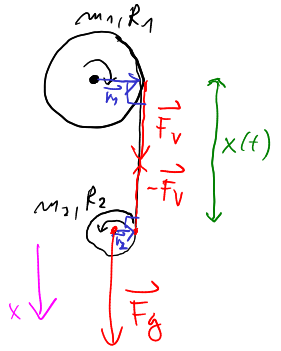
3. 39



$m_1 = 1\,kg$
$R_1 = 10\,cm$
$m_2 = 200\,g$
$R_2 = 3\,cm$
$N_1 = 1$
$t_1 = ?$

grad: $M_1 = J_1 \cdot \alpha_1$

$\vec{M} = \vec{r} \times \vec{F}$

$F_V \cdot R_1 = \frac{1}{2} m_1 R_1^2 \alpha_1$

$\alpha_1 = \frac{2F_V}{m_1 R_1}$ → $\varphi_1(t) = \frac{\alpha_1 t^2}{2}$

$2\pi N_1 = \frac{\alpha_1 t_1^2}{2}$

$t_1 = \sqrt{\frac{4\pi N_1}{\alpha_1}}$

valj: $M_2 = J_2 \alpha_2$

$F_V \cdot R_2 = \frac{1}{2} m_2 R_2^2 \alpha_2$

$\alpha_2 = \frac{2F_V}{m_2 R_2}$

valj: $\vec{F_g} + (-\vec{F_V}) = m_2 \vec{a_2}^*$

X: $m_2 g - F_V = m_2 a_2^*$

$x(t) = x(0) + R_1 \varphi_1(t) + R_2 \varphi_2(t)$ / $\frac{d^2}{dt^2}$ $(\ddot{\ })$

$a^* = 0 + R_1 \alpha_1 + R_2 \alpha_2$

$g - \frac{F_V}{m_2} = \frac{2F_V}{m_1} + \frac{2F_V}{m_2}$

$g = \frac{3F_V}{m_2} + \frac{2F_V}{m_1}$

$F_V = \frac{g}{\frac{3}{m_2} + \frac{2}{m_1}} = \frac{g m_1 m_2}{3m_1 + 2m_2}$

$\alpha_1 = \frac{2F_V}{m_1 R_1} = \frac{2g m_2}{R_1(3m_1 + 2m_2)}$

$t_1 = \sqrt{\frac{4\pi N_1 R_1 (3m_1 + 2m_2)}{2 g m_2}} = \sqrt{\frac{2\pi N_1 R_1 (3m_1 + 2m_2)}{g m_2}} =$

$= \sqrt{\frac{2\pi \cdot 1 \cdot 0{,}1\,\cancel{m} \cdot 3{,}4\,\cancel{kg}\,s^2}{9{,}81\,\cancel{m} \cdot 0{,}2\,\cancel{kg}}} = \underline{1{,}04 \quad s}$

3.43

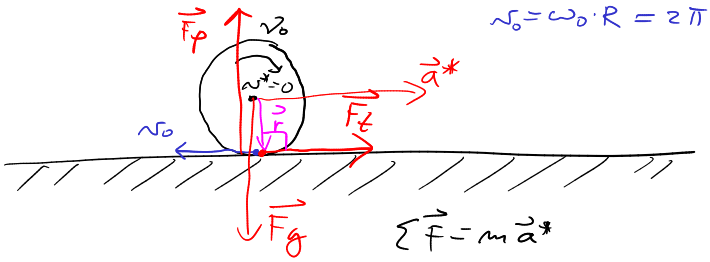
$R = 10\,c$

$\nu_0 = 10\,l$

$k_t = 0{,}1$

$t_1 = ?$

$s^* = ?$



$\nu_0 = \omega_0 \cdot R = 2\pi\nu_0 \cdot R$

$\sum \vec{F} = m\vec{a}^*$

$\hookrightarrow F_p = F_g = mg$

$F_t = F_p k_t = mgk_t = ma^*$

$\boxed{a^* = gk_t}$

$\sum \vec{M} = J\vec{\alpha}$

$-F_t \cdot R = \frac{1}{2}mR^2\alpha$

$\boxed{\alpha = \frac{-2F_t}{mR} = \frac{-2mgk_t}{mR} = \frac{-2gk_t}{R}}$

$\nu^* = a^* t = gk_t t$

$\nu = \omega R$ ← obodna hitrost

$\omega = \omega_0 + \alpha t = 2\pi\nu_0 - \frac{2gk_t \cdot t}{R}$

$\nu = 2\pi\nu_0 R - 2gk_t t$

ν' ← hitrost točke na površini valja glede na tla

$\nu' = \nu - \nu^* = (2\pi\nu_0 R - 2gk_t \cdot t) - (gk_t t) = 2\pi\nu_0 R - 3gk_t \cdot t$

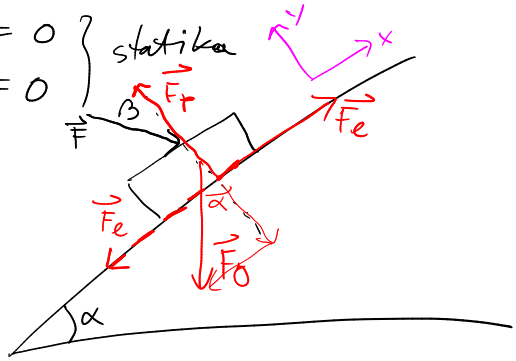
$\nu' = 0 = 2\pi\nu_0 R - 3gk_t \cdot t_1$

$t_1 = \frac{2\pi\nu_0 R}{3gk_t} = \frac{2\pi \cdot 10 \cdot \cancel{0{,}1\,m}\,s^2}{\cancel{8} \cdot 3 \cdot 9{,}81\,\cancel{m} \cdot \cancel{0{,}1}} = \underline{2{,}13\ s}$

$s^* = \frac{a^* t_1^2}{2} = \frac{gk_t \cdot t_1^2}{2} = \frac{9{,}81\,m \cdot 0{,}1 \cdot 2{,}13^2\,\cancel{s^2}}{\cancel{s^2} \cdot 2} = \underline{2{,}23\ m}$

4.1

$\Sigma \vec{F} = 0$
$\Sigma \vec{M} = 0$
} statika



$\alpha = 30°$
$m = 50\ kg$
$\beta = 10°$
$k_\ell = 0{,}1$

$F = ?$

$F_{min} < F < F_{max}$
$\|$ $\|$
? ?

$F_\ell \leq F_p k_\ell$

$$\vec{F}_G + \vec{F}_p + \vec{F} + \vec{F}_\ell = 0$$

x: $-mg\sin\alpha + F\sin\beta \pm F_p k_\ell = 0$ ($\to F_{min}$ / $\to F_{max}$)

y: $-mg\cos\alpha + F_p - F\cos\beta = 0$

$\hookrightarrow F_p = mg\cos\alpha + F\cos\beta$

$$-mg\sin\alpha + F\sin\beta \pm k_\ell(mg\cos\alpha + F\cos\beta) = 0$$

$$-mg\sin\alpha \pm mg\cos\alpha\, k_\ell = -F\sin\beta \mp k_\ell F\cos\beta$$

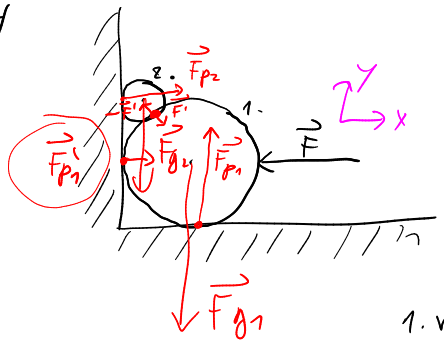
$$F = \frac{mg(-\sin\alpha \pm k_\ell\cos\alpha)}{-\sin\beta \mp k_\ell\cos\beta}$$

zg. predznak za F_{min}
sp. $-$ " za F_{max}

$$F_{min} = mg\,\frac{-\sin\alpha + k_\ell\cos\alpha}{-\sin\beta - k_\ell\cos\beta} = 50\,kg \cdot 9{,}81\,\frac{m}{s^2} \cdot \frac{-\sin 30° + 0{,}1\cos 30°}{-\sin 10° - 0{,}1\cos 10°} = \underline{745\ N}$$

$$F_{max} = mg\,\frac{-\sin\alpha - k_\ell\cos\alpha}{-\sin\beta + k_\ell\cos\beta} = \ldots\ldots\ldots = \underline{3828\ N}$$

4.4



$F_{min} = ?$

$m_1 = 3\,kg \rightarrow \rho_1$

$R_1 = 4\,cm$ ||

$m_2 = ? \rightarrow \rho_2$

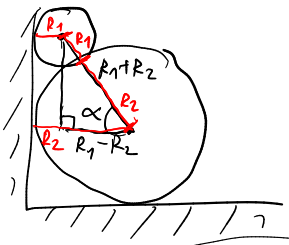
$R_2 = 1\,cm$

$\rho_1 = \rho_2 = \rho$ $\quad \rho = \frac{m}{V}$

$= 0$ (pri $F = F_{min}$)

1. valj $\quad \vec{F} + \vec{F}_{g1} + \vec{F}_{p1} + \cancel{\vec{F}'_{p1}} + \vec{F}' = 0$

2. valj $\quad \vec{F}_{g2} + \vec{F}_{p2} - \vec{F}' = 0$



$\cos\alpha = \frac{R_1 - R_2}{R_1 + R_2}$

x: $\quad -F + F'\cos\alpha = 0$ (1. valj)

$\quad -F'\cos\alpha + F_{p2} = 0$ (2. valj)

y: $\quad -m_1 g + F_{p1} - F'\sin\alpha = 0$ (1. valj)

$\quad -m_2 g + F'\sin\alpha = 0$ (2. valj)

F
F'
α

$F' = \frac{F}{\cos\alpha}$ $\qquad m_2 g = F \operatorname{tg}\alpha$

$F = m_2 g \operatorname{ctg}\alpha$

$\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\frac{R_1-R_2}{R_1+R_2}}{\sqrt{1-\left(\frac{R_1-R_2}{R_1+R_2}\right)^2}} =$

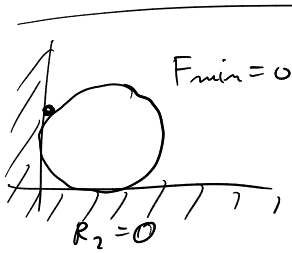
$= \frac{\frac{R_1-R_2}{R_1+R_2}}{\sqrt{\frac{(R_1+R_2)^2-(R_1-R_2)^2}{(R_1+R_2)^2}}} = \frac{R_1-R_2}{\sqrt{4R_1R_2}}$

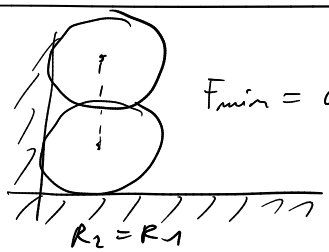
$m_2 = \rho V_2 \rightarrow m_2 = m_1 \frac{V_2}{V_1}$

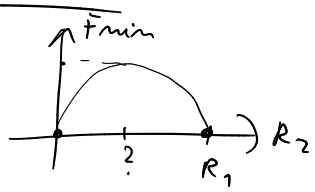
$m_1 = \rho V_1$ $\qquad m_2 = m_1 \cdot \frac{\pi R_2^2 \cdot h_2}{\pi R_1^2 \cdot h_1}$ $\qquad h_1 = h_2$

$$F_{min} = m_1 \left(\frac{R_2}{R_1}\right)^2 g \frac{R_1 - R_2}{\sqrt{4R_1R_2}} = 3\,kg \cdot \left(\frac{1\,cm}{4\,cm}\right)^2 \cdot 9{,}81\,\frac{m}{s^2} \cdot \frac{3\,cm}{\sqrt{4 \cdot 4\,cm \cdot 1\,cm}} =$$

$$= \frac{3 \cdot 10 \cdot 3}{16 \cdot 4}\,N = \frac{90}{64}\,N = 1{,}4\,N$$







$\frac{dF_{min}}{dR_2} = 0$

$$F_{min} = \frac{m_1 g}{2}\left(\left(\frac{R_2}{R_1}\right)^{3/2} - \left(\frac{R_2}{R_1}\right)^{5/2}\right)$$

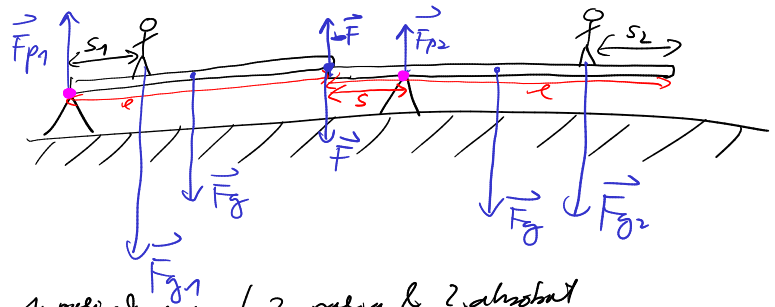
$$\frac{dF_{min}}{dR_2} = \frac{m_1 g}{2}\left(\frac{\frac{3}{2}R_2^{1/2}}{R_1^{3/2}} - \frac{\frac{5}{2}R_2^{3/2}}{R_1^{5/2}}\right) = 0$$

$$\frac{m_1 g}{2}\frac{3}{2}\frac{R_2^{1/2}}{R_1^{3/2}}\left(1 - \frac{5}{3}\frac{R_2}{R_1}\right) = 0$$

$\left(1 - \frac{5}{3}\frac{R_2}{R_1}\right) = 0$

$R_2 = \frac{3}{5}R_1 = \frac{3}{5} \cdot 4\,cm = 2{,}4\,cm$

4.6



$\ell = 3\,m$
$s = 1\,m$
$F_g = 150\,N$
$s_1 = 1\,m$
$F_{g1} = 800\,N$
$F_{g2} = 600\,N$
$s_2 = ?$

1. Paar mit 1. Abstand / 2. Paar & 2. Abstand

$\sum \vec{F} = 0$ $\quad$ $\sum \vec{F} = 0$

$\sum \vec{n} = 0$ $\quad$ $\sum \vec{n} = 0$

$F_{g1} \cdot s_1 + F_g \frac{\ell}{2} - F\ell = 0$

$F_g\left(\frac{\ell}{2} - s\right) + F_{g2}(\ell - s - s_2) - Fs = 0$

$\hookrightarrow F = \frac{F_g}{2} + F_{g1}\frac{s_1}{\ell}$

$F_g\left(\frac{\ell}{2} - s\right) + F_{g2}(\ell - s - s_2) - \frac{F_g}{2}s - F_{g1}\frac{s_1}{\ell}s = 0$

$$s_2 = \frac{F_g\left(\frac{\ell}{2} - \frac{3}{2}s\right) + F_{g2}(\ell - s) - F_{g1}\frac{s_1 \cdot s}{\ell}}{F_{g2}}$$

$$s_2 = \frac{150\,N\left(\frac{3\,m}{2} - \frac{3}{2}1\,m\right) + 600\,N\,(3\,m - 1\,m) - 800\,N\,\frac{1\,m \cdot 1\,m}{3\,m}}{600\,N}$$

$$s_2 = 2\,m - \frac{800}{3 \cdot 600}\,m = \left(2 - \frac{4}{9}\right)m = \frac{14}{9}\,m$$

$v_2 = 0{,}1\,m/s = \dot{s}_2$

$v_1 = -\dot{s}_1 = ?$

$\frac{d}{dt}$: $v_2 = \dot{s}_2 = -\frac{F_{g1}}{F_{g2}}\frac{s}{\ell}\dot{s}_1 = \frac{F_{g1}}{F_{g2}}\frac{s}{\ell}v_1$

$$v_1 = \frac{F_{g2}}{F_{g1}}\frac{\ell}{s}v_2 = \frac{600\,N}{800\,N}\cdot\frac{3\,m}{1\,m}\cdot 0{,}1\,m/s =$$

$$= \frac{9}{4}\cdot 0{,}1\,m/s = 0{,}225\,m/s$$

4.7

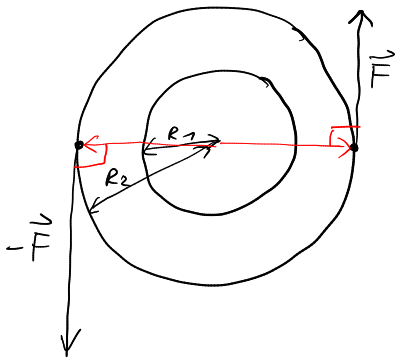
$F_g = 500N$

$R_1 = 0,5m$

$R_2 = 0,7\ m$

$k_t = 0,6$

$F = ?$

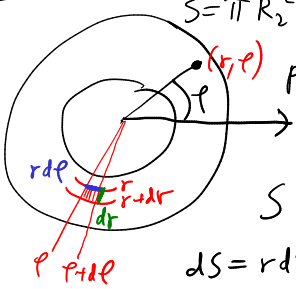


$M_r = 2FR_2$

$\sum \vec{M} = 0$

$\vec{M}_r + \vec{M}_e = 0$

$M_r = M_e$

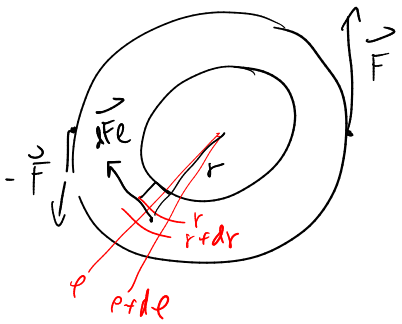


$dS = rd\varphi \cdot dr$

$$S = \int dS = \int_0^{2\pi}\int_{R_1}^{R_2} r\,dr\,d\varphi = \int_{R_1}^{R_2}\left(rdr \int_0^{2\pi} d\varphi\right) = \int_{R_1}^{R_2}\left(rdr\ \underbrace{\varphi\Big|_0^{2\pi}}_{2\pi}\right) = 2\pi \int_{R_1}^{R_2} r\,dr =$$

$$= 2\pi \frac{r^2}{2}\Big|_{R_1}^{R_2} = \pi\left(R_2^2 - R_1^2\right)$$

$$M_e = \int dM_e = \int r\,dF_e = \int r\,dF_p \cdot k_e = k_e \int r \cdot dF_g =$$



$$= k_e \int r \cdot F_g \frac{dS}{S} =$$

$$= k_e F_g \cdot \int_{R_1}^{R_2} r \frac{r\,d\varphi\,dr}{\pi(R_2^2 - R_1^2)} =$$

$$= \frac{k_e F_g}{\pi(R_2^2 - R_1^2)} \int_{R_1}^{R_2} r^2 dr \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} = \frac{2k_e F_g}{R_2^2 - R_1^2} \cdot \frac{r^3}{3}\Big|_{R_1}^{R_2} =$$

$$= \frac{2}{3} k_e F_g \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

$M_e = M_r$

$$\frac{2}{3} k_e F_g \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} = 2FR_2 \qquad F = \frac{k_e F_g}{3R_2} \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

$$F = \frac{0,6 \cdot 500N}{3 \cdot 0,7m} \frac{(0,7m)^3 - (0,5m)^3}{(0,7m)^2 - (0,5m)^2} =$$

$$= 130\ N$$