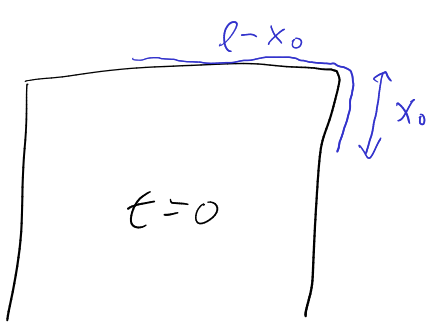
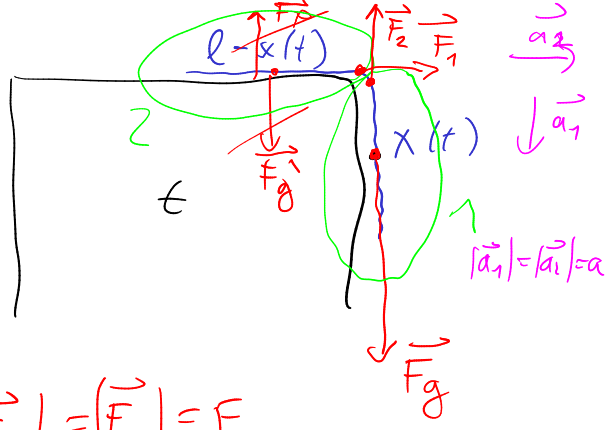
3. g

m
ℓ
x_0

$x(t) = ?$





$$|\vec{F}_1| = |\vec{F}_2| = F$$

2. Newtonov zakon:

(1) $m\frac{x(t)}{\ell}g - F = m\frac{x(t)}{\ell}a$ ($m\frac{x(t)}{\ell} = m_1$)

(2) $F = m\frac{\ell - x(t)}{\ell}a$ ($m\frac{\ell - x(t)}{\ell} = m_2$)

$$\sum \quad \frac{m\,x(t)}{\ell}g = m\,a$$

$$\boxed{\frac{g}{\ell}x(t) = x''(t)} \qquad x(t) = ?$$

navadna linearna diferencialna enačba druga reda
- odvod samo po t
- $x(t)$ samo v 1. potenci
- najvišji odvod je 2. odvod

$$x(t) = A\,f_1(t) + B\,f_2(t)$$

f_1 in f_2 linearno neodvisni

$x''(t) = x(t)$

↳ $x(t) = e^t \quad x'(t) = e^t \quad x''(t) = e^t$ ✓

$x(t) = e^{-t} \quad x'(t) = -e^{-t} \quad x''(t) = e^t$ ✓

$$x(t) = Ae^t + Be^{-t}$$

$x(t) = \cosh t \,/\, \sinh t$
$x''(t) = \cosh t \,/\, \sinh t$

$$x(t) = Ae^{\sqrt{\frac{g}{\ell}}t} + Be^{-\sqrt{\frac{g}{\ell}}t}$$

2 začetna pogoja: $x(0) = x_0 = A + B$

$$x'(0) = 0 = \left(A\sqrt{\tfrac{g}{\ell}}e^{\sqrt{\frac{g}{\ell}}t} + B\left(-\sqrt{\tfrac{g}{\ell}}\right)e^{-\sqrt{\frac{g}{\ell}}t}\right)\Big|_{t=0} = \sqrt{\tfrac{g}{\ell}}(A - B)$$

$A - B = 0 \rightarrow A = B$

$A + B = x_0 \rightarrow A = B = \frac{x_0}{2}$

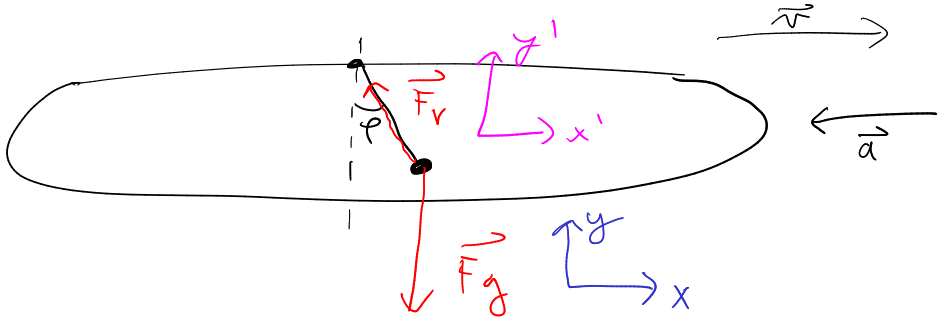
$$x(t) = \frac{x_0}{2}\left(e^{\sqrt{\frac{g}{\ell}}t} + e^{-\sqrt{\frac{g}{\ell}}t}\right) = x_0\cosh\left(\sqrt{\tfrac{g}{\ell}}t\right) \quad ; \quad x(t) \le \ell$$

3.51

$a = 0{,}7\ m/s^2$

$l = 1\ m$

$\varphi = ?$



↳ 2. N. Z.

$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_g + \vec{F}_v = m\vec{a}$$

x: $0 - F_v \sin\varphi = -ma$ → $+F_v \sin\varphi = +ma$

y: $-mg + F_v \cos\varphi = 0$ → $F_v \cos\varphi = mg$

: $\operatorname{tg}\varphi = \frac{a}{g}$

$\varphi = \operatorname{arctg}\frac{a}{g} = 4^\circ$

↳ 2. N. Z.

$$\sum \vec{F} + \vec{F}_s = 0$$

$$\vec{F}_g + \vec{F}_v - m\vec{a} = 0$$

3.52

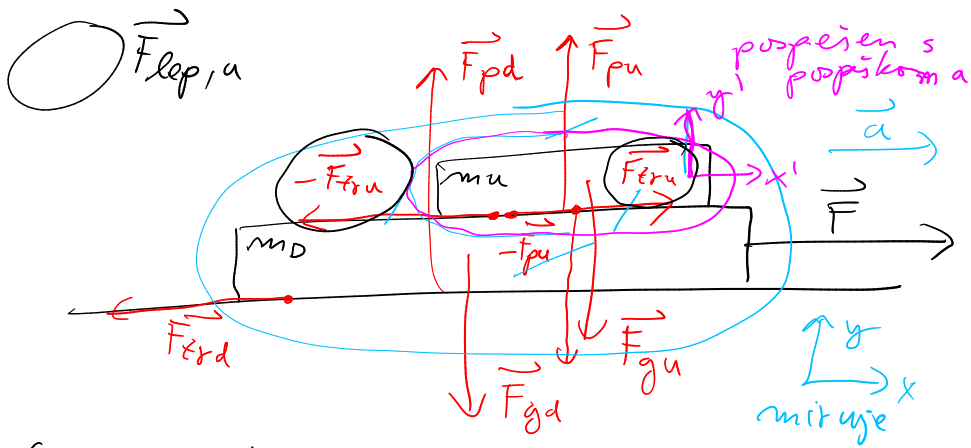
$m_D = 2\,kg$

$m_u = 1\,kg$

$k_1 = 0.4$

$k_2 = 0.3$

F_{min}



uteč mi zdrsnila ($F < F_{min}$)

2NZ: $(m_D + m_u)\vec{a} = \vec{F}_{gd} + \vec{F}_{gu} + \vec{F} + \vec{F}_{trd} + \vec{F}_{pd}$

x: $(m_D + m_u)a = 0 + 0 + F - F_{trd} + 0$

y: $0 = -m_d g - m_u g + 0 + 0 + F_{pd}$

$F_{trd} = F_{pd} \cdot k_1$

$F_{trd} = (m_d + m_u) g k_1$

$(m_D + m_u)a = F - (m_d + m_u) g k_1$

$$a = \frac{F}{m_d + m_u} - g k_1$$

2NZ za uteč:

sistemska sila

$\vec{F}_{pu} + \vec{F}_{gu} + \vec{F}_{lep,u} - m_u\vec{a} = 0$

x': $0 + 0 + F_{lep,u} - m_u a = 0$

y': $F_{pu} - m_u g = 0$

$F_{lep,u} \le F_{pu} \cdot k_2$

uteč zdrsne, ko $F_{lep,u} = k_2 F_{pu}$

$F_{lep,u} = m_u g k_2$

$m_u g k_2 = m_u a$

mejni pospešek

$a = g k_2$

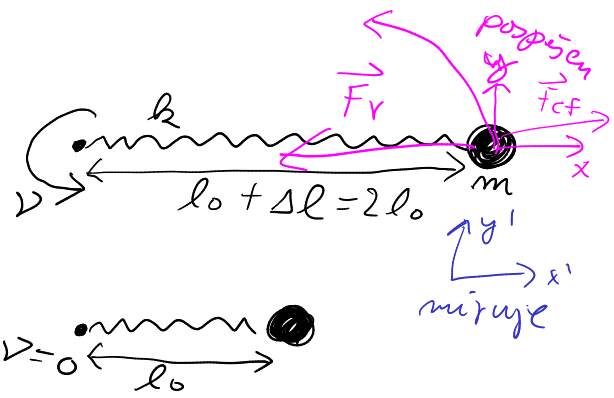
$$g k_2 = \frac{F_{min}}{m_d + m_u} - g k_1$$

$$F_{min} = (m_d + m_u) g (k_1 + k_2) = 3\,kg \cdot 10\,\frac{m}{s^2} \cdot 0.7 = \underline{\underline{21\,N}}$$

3.56 $k = 20\ \frac{N}{cm}$

$m = 100\,g$ $\quad \Delta l = l_0$

$\nu = ?$



2NZ: $\quad \sum \vec{F} + \vec{F}_s = 0$

$F_v + \vec{F}_{cf} = 0$

centrifugalna sila

$-k\Delta l + m R \omega^2 = 0$ $\quad R = 2l_0$, $\Delta l = l_0$, $\omega = 2\pi\nu$

$-k l_0 + m\, 2 l_0 (2\pi\nu)^2 = 0$

$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{2m}} = \frac{1}{2\pi}\sqrt{\frac{20\,N}{0.01\,m \cdot 2 \cdot 0.1\,kg}} = 15{,}9\ \frac{1}{s} =$

$= 15{,}9\ Hz$ $\quad Hz = \frac{1}{s}$

2NZ: $\quad \sum \vec{F} = m\vec{a}$

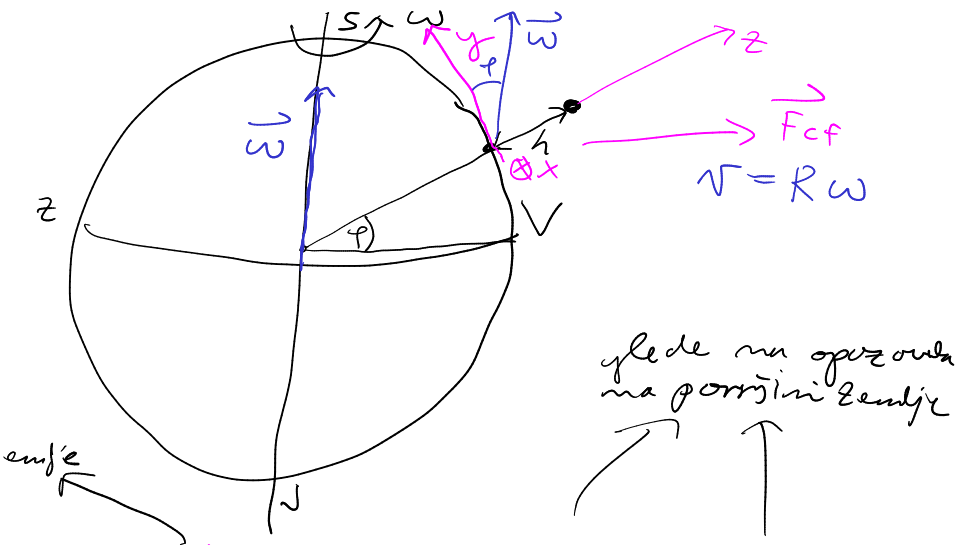
$\vec{F}_N = m\vec{a}_R$

$-k\Delta l = -m(R\omega^2)$

3.66 $\varphi = 30°$

$h = 125\,m$

$\Delta x = ?$



glede na središče Zemlje

glede na opazovalca na površini Zemlje

$$\sum \vec{F} - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m\vec{\omega}\times\vec{v} = m\vec{a}$$

centrifugalna sila — coriolisova sila

↳ je v ravnini yz

zanemarimo

$$\vec{F}_g - 2m\vec{\omega}\times\vec{v} = m\vec{a}$$

1.) zanemarimo tudi coriolisovo silo

$$\vec{F}_g = m\vec{a}$$

$$z(t) = h - \frac{gt^2}{2}$$

2.) upoštevamo coriolisovo silo kot popravek

$$-2m\vec{\omega}\times\vec{v}$$

$$\vec{v} = (0, 0, -gt)$$

$$\vec{\omega} = (0, \omega\cos\varphi, \omega\sin\varphi)$$

$\omega = \frac{2\pi}{t_0}$

$$\vec{\omega}\times\vec{v} = (-\omega\cos\varphi\cdot gt, \dots, \dots)$$

$$\vec{F}_{cor} = (2m\omega\cos\varphi\, gt, \dots, \dots)$$

$$m a_x(t) = F_{cor} = 2m\omega\cos\varphi\, gt$$

$$v_x(t) = \int_0^t a_x(t')dt' = 2\omega\cos\varphi\, g \int_0^t t'dt' =$$

$$= \omega\cos\varphi\, g\frac{t^2}{2}$$

$$x(t) = \int_0^t v_x(t')dt' = \omega\cos\varphi\, g \int_0^t t'^2 dt'$$

$$x(t) = \omega\cos\varphi\, g\frac{t^3}{3}$$

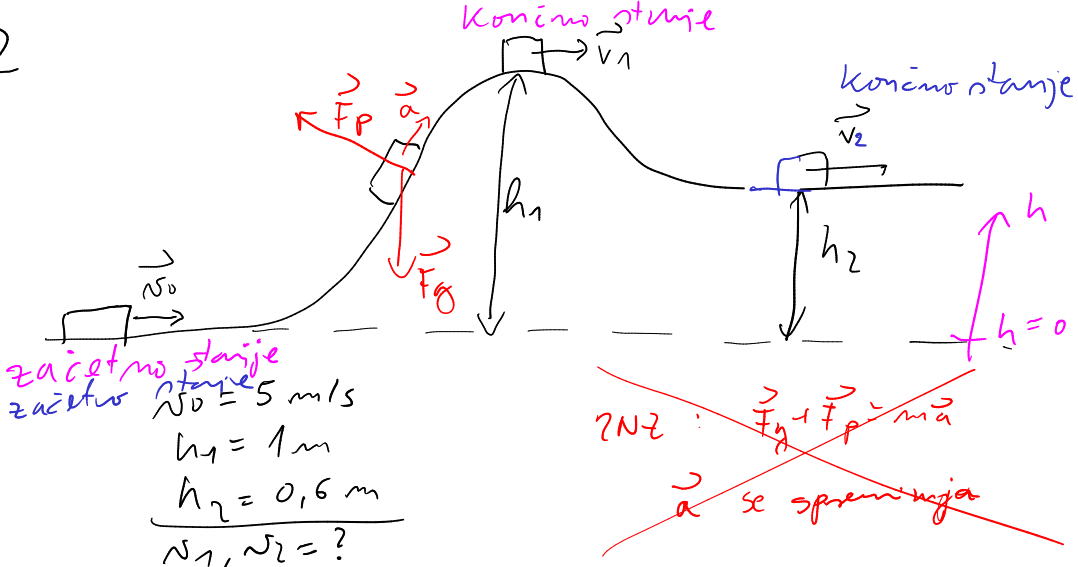
$z: \; h - \frac{gt_p^2}{2} = z(t_p) = 0$

$$t_p = \sqrt{\frac{2h}{g}}$$

$$x(t_p) = \Delta x = \frac{\omega\cos\varphi\, g}{3}\left(\frac{2h}{g}\right)^{3/2} = \frac{1}{3}\omega\cos\varphi\sqrt{\frac{8h^3}{g}}$$

$$\Delta x = \frac{1}{3}\,\frac{2\pi}{24\cdot 3600\,s}\cos 30°\sqrt{\frac{8\cdot(125\,m)^3}{9{,}81\,m}} = 0{,}026\,m$$

5.2



začetno stanje
$v_0 = 5\ m/s$
$h_1 = 1\ m$
$h_2 = 0{,}6\ m$

$v_1, v_2 = ?$
$h_{1\,max} = ?$

$W_k = \frac{1}{2} m v^2$

$W_p = mgh$

$W_{pr} = \frac{1}{2} k \Delta l^2$

2NZ: $\vec{F}_g + \vec{F}_p = m\vec{a}$

$\vec{a}$ se spreminja

energijski zakon

$\Delta W = A$

$\Delta W_k + \Delta W_p + \Delta W_{pr} = A'$

A' je delo sile podlage

$\vec{F}_p \perp d\vec{s}$ $\quad \vec{F}_p \cdot d\vec{s} = 0$

samo deko sil, ki ga še nismo vključili v spremembo energije

$A = \int \vec{F} \cdot d\vec{s}$

$A = \vec{F} \cdot \vec{s}$ $\quad \vec{F} = konst$

$A = Fs$ $\quad \vec{F}$ konst. in $\vec{F} \parallel \vec{s}$

$\Delta W_k + \Delta W_p = 0$

$(W_k^{kon} - W_k^{zač}) + (W_p^{kon} - W_p^{zač}) = 0$

$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 + mgh_1 - mg \cdot 0 = 0$

$v_1 = \sqrt{v_0^2 - 2gh_1} = \sqrt{5^2 \frac{m^2}{s^2} - 2 \cdot 9{,}8 \frac{m}{s^2} \cdot 1\,m} =$

$= 2{,}32\ m/s$

$v_2 = \sqrt{v_0^2 - 2gh_2} = \sqrt{5^2 \frac{m^2}{s^2} - 2 \cdot 9{,}81 \frac{m}{s^2} 0.6\,m} =$

$= 3{,}64\ m/s$

$v_1 = 0 = \sqrt{v_0^2 - 2gh_{1max}}$

$h_{1max} = \frac{v_0^2}{2g} = \frac{5^2\ m^2/s^2}{2 \cdot 9{,}81\ m/s^2} = 1{,}27\ m$