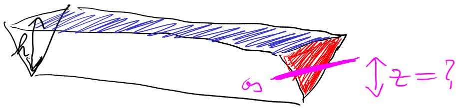
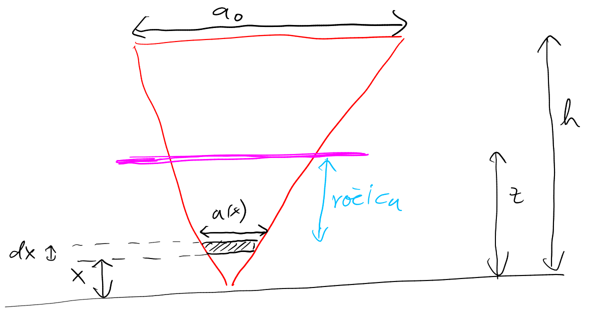
8.4 $\quad \underline{h = 2\,m}$





hidrostatski tlak:

$$p(y) = p_0 + \rho g y$$

y ↑ globina $\qquad \rho$ ↑ gostota tekočine

$$p(x) = p_0 + \rho g (h-x) \qquad \text{tlak vode} \qquad \rho = 1000\,kg/m^3$$

$$p_{zraka}(x) = p_0 + \underbrace{\cancel{\rho_{zraka}\, g (h-x)}}_{\text{zanemarimo}} \qquad \text{tlak zraka} \qquad \rho_{zraka} \approx 1\,kg/m^3$$

$$\Delta p(x) = p(x) - p_{zraka}(x) = \rho g (h-x)$$

$$p = \frac{F}{S} \leftarrow \quad dF(x) = \Delta p(x) \cdot dS = \Delta p(x) \cdot dx \cdot a(x) \qquad a(x) = a_0 \cdot \frac{x}{h}$$

$$dF(x) = \rho g (h-x) \cdot dx \cdot a_0 \frac{x}{h}$$

$$dM(x) = dF(x) \cdot (z-x)$$

$$dM(x) = \rho g (h-x)\, dx\, a_0 \frac{x}{h} (z-x)$$

$$M = \int dM = \int_0^h \rho g (h-x)\, \underline{dx}\, a_0 \frac{x}{h} (z-x) =$$

$$= \frac{\rho g a_0}{h} \int_0^h (h-x)\, x\, (z-x)\, dx =$$

$$= \frac{\rho g a_0}{h} \int_0^h (hzx - hx^2 - x^2 z + x^3)\, dx =$$

$$= \rho g \frac{a_0}{h} \left[\underline{hz \frac{h^2}{2}} - h \cdot \frac{h^3}{3} \underline{- z \frac{h^3}{3}} + \frac{h^4}{4} \right] =$$

$$= \rho g \frac{a_0}{h} \left[h^3 z \left(\frac{1}{2} - \frac{1}{3}\right) + h^4 \left(\frac{1}{4} - \frac{1}{3}\right) \right] =$$

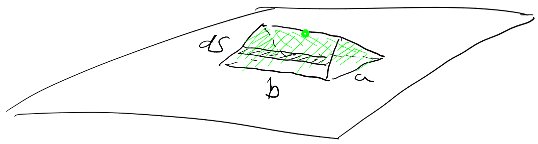
$$= \rho g \frac{a_0}{h} \left[\frac{h^3 z}{6} - \frac{h^4}{12} \right] = \rho g \frac{a_0}{h} \frac{h^3}{12} (2z - h)$$

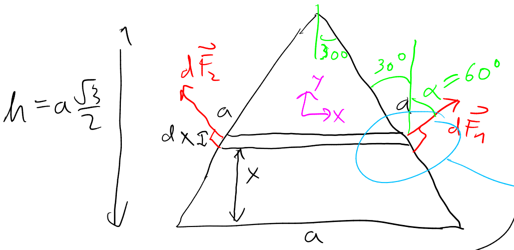
$$M = 0 \longrightarrow 2z - h = 0 \rightarrow z = \frac{h}{2} = \underline{\underline{1\,m}}$$

8.5 $\quad b = 10\,cm$

$a = 10\,cm$

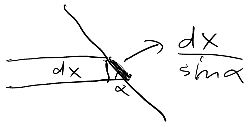
$m = ?$





$$d\vec{F} = d\vec{F}_1 + d\vec{F}_2 = (0, 2\,dF_1 \cdot \cos\alpha)$$

$$|d\vec{F}_1| = |d\vec{F}_2| = dF_1$$



$$p(x) = p_0 + \rho g (h - x)$$

$$\Delta p(x) = \rho g (h - x) \qquad dF_1 = \Delta p(x) \cdot dS = \Delta p(x) \cdot b \cdot \frac{dx}{\sin\alpha} = \rho g (h-x)\, b \frac{dx}{\sin\alpha}$$

$$\rightarrow d\vec{F} = (0, 2\rho g (h-x)\, b\, dx\, \mathrm{ctg}\,\alpha)$$

$$\vec{F} = \int d\vec{F} = \Big(0, 2\rho g b\, \mathrm{ctg}\,\alpha \underbrace{\int_0^h (h-x)\,dx}_{\left(hx - \frac{x^2}{2}\right)\Big|_0^h = h^2 - \frac{h^2}{2} = \frac{h^2}{2}}\Big)$$

$$\vec{F} = \left(0, \cancel{2}\rho g b\, \mathrm{ctg}\,\alpha \frac{h^2}{\cancel{2}}\right) \qquad h = \frac{a\sqrt{3}}{2}$$

$$\vec{F} = \left(0, \tfrac{3}{4}\rho g b\, \mathrm{ctg}\,\alpha\, a^2\right)$$

$\sum \vec{F} = 0$ $\quad$ najmanjša masa pride bo takrat, ko $\vec{F}_p = 0$

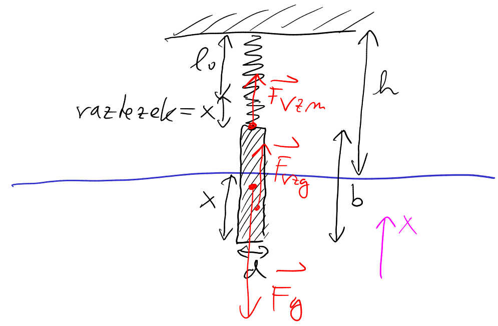
$$\vec{F} + \vec{F}_g + \cancel{\vec{F}_p} = 0$$

$$\frac{3}{4}\rho \cancel{g} b\, \mathrm{ctg}\,\alpha\, a^2 - m\cancel{g} = 0 \longrightarrow m = \frac{3}{4}\rho b a^2 \mathrm{ctg}\,\alpha \qquad \mathrm{ctg}\,60° = \frac{\cos 60°}{\sin 60°} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$m = \frac{\sqrt{3}}{4}\rho b a^2$$

$$m = \frac{\sqrt{3}}{4}\, 1000 \frac{kg}{\cancel{m^3}} \cdot 0{,}1\cancel{m}\, (0{,}1\cancel{m})^2 = \frac{\sqrt{3}}{4}\,kg = \underline{0{,}43\,kg}$$

8.7



$h = 40\,cm$ $\qquad k = 12\ N/m$

$l_0 = 20\,cm$

$b = 20\,cm$

$d = 2\,cm$

$\rho_{Al} = 2{,}7\,g/cm^3 = 2{,}7\,\frac{10^{-3}\,kg}{(10^{-2}\,m)^3} = 2700\,\frac{kg}{m^3}$

$\rho_V = 1000\,kg/m^3$

$x = ?$

$$\sum \vec{F} = \underline{0}$$

$$\vec{F}_g + \vec{F}_{vzg} + \vec{F}_{vzm} = 0$$

x: $-mg + V_{pot}\,\rho_V \cdot g + kx = 0$

$$-V_{valja} \cdot \rho_{Al}\, g + V_{pot}\,\rho_V\, g + kx = 0$$

$$-\tfrac{1}{4}\pi d^2 b\, \rho_{Al}\, g + \tfrac{1}{4}\pi d^2 \cdot \underline{x}\, \rho_V\, g + k\,\underline{x} = 0$$

$$\frac{\cancel{kg}\,m/\cancel{s^2}}{\cancel{kg}/\cancel{s^2}} = m \qquad N/m = kg/s^2$$

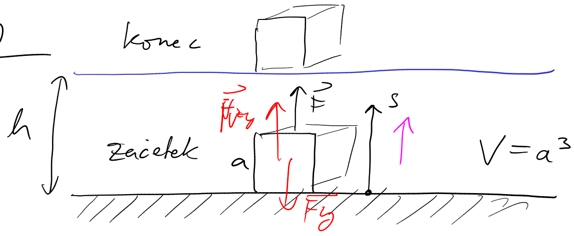
$$x = \frac{+\tfrac{1}{4}\pi d^2 b\, \rho_{Al}\, g}{\tfrac{1}{4}\pi d^2 \rho_V\, g + k} = \frac{\tfrac{1}{4}\pi (0{,}02\,\cancel{m})^2 \cdot 0{,}2\,\cancel{m} \cdot 2700\,kg/\cancel{m^3} \cdot 9{,}81\,m/s^2}{\tfrac{1}{4}\pi (0{,}02\,\cancel{m})^2\, 1000\,kg/\cancel{m^3} \cdot 9{,}81\,\cancel{m}/s^2 + 12\,N/m} =$$

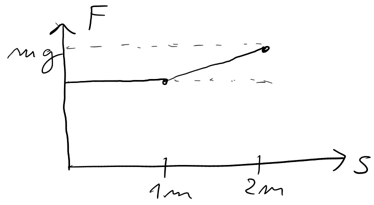
$$= \underline{0{,}11\ m}$$

8.10

$m = 2500\,kg$
$h = 2\,m$
$a = 1\,m$

$A = ?$





$$A = \int \vec{F} \cdot d\vec{s} = \int F\,ds$$

kocka je potopljena: $s \in [0, 1\,m]$

$$\vec{F}_g + \vec{F}_{vzg} + \vec{F} = 0$$
$$-mg + \rho_v \cdot V g + F = 0$$
$$F = mg - \rho_v a^3 g$$

kocka delno potopljena: $s \in [1\,m, 2\,m]$

$$\vec{F}_g + \vec{F}_{vzg} + \vec{F} = 0$$
$$-mg + \rho_v a^2 (h-s) g + F = 0$$
$$F = mg - \rho_v a^2 (h-s) g$$

$$A = \int_0^{h-a} (mg - \rho_v a^3 g)\,ds + \int_{h-a}^{h} (mg - \rho_v a^2 (h-s) g)\,ds =$$
$$= (mg - \rho_v a^3 g)(h-a) + (mg - \rho_v a^2 h g)\,a + \rho_v a^2 g \int_{h-a}^{h} s\,ds =$$
$$= mg(h - \cancel{a} + \cancel{a}) - \rho_v a^2 g (a(h-a) + ha) + \frac{\rho_v a^2 g}{2} (h^2 - (h-a)^2) =$$
$$= mgh - \rho_v a^2 g (2ah - a^2) + \frac{\rho_v a^2 g}{2} (\cancel{h^2} - \cancel{h^2} + 2ha - a^2) =$$
$$= mgh - \rho_v g a^2 \left[2ah - a^2 - ha + \frac{a^2}{2}\right] =$$
$$= mgh - \rho_v g a^2 \left[ah - \frac{a^2}{2}\right] =$$
$$= mgh - \rho_v g a^3 \left(h - \frac{a}{2}\right)$$

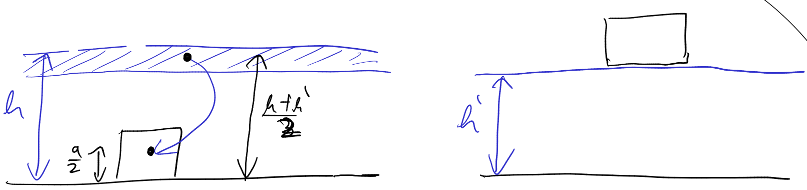
energijski zakon:

$$\Delta W = A$$
$$\Delta W_p = A$$
$$mgh - \rho_v a^3 \left(h - \frac{a}{2}\right) g = A$$

$$A = 2500\,kg \cdot 9{,}81\,m/s^2 \cdot 2\,m - 1000\,kg/m^3 \cdot 1\,m^3 \cdot 1{,}5\,m \cdot 9{,}81\,m/s^2 = 34{,}3\,kJ$$

$S = 7\,m^2$

$$A = \Delta W_p$$
$$A = mgh' - \rho_v a^3 \left[\frac{h+h'}{2} - \frac{a}{2}\right]$$



$V_v = Sh - a^3$ $\quad$ $V_v = Sh'$

$$Sh - a^3 = Sh' \rightarrow h' = h - \frac{a^3}{S}$$

$$A = mg\left(h - \frac{a^3}{S}\right) - \rho_v a^3 \left[h - \frac{a^3}{2S} - \frac{a}{2}\right] =$$
$$= 2500\,kg \cdot 9{,}81\,\frac{m}{s} \left[2\,m - \frac{1}{7}\,m\right] - 1000\,kg/m^3 \cdot 1\,m^3 \left[2\,m - \frac{1}{14}\,m - \frac{1}{2}\,m\right] 9{,}81\,\frac{m}{s^2} =$$
$$= 31{,}5\,kJ$$

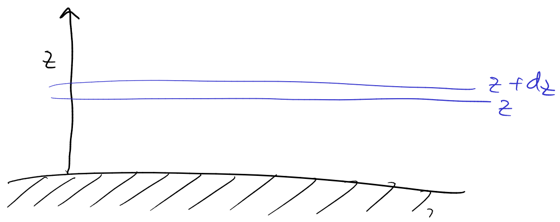
9.16

$g(z) = g = 9{,}81\,\mathrm{m/s^2}$

$p_0 = 10^5\,\mathrm{Pa}$

$T = 20^\circ C$

$p(z) = ?$



voda: $p = p_0 + \rho g h$

① $p(z) = p(z+dz) + \rho(z) g\, dz$

$\cancel{p(z)} = \cancel{p(z)} + dp + \rho(z) g\, dz$

$dp = -\rho(z) g\, dz$

$pV = \frac{m}{M} RT \quad / : V$

$p = \frac{m}{V}\frac{RT}{M} = \rho \frac{RT}{M}$, $\frac{m}{V} = \rho$

$\rho = \frac{pM}{RT}$

$dp = -\frac{p(z) M g}{RT} dz \quad / : p$

$\frac{dp}{p} = -\frac{Mg}{RT} dz \quad / \int$

$\int_{p_0}^{p(z)} \frac{dp}{p} = -\frac{Mg}{RT}\int_0^z dz$

$\ln \frac{p(z)}{p_0} = -\frac{Mg}{RT} z$

$p(z) = p_0 e^{-\frac{Mgz}{RT}}$

$\boxed{p_1(z) = p_0 e^{-\frac{z}{z_0}}}$

$T \neq T(z)$
$g \neq g(z)$

$z_0 = \frac{RT}{Mg} = \frac{8300\,\cancel{J} \cdot 293\,\cancel{K}\,\cancel{kmol}\,s^2}{\cancel{kmol}\,\cancel{K} \cdot 29\,kg \cdot 9{,}81\,m} =$

$= 8\,500\ \mathrm{m}$

$p_1(8500\,\mathrm{m}) = p_0 \cdot \frac{1}{e} \approx \frac{1}{3} p_0$

② $g(z) = g \frac{r_0^2}{(r_0+z)^2}$

$g = 9{,}81\,\mathrm{m/s^2}$

$r_0 = 6400\,\mathrm{km}$

$dp = -\rho(z) g(z)\, dz$

$dp = -\frac{pM}{RT} g \frac{r_0^2}{(r_0+z)^2} dz$

$\frac{dp}{p} = -\frac{Mg}{RT}\frac{r_0^2}{(r_0+z)^2} dz$, $\frac{Mg}{RT} = \frac{1}{z_0}$

$\int_{p_0}^{p(z)} \frac{dp}{p} = -\int_0^z \frac{r_0^2}{(r_0+z)^2}\frac{dz}{z_0}$

$\int \frac{du}{u^2} = -\frac{1}{u} + C$

$\ln \frac{p(z)}{p_0} = \frac{r_0^2}{z_0} \frac{1}{r_0+z}\Big|_0^z$

$\ln \frac{p(z)}{p_0} = \frac{r_0^2}{z_0}\left(\frac{1}{r_0+z} - \frac{1}{r_0}\right) =$

$= \frac{r_0^2}{z_0} \frac{r_0 - (r_0+z)}{(r_0+z) r_0} =$

$= \frac{r_0^2}{z_0} \frac{-z}{(r_0+z) r_0}$

$T \neq T(z)$

$\boxed{p_2(z) = p_0 e^{-\frac{r_0 z}{z_0 (r_0+z)}}}$

$p_2(z) = p_0 e^{-\frac{z}{z_0}\cdot\frac{r_0}{r_0+z}}$

$\frac{r_0}{r_0+z} = \frac{1}{1+\frac{z}{r_0}} \approx 1 - \frac{z}{r_0} + O\left(\left(\frac{z}{r_0}\right)^2\right)$

$z \ll r_0$

$\downarrow$

$p_2(z) \doteq p_0 e^{-\frac{z}{z_0}\left(1 - \frac{z}{r_0}\right)} =$

$= \underbrace{p_0 e^{-\frac{z}{z_0}}} e^{\frac{z^2}{z_0 r_0}} = p_1(z) \underbrace{e^{\frac{z^2}{z_0 r_0}}}_{\geq 1} \geq p_1(z)$