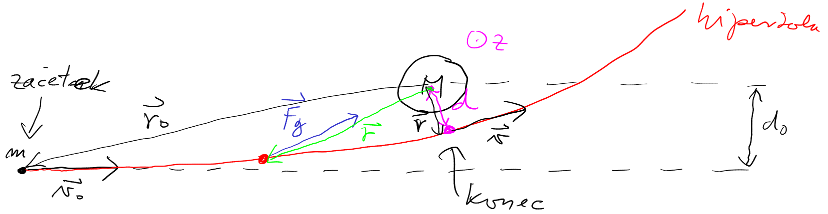
6.20

$M = 5 \times 10^{23}\ kg$

$v_0 = 6\ km/s$

$d_0 = 5000\ km$

$d = ?$



$$\Delta W_k + \Delta W_p = 0$$

$$\left(\frac{\cancel{m} v^2}{2} - \frac{\cancel{m} v_0^2}{2}\right) + \left(-\frac{GM\cancel{m}}{d} - \cancel{\left(-\frac{GMm}{\infty}\right)}\right) = 0$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} - \frac{GM}{d} = 0$$

$$\Delta \vec{\Gamma} = \int \vec{M}\, dt = 0$$

$$\vec{r} \parallel \vec{F}_g \rightarrow \vec{M} = 0 = \vec{r} \times \vec{F}_g$$

$$\vec{\Gamma} = \vec{r} \times m\vec{v}$$

$$\vec{r} \times m\vec{v} - \vec{r}_0 \times m\vec{v}_0 = 0$$

$$\downarrow \vec{r} \perp \vec{v}$$

$z:$ $$d \cancel{m} v - d_0 \cancel{m} v_0 = 0$$

$$d v - d_0 v_0 = 0 \rightarrow v = v_0 \frac{d_0}{d}$$

$$\frac{1}{2}\left(v_0 \frac{d_0}{d}\right)^2 - \frac{v_0^2}{2} - \frac{GM}{d} = 0 \quad / \cdot 2d^2$$

$$v_0^2 d_0^2 - v_0^2 d^2 - 2GMd = 0 \quad / \cdot \frac{1}{-v_0^2}$$

$$d^2 + \frac{2GM}{v_0^2} d - d_0^2 = 0$$

$$d = -\frac{GM}{v_0^2} \pm \sqrt{\left(\frac{GM}{v_0^2}\right)^2 + d_0^2}$$

$$\frac{N s^2}{kg} = \frac{\cancel{kg}\, m\, \cancel{s^2}}{\cancel{s^2}\, \cancel{kg}} = m$$

$$d = -\underbrace{\left(\frac{6{,}7 \cdot 10^{-11}\, N m^2 \cdot 5 \times 10^{23}\, \cancel{kg}}{kg^2 \cdot (6 \times 10^3\, m/s)^2}\right)}_{A} + \sqrt{A^2 + (5 \times 10^6\, m)^2}$$

$$d = 4155\ km$$

9.5

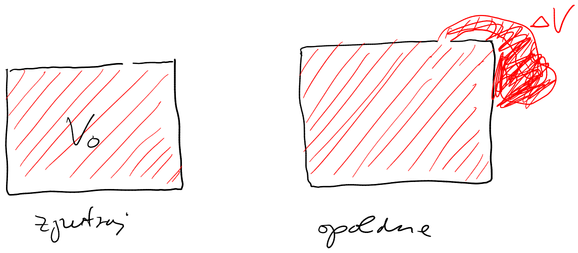
$V_0 = 5000\,\ell$

$\Delta T = 23^\circ C$

$\beta_m = 9{,}5 \times 10^{-5}\,K^{-1}$

$\alpha_j = 1{,}1 \times 10^{-5}\,K^{-1}$

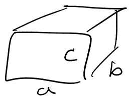
$\Delta V = ?$



$$\frac{\Delta V_m}{V_0} = \beta_m \Delta T \qquad \Delta V_m = V_0 \beta_m \Delta T$$

$$\frac{\Delta \ell}{\ell} = \alpha \Delta T \rightarrow \Delta \ell = \ell \alpha \Delta T$$

cisterna je kvader: $V_0 = abc \longrightarrow (a + a\alpha\Delta T)(b + b\alpha\Delta T)(c + c\alpha\Delta T) =$



$$= abc(1+\alpha\Delta T)(1+\alpha\Delta T)(1+\alpha\Delta T) =$$

$$= V_0(1+\alpha\Delta T)^3 =$$

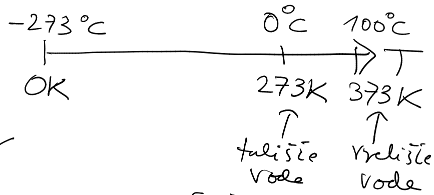
zanemarimo

$$= V_0(1 + 3\alpha\Delta T + \cancel{3(\alpha\Delta T)^2} + \cancel{(\alpha\Delta T)^3}) \doteq \qquad \alpha\Delta T \ll 1$$

$$\doteq V_0(1 + \underbrace{3\alpha}_{\beta_j = 3\alpha_j} \Delta T)$$

$$\frac{\Delta V_j}{V_0} = 3\alpha_j \cdot \Delta T \qquad \Delta V_j = V_0 \cdot 3\alpha_j \cdot \Delta T$$

$$\Delta V = \Delta V_m - \Delta V_j = V_0(\beta_m - 3\alpha_j)\Delta T$$



$$\Delta V = 5000\,\ell \cdot \left(\frac{9{,}5 \cdot 10^{-5}}{\cancel{K}} - 3\,\frac{1{,}1 \times 10^{-5}}{\cancel{K}}\right) \cdot 23\,\cancel{^\circ C}$$

$\Delta T\,[^\circ C] = \Delta T\,[K]$

$\Delta T = 23^\circ C = 23K$

$$\Delta V = 7{,}13\ \ell$$

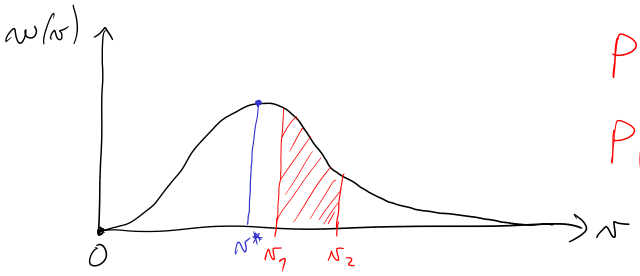
9.57 $\quad w(v) = A v^2 e^{-\frac{m v^2}{2k_BT}}$

$A = ?$

$\langle v \rangle = ?$

$\langle v^2 \rangle = ?$

$v^* = ?$ (najverjetnejša velikost hitrosti)



$$P(v_1 < v < v_2) = \int_{v_1}^{v_2} w(v)\,dv$$

$$P(0 < v < \infty) = \int_0^\infty w(v)\,dv = 1$$

$$\int_0^\infty w(v)\,dv = 1 = \int_0^\infty A v^2 e^{-\frac{m v^2}{2k_BT}}\,dv \qquad A = ?$$

$$1 = A\int_0^\infty v\,dv \cdot v \cdot e^{-\frac{m v^2}{2k_BT}} =$$

$$= A\int_0^\infty \frac{k_BT}{m}\,ds \cdot \sqrt{\frac{2k_BT}{m}}\sqrt{s}\, e^{-s} =$$

$$= A\sqrt{2}\left(\frac{k_BT}{m}\right)^{3/2} \int_0^\infty s^{1/2} e^{-s}\,ds =$$

$$= A\sqrt{2}\left(\frac{k_BT}{m}\right)^{3/2} \Gamma\left(\frac{3}{2}\right) =$$

$$\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}$$

$$= A\sqrt{\frac{\pi}{2}}\left(\frac{k_BT}{m}\right)^{3/2} = 1$$

$$A = \sqrt{\frac{2}{\pi}}\left(\frac{m}{k_BT}\right)^{3/2}$$

$$s = \frac{m v^2}{2k_BT} \rightarrow v = \sqrt{\frac{2k_BT}{m}s}$$

$$ds = \frac{m v\,dv}{k_BT}$$

$$v\,dv = \frac{k_BT}{m}\,ds$$

$$\int_0^\infty s^n e^{-s}\,ds = \Gamma(n+1)$$

$n = 0, 1, 2,$

$\Gamma(n+1)$

$\Gamma(n+1) = n\,\Gamma(n)$

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$$\longrightarrow \boxed{w(v) = \sqrt{\frac{2}{\pi}}\left(\frac{m}{k_BT}\right)^{3/2} v^2 e^{-\frac{m v^2}{BT}}}$$

$\left.\begin{matrix}\langle v \rangle = ? \\ \langle v^2 \rangle = ?\end{matrix}\right\}\ \langle v^n \rangle = ?$

$$\langle f(v) \rangle \quad \int_0^\infty w(v) f(v)\,dv$$

$$\langle v^n \rangle = \int_0^\infty w(v)\,v^n\,dv =$$

$$= A\int_0^\infty v^2 e^{-\frac{m v^2}{2k_BT}} v^n\,dv =$$

$$= A\int_0^\infty v\,dv \cdot v^{n+1} e^{-\frac{m v^2}{2k_BT}} =$$

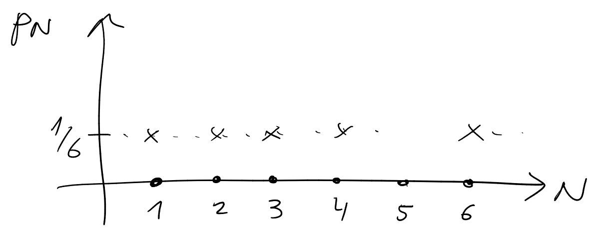
$$A\int_0^\infty \left(\frac{k_BT}{m}\right) ds \cdot \left(\frac{2k_BT}{m}\right)^{\frac{n+1}{2}} s^{\frac{n+1}{2}} e^{-s} =$$

$$= A\left(\frac{k_BT}{m}\right)^{\frac{n+3}{2}} 2^{\frac{n+1}{2}} \int_0^\infty s^{\frac{n+1}{2}} e^{-s}\,ds =$$

$$= \sqrt{\frac{2}{\pi}}\left(\frac{k_BT}{m}\right)^{-3/2}\left(\frac{k_BT}{m}\right)^{\frac{n+3}{2}} 2^{\frac{n+1}{2}}\,\Gamma\left(\frac{n+3}{2}\right) =$$

$$= \sqrt{\frac{2}{\pi}}\left(\frac{k_BT}{m}\right)^{\frac{n}{2}} 2^{\frac{n+1}{2}}\,\Gamma\left(\frac{n+3}{2}\right)$$

mečemo kocko



$$\langle N \rangle = \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \dots + \frac{1}{6}\cdot 6 =$$

$$= \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = 3{,}5$$

$$= \sum_{N=1}^{6} p_N \cdot N$$

$$\langle N^2 \rangle = \sum_{N=1}^{6} p_N \cdot N^2$$

$$\langle f(N) \rangle = \sum_{N=1}^{6} p_N \cdot f(N)$$

$$\langle v \rangle = \sqrt{\frac{2}{\pi}} \left(\frac{k_B T}{m}\right)^{1/2} 2^1 \, \Gamma(2) = \sqrt{\frac{2}{\pi}} \left(\frac{k_B T}{m}\right)^{1/2} 2 \cdot 1! = \underline{\sqrt{\frac{8 k_B T}{\pi m}} = \langle v \rangle}$$

$n=1$

$$\langle v^2 \rangle = \sqrt{\frac{2}{\pi}} \left(\frac{k_B T}{m}\right)^{2/2} 2^{3/2} \, \Gamma\left(\frac{5}{2}\right) = \frac{k_B T}{m} \cancel{\sqrt{\frac{2}{\pi}}} \cancel{2^{3/2}} \cdot \frac{3}{\cancel{2}} \cdot \frac{1}{\cancel{2}} \cdot \cancel{\sqrt{\pi}} =$$

$n=2$

$$\Gamma\left(\frac{5}{2}\right) = \frac{3}{2} \Gamma\left(\frac{3}{2}\right) = \frac{3}{2} \cdot \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{3}{2} \cdot \frac{1}{2} \cdot \sqrt{\pi}$$

$$= \underline{\frac{3 k_B T}{m} = \langle v^2 \rangle} \longrightarrow \langle W_k \rangle = \left\langle \frac{1}{2} m v^2 \right\rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

$$\frac{dw(v)}{dv} = 0$$

$$w(v) = A v^2 e^{-\frac{m v^2}{2 k_B T}} \qquad v \to v^*$$

$$w'(v^*) = \cancel{A}\left(2 \cancel{v} \cancel{e^{-\frac{m v^2}{2 k_B T}}} + v^2 \left(-\frac{m \cancel{v}}{k_B T}\right) \cancel{e^{-\frac{m v^2}{2 k_B T}}}\right) = 0$$

$$2 - \frac{m (v^*)^2}{k_B T} = 0 \longrightarrow \underline{v^* = \sqrt{\frac{2 k_B T}{m}}}$$

9.8

$m_0 = 5\,kg$

$V = 10\,l$

$p = 10^5\,Pa$

$T_1 = 30°C$ (poleti)

$T_2 = -20°C$ (pozimi)

$M = 58\,kg/kmol$

plinska enačba

$$pV = \frac{m}{M}RT$$

$R = 8300\,J/kmol\,K$

T v Kelvinih!

$$m = \frac{pVM}{RT}$$

$$m_1 = \frac{pVM}{RT_1}$$

$$m_2 = \frac{pVM}{RT_2}$$

masi, ki ostaneta v jeklenki, ko tlak pade na $10^5\,Pa$

$p = \frac{F}{S}$

$Pa = N/m^2$

masa plina, ki ga iz jeklenke dobimo poleti

$$\Delta m = (m_0 - m_1) - (m_0 - m_2) = m_2 - m_1$$

$$\Delta m = \frac{pVM}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right) = \frac{10^5\,Pa \cdot 10\,l \cdot \cancel{kmol}\,\cancel{K}\,58\,kg}{8300\,J \cdot \cancel{kmol}}\left(\frac{1}{253\,\cancel{K}} - \frac{1}{303\,\cancel{K}}\right) =$$

$$= \frac{10^5\,\cancel{N} \cdot 10^{-2}\,\cancel{m^3}\,58\,kg}{\cancel{m^2}\,8300\,\cancel{J}}\left(\frac{1}{253} - \frac{1}{303}\right) =$$

$$= \underline{4{,}55\,g}$$

9,10

$S = 100\ cm^2$
$V_0 = 1\ dm^3$
$p_0 = 10^5\ Pa$
$T_0 = 20°C$
$k = 0{,}1\ N/cm$
$T = 80°C$

$V = ?$

začetek

V_0, p_0, T_0

konec

V, p, T

$V - V_0 = S\Delta X$

$$p_0 V_0 = \frac{m}{M} R T_0 \quad /:T_0$$
$$\frac{p_0 V_0}{T_0} = \frac{mR}{M}$$

$$pV = \frac{m}{M} R T \quad /:T$$
$$\frac{pV}{T} = \frac{m}{M} R$$

$$\boxed{\frac{p_0 V_0}{T_0} = \frac{pV}{T}}$$
$$\rightarrow p = p_0 \frac{V_0}{T_0} \frac{T}{V}$$

$$pS - p_0 S = (p - p_0) S = k \Delta X$$
$$\boxed{(p - p_0) S = k \frac{V - V_0}{S}}$$

$$\left(p_0 \frac{V_0}{T_0} \frac{T}{V} - p_0\right) S = k \frac{V - V_0}{S} \quad /\cdot \frac{VS}{k}$$
$$\frac{p_0 S^2}{k}\left(\frac{V_0}{T_0} T - V\right) = V^2 - V_0 V$$
$$V^2 + V\left(\frac{p_0 S^2}{k} - V_0\right) - \frac{p_0 S^2}{k} \frac{V_0 T}{T_0} = 0$$

$$V = \frac{-\left(\frac{p_0 S^2}{k} - V_0\right) \pm \sqrt{\left(\frac{p_0 S^2}{k} - V_0\right)^2 + 4 \frac{p_0 S^2}{k} \frac{V_0 T}{T_0}}}{2}$$

$$A = \frac{p_0 S^2}{k} - V_0 = \frac{10^5 \cancel{Pa} \cdot (10^{-2} m^2)^2 m}{10 \cancel{N}\ m^2} - 10^{-3} m^3 = \underline{0{,}999\ m^3}$$

$$B = \frac{4 p_0 S^2}{k} \frac{V_0 T}{T_0} = 4 \cdot 1 m^3 \cdot 10^{-3} m^3 \cdot \frac{353 \cancel{K}}{293 \cancel{K}} = \underline{4 \cdot \frac{353}{293} 10^{-3} m^3}$$

$$\frac{p_0 S^2}{k} = 1 m^3$$

$$V = \frac{-0{,}999 m^3 \pm \sqrt{(0{,}999 m^3)^2 + 4 \cdot \frac{353}{293} 10^{-3} m^6}}{2} = \underline{\underline{1{,}2\ dm^3}}$$

9.11

$S = 100\,cm^2$

$h = 20\,cm$

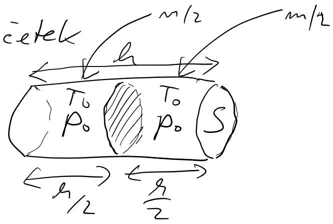
$p_0 = 10^5\,Pa$

$T_0 = 20°C$

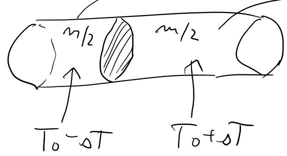
$\Delta T = 20°C$

$\frac{V_1}{V_2} = ?$

začetek



konec



$p_1 = p_2$

$V_1 \quad V_2$

$$p_1 V_1 = \frac{m/2}{M}(T_0 - \Delta T)$$

$$p_2 V_2 = \frac{m/2}{M}(T_0 + \Delta T)$$

$$: \qquad \frac{V_1}{V_2} = \frac{T_0 - \Delta T}{T_0 + \Delta T}$$

$$\frac{V_1}{V_2} = \frac{273\,\cancel{K}}{313\,\cancel{K}} = \underline{0{,}87}$$

9.12

$V = 10\ dm^3$

$p_0 = 10^5\ Pa$

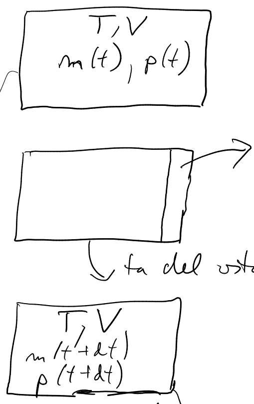
$t = 1\ min$

$\phi_V = 0{,}2\ dm^3/s$

$T = konst.$

$p = ?$

ob času t:



ta del zraka črpalka odstrani (dV)

$$\frac{dV}{dt} = \phi_V$$

ta del ostane v posodi in se razširi na celotno posodo

ob času t + dt

$p(t+dt) < p(t)$

$m(t+dt) < m(t)$

$$p(t+dt)V = \frac{m(t+dt)}{M}RT$$

$$p(t)V = \frac{m(t)}{M}RT$$

$$m(t+dt) = m(t)\left(1 - \frac{dV}{V}\right) = m(t)\left(1 - \frac{dV}{dt}dt\frac{1}{V}\right) = m(t)\left(1 - \frac{\phi_V}{V}dt\right)$$

$$p(t+dt) = p(t) + dp$$

$$\left(p(t) + dp\right)V = \frac{RT}{M}\left[m(t) - m(t)\frac{\phi_V}{V}dt\right]$$

$$\cancel{p(t)V} + dp\,V = \cancel{\frac{RT}{M}m(t)} - \frac{RT}{M}m(t)\frac{\phi_V}{V}dt$$

$p(t)\cdot V$

$$dp\,V = -p(t)\phi_V\,dt \quad / : p(t)$$

$$\frac{dp}{p(t)} = -\frac{\phi_V}{V}dt \quad / \int$$

$$\int_{p_0}^{p}\frac{dp'}{p'} = \int_0^t -\frac{\phi_V}{V}dt'$$

$$\ln p'\Big|_{p_0}^{p} = -\frac{\phi_V}{V}t'\Big|_0^t$$

$$\ln\frac{p}{p_0} = -\frac{\phi_V}{V}t$$

$$\boxed{p = p_0 e^{-\frac{\phi_V}{V}t}}$$

$$p = 10^5\,Pa \cdot e^{-\frac{0{,}2\,dm^3 \cdot 60\cancel{s}}{\cancel{s}\cdot 10\,dm^3}} = 10^5\,Pa \cdot e^{-1{,}2} = 30\ kPa$$