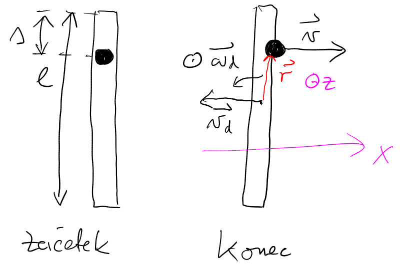
5.35



$v = 1\,m/s$
$l = 3\,m$
$s = 0{,}5\,m$
$m_d = 20\,kg$
$m = 60\,kg$

$v_d = ?$
$\omega_d = ?$
$A = ?$

$$\Delta \vec{G} = \int \vec{F}\,dt = 0$$

$$0 = \vec{G}_z = \vec{G}_k = m\vec{v} + m_d \vec{v}_d$$

$$x: \quad m v + m_d v_d = 0$$

$$v_d = -\frac{m}{m_d} v = -\frac{60\,kg}{20\,kg}\cdot 1\,m/s = \underline{\underline{-3\,m/s}}$$

$$\Delta \vec{\Gamma} = \int \vec{M}\,dt = 0$$

$$0 = \vec{\Gamma}_z = \vec{\Gamma}_k = \vec{r} \times m\vec{v} + J\vec{\omega}_d$$

vrtilno količino računamo glede na težišče palice

$$z: \quad -r m v + J\omega_d = 0 \qquad r = \frac{l}{2} - s \qquad J = \frac{1}{12} m_d l^2$$

$$\omega_d = \frac{\left(\frac{l}{2} - s\right) m v}{\frac{1}{12} m_d l^2}$$

$$\omega_d = \frac{1\cancel{m}\cdot 60\cancel{kg}\cdot 1\cancel{m}/s}{\frac{1}{12}\cdot 20\cancel{kg}\cdot 3^2\cancel{m^2}} = \underline{\underline{4\,s^{-1}}}$$

$$A = \Delta W = W_k - \cancel{W_z}^{\,0} = W_k = \frac{1}{2} m v^2 + \frac{1}{2} m_d v_d^2 + \frac{1}{2} J \omega_d^2 =$$

$$= \frac{1}{2}\, 60\,kg\,(1\,m/s)^2 + \frac{1}{2}\, 20\,kg\cdot(3\,m/s)^2 + \frac{1}{2}\cdot\frac{1}{12}\, 20\,kg\cdot(3\,m)^2\cdot(4\,s^{-1})^2 =$$

$$= 30\,J + 90\,J + 120\,J = \underline{\underline{240\,J}}$$

6.4 $\vec{F}_g = \frac{G \cdot m_1 \cdot m_2}{r^3}\vec{r}$ $\qquad F_g = \frac{G m_1 m_2}{r^2}$; $W_p = -\frac{G m_1 m_2}{r}$

$$G = 6{,}7 \times 10^{-11}\ \frac{Nm^2}{kg^2}$$

$R = 6400\ km$

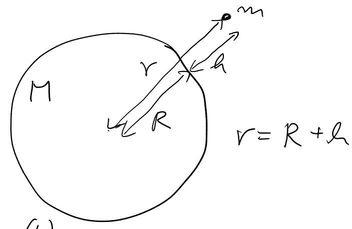
$g(0) = 9{,}81\ m/s^2$

$g(h) = ?$

$g(h_1) = \frac{1}{2} g(0) \qquad h_1 = ?$

$\Delta W_{pot} \sim mgh$ za $h \ll R$

$$F_g = \frac{GMm}{r^2} = m\,g(h)$$

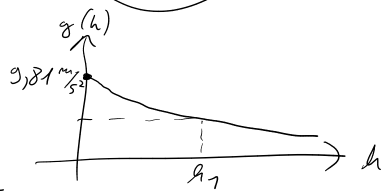


$r = R + h$

$$g(h) = \frac{GM}{(R+h)^2}$$

$$g(h) = \underbrace{\frac{GM}{R^2}}_{g(0)} \frac{R^2}{(R+h)^2}$$

$$g(h) = g(0)\left(\frac{R}{R+h}\right)^2$$



$$g(h_1) = \frac{1}{2} g(0) = g(0)\left(\frac{R}{R+h_1}\right)^2$$

$$\frac{1}{2} = \left(\frac{R}{R+h_1}\right)^2 \longrightarrow h_1 = R\sqrt{2} - R = R(\sqrt{2}-1) = 6400\,km\cdot(\sqrt{2}-1) =$$

$$= 2655\ km$$

$$\Delta W_{pot} = W_{pot}(h) - W_{pot}(0) =$$

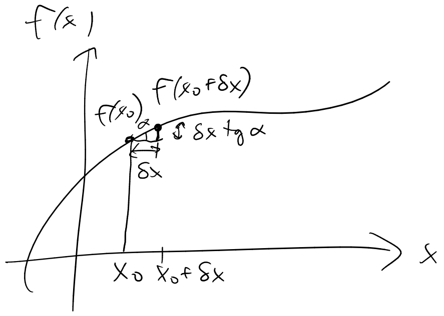
$$= -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R} - \frac{GMm}{R+h} = \frac{GM}{R^2} m \cdot R - \frac{GM}{R^2}\cdot m \cdot \frac{R^2}{R+h} =$$

$$= m\,g(0)\left(R - \frac{R^2}{R+h}\right) = m\,g(0)\,R\left(1 - \frac{R}{R+h}\right) =$$

$$= m\,g(0)\,R\left(1 - \frac{1}{1+\frac{h}{R}}\right) \doteq m\,g(0)\,\cancel{R}\left(\cancel{1} - \left(\cancel{1} - \frac{h}{\cancel{R}}\right)\right) = m\,g(0)\,h$$

$h \ll R$

$\frac{h}{R} \ll 1$



$$f(x_0+\delta x) \doteq f(x_0) + \underbrace{tg\,\alpha}_{f'(x_0)} \cdot \delta x$$

$$f(x_0+\delta x) \doteq f(x_0) + f'(x_0)\cdot\delta x + \frac{1}{2} f''(x_0)\,\delta x^2 + \dots$$

Taylorjev razvoj

$$f(x) = \frac{1}{1+x}$$

$x_0 = 0$

$$f(x) \doteq f(0) + f'(0)\cdot x = 1 + (-1)x = 1 - x$$

$$f'(x) = -\frac{1}{(1+x)^2}$$

6.5 $R = 6400\ km$

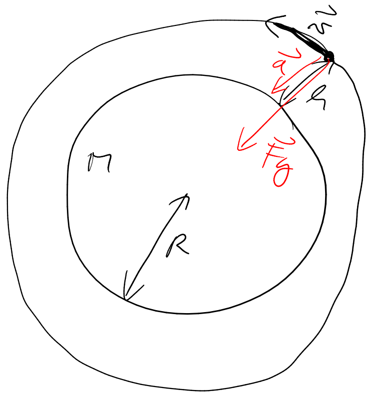
$M = 6 \times 10^{24}\ kg$

$v_1 = 5\ km/s$

a) $v = ?$ za $h \ll R$ (1. kozmična hitrost)

b) $h_1 = ?$ $(v = v_1)$

c) $h_2 = ?$ (geostacionaren satelit)



$r = R + h$

$\vec{a} = \vec{a}_R$ $\qquad a_R = \frac{v^2}{r} = r\omega^2$

a) $h \ll R$

$\sum \vec{F} = m\vec{a}$

$\vec{F}_g = m\vec{a} \quad \rightarrow \quad F_g = ma$

$\cancel{m}\, g(h) = \cancel{m}\, a_R$

$\boxed{a_R = g(h)}$

$h \ll R \rightarrow g(h) \doteq g(0) = g$

$$\frac{v^2}{R+h} \doteq \frac{v^2}{R} = g \qquad v = \sqrt{gR} = \sqrt{9{,}81\ m/s^2 \cdot 6{,}4 \cdot 10^6\ m} =$$

$$\doteq \underline{8 \cdot 10^3\ m/s} = \underline{8\ km/s}$$

b) $a_R = g(h)$

$$\frac{v_1^2}{R+h_1} = g\left(\frac{R}{R+h_1}\right)^2 = g\frac{R^2}{(R+h_1)^2}$$

$$v_1^2 = g\frac{R^2}{R+h_1} \longrightarrow h_1 = \frac{gR^2}{v_1^2} - R = R\left(\frac{gR}{v_1^2} - 1\right) =$$

$$= 6400\ km \left(\frac{9{,}81\ \cancel{m}/\cancel{s^2} \cdot 6{,}4 \cdot 10^6\ \cancel{m}}{5000^2\ \cancel{m^2}/\cancel{s^2}} - 1\right) =$$

$$= \underline{9670\ km}$$

c) $\omega = \omega_z = \frac{2\pi}{T_z}$ $\qquad T_z = 1$ dan

$a_R = g(h_2)$

$$(R+h_2)\omega^2 = g\frac{r^2}{(R+h_2)^2}$$

$$\left(\frac{2\pi}{T_z}\right)^2 = \frac{gR^2}{(R+h_2)^3} \longrightarrow h_2 = \sqrt[3]{\frac{gR^2}{(2\pi/T_z)^2}} - R = \sqrt[3]{\frac{gR^2 4\pi^2}{T_z^2}} - R$$

$$h_2 = R\left(\sqrt[3]{\frac{gT_z^2}{4\pi^2 R}} - 1\right)$$

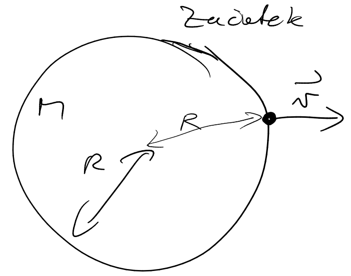
$$h_2 = 6400\ km \left(\sqrt[3]{\frac{9{,}81\ \cancel{m}\ (24 \cdot 60 \cdot 60)^2 \cancel{s^2}}{\cancel{s^2} \cdot 4\pi^2 \cdot 6{,}4 \cdot 10^6\ \cancel{m}}} - 1\right) =$$

$$= \underline{\underline{35950\ km}}$$

6.6

$R = 6400\,km$
$M = 6\times10^{24}\,kg$

$v = ?$ (2. kozmična hitrost)
$R_1 = ?$ $(v = c)$
$c = 3\times10^{8}\,m/s$



$\Delta W_k + \Delta W_p = 0$

$$\left(0 - \tfrac{1}{2}\cancel{m}v^2\right) + \left(\underbrace{\cancel{-\frac{GM\cancel{m}}{\infty}}}_{=0} - \left(-\frac{GM\cancel{m}}{R}\right)\right) = 0$$

$\cancel{W_p = mgh}$

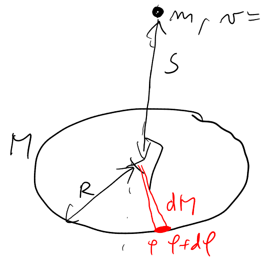
$$v = \sqrt{\frac{2GM}{R}} = \sqrt{2\underbrace{\frac{GM}{R^2}}_{g}R} = \sqrt{2gR}$$

$$v \doteq \sqrt{2\cdot 9{,}81\,\tfrac{m}{s^2}\cdot 6{,}4\cdot10^6\,m} = 11{,}3\ km/s$$

$$c = \sqrt{\frac{2GM}{R_1}} = \sqrt{2\underbrace{\frac{GM}{R^2}}_{g}\frac{R^2}{R_1}} = \sqrt{2g\frac{R^2}{R_1}} = \sqrt{2gR\frac{R}{R_1}} = \underbrace{\sqrt{2gR}}_{=v}\sqrt{\frac{R}{R_1}}$$

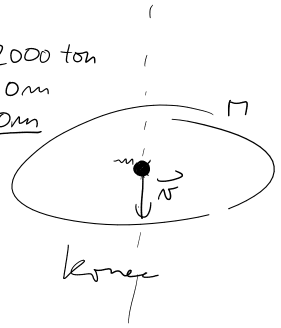
$$c = v\sqrt{\frac{R}{R_1}} \longrightarrow R_1 = R\left(\frac{v}{c}\right)^2 = 6400\,km\left(\frac{11{,}3\ \cancel{km/s}}{3\cdot10^5\ \cancel{km/s}}\right)^2 = 8{,}9\,mm$$

6.10



začetek

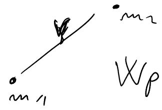
$M = 2000$ ton
$R = 10$ m
$s = 50$ m
$v = ?$



konec

$$\Delta W = \Delta W_k + \Delta W_p = 0$$

$$\left(\frac{1}{2}\cancel{m}v^2 - 0\right) + \left(-\frac{G\cancel{m}M}{R} - \left(-\frac{G\cancel{m}M}{\sqrt{R^2+s^2}}\right)\right) = 0$$



$$W_p = -\frac{G m_1 m_2}{r}$$

$$W_p = \int -\frac{G m\, dM}{\sqrt{R^2+s^2}} = \int_0^{2\pi} -\frac{G m M \frac{d\varphi}{2\pi}}{\sqrt{R^2+s^2}} =$$

$$dM = M \cdot \frac{d\varphi}{2\pi}$$

$$= -\frac{G m M}{\sqrt{R^2+s^2}} \cancel{\frac{1}{2\pi}} \underbrace{\cancel{\int_0^{2\pi} d\varphi}}_{2\pi} = -\frac{G m M}{\sqrt{R^2+s^2}}$$

$$\frac{1}{2}v^2 - \frac{GM}{R} + \frac{GM}{\sqrt{R^2+s^2}} = 0$$

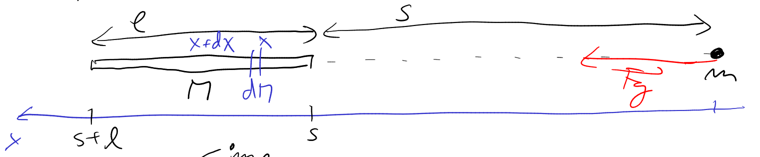
$$\frac{Nm}{kg} = \frac{\cancel{kg}\, m \cdot m}{s^2 \cdot \cancel{kg}} = \frac{m^2}{s^2}$$

$$v = \sqrt{\frac{2GM}{R}\left(1 - \frac{R}{\sqrt{R^2+s^2}}\right)}$$

$$v = \sqrt{\frac{2GM}{R}\left(1 - \frac{1}{\sqrt{1+\left(\frac{s}{R}\right)^2}}\right)} = \sqrt{\frac{2 \cdot 6{,}7 \times 10^{-11}\, Nm^2 \cdot 2 \times 10^6\, \cancel{kg}}{kg^{\cancel{2}} \cdot 10\, \cancel{m}}\left(1 - \frac{1}{\sqrt{1+\left(\frac{50\,\cancel{m}}{10\,\cancel{m}}\right)^2}}\right)} =$$

$$= \underline{\underline{4{,}6 \text{ mm/s}}}$$

6.11



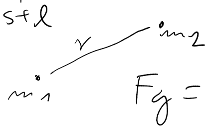
$l = 10\,m$

$s = 20\,m$

$M = 1000\,kg$

$m = 1\,kg$

$F_g = ?$



$$F_g = \frac{G m_1 m_2}{r^2}$$

$$F_g = \int dF_g = \int \frac{G m \cdot dM}{x^2} = \qquad dM = \frac{dx}{l} M$$

$$= \int_s^{s+l} \frac{G m \cdot \frac{dx}{l} M}{x^2} =$$

$$= \frac{G m M}{l} \int_s^{s+l} \frac{dx}{x^2} = \frac{G m M}{l} \left(-\frac{1}{x}\right)\Big|_s^{s+l} =$$

$$= \frac{G m M}{l}\left(-\frac{1}{s+l} + \frac{1}{s}\right) =$$

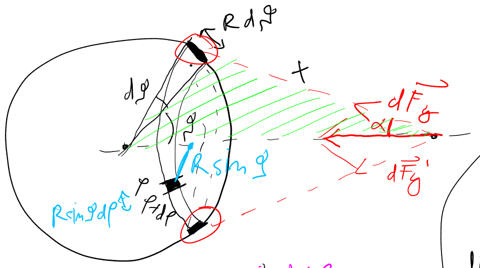
$$= \frac{G m M}{\cancel{l}} \, \frac{\cancel{-s} + \cancel{s} + \cancel{l}}{s(s+l)} = \frac{G m M}{s(s+l)}$$

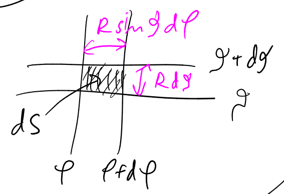
$$F_g = \frac{6{,}7 \times 10^{-11}\, N m^2 \cdot 1\,kg \cdot 1000\,kg}{kg^2 \cdot 20\,m \cdot (20\,m + 10\,m)} = \underline{\underline{1{,}1 \times 10^{-10}\ N}}$$

6.16

$\vec{F}_g = m\vec{g}$

$g(r) = ?$

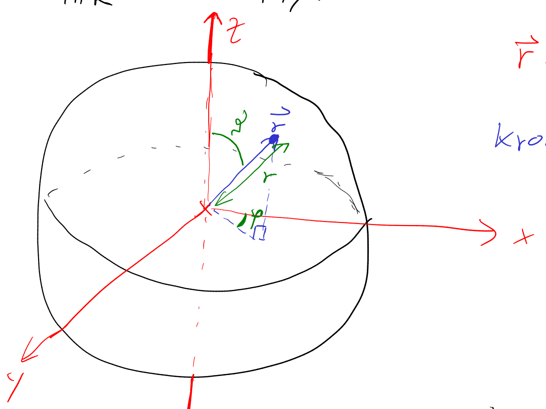




$\vec{F}_g = \int d\vec{F}_g$

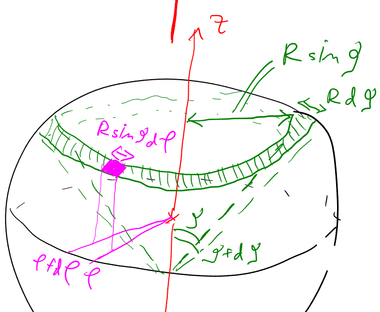
$F_g = \int dF_g \cos\alpha$

$$dM = M \cdot \frac{dS}{4\pi R^2} = M\,\frac{R^2 \sin\vartheta\, d\vartheta\, d\varphi}{4\pi R^2} = \frac{M}{4\pi}\sin\vartheta\, d\vartheta\, d\varphi$$



$\vec{r} = (x, y, z)$ Kartezične koordinate

krogelne koordinate (r, ϑ, φ)



$\vartheta \in [0, \pi]$

$\varphi \in [0, 2\pi)$

$$F_g = \int dF_g \cos\alpha =$$

$$= \int \frac{Gm\,dM}{x^2}\cos\alpha =$$

$$= \int_0^{\pi}\int_0^{2\pi} \frac{GmM\cdot\frac{1}{4\pi}\sin\vartheta\, d\vartheta\, d\varphi}{r^2+R^2-2rR\cos\vartheta}\cos\alpha =$$

= prelet

lažja pot:

$$W_{pot} = \int -\frac{Gm\,dM}{x} =$$

$$= \int_0^{\pi}\int_0^{2\pi} \frac{-Gm\,\frac{1}{4\pi}M\sin\vartheta\, d\vartheta\, d\varphi}{\sqrt{r^2+R^2-2rR\cos\vartheta}} =$$

$$\int_0^{2\pi} d\varphi = 2\pi$$

$$= -\frac{1}{2}GmM\int_0^{\pi}\frac{\sin\vartheta\, d\vartheta}{\sqrt{r^2+R^2-2rR\cos\vartheta}} =$$

$$r^2+R^2-2rR\cos\vartheta = u$$

$$2rR\sin\vartheta\, d\vartheta = du$$

$$\sin\vartheta\, d\vartheta = \frac{du}{2rR}$$

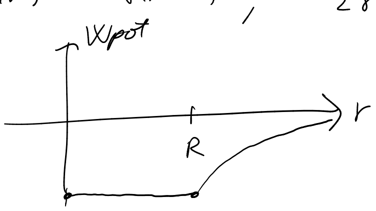
$$= -\frac{1}{2}\frac{GmM}{2rR}\int_{(r-R)^2}^{(r+R)^2}\frac{du}{\sqrt{u}} =$$

$$\vartheta = 0 \rightarrow u = r^2+R^2-2rR = (r-R)^2$$

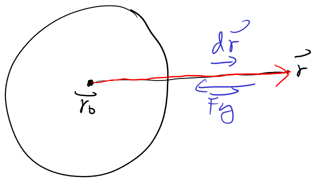
$$\vartheta = \pi \rightarrow u = (r+R)^2$$

$$= -\frac{1}{\cancel{2}}\frac{GmM}{2rR}\,\cancel{2}\sqrt{u}\,\Big|_{(r-R)^2}^{(r+R)^2} =$$

$$= -\frac{GmM}{2rR}\left(\sqrt{(r+R)^2}-\sqrt{(r-R)^2}\right) = -\frac{GmM}{2rR}\left(r+R-|r-R|\right) = \begin{cases} r>R; & -\dfrac{GmM}{r} \\ r<R; & -\dfrac{GmM}{R} \end{cases}$$



$$W_{pot}(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}_g \cdot d\vec{r}$$



$$W_{pot} = + \int_0^r F_g(r') dr' \quad / \frac{d}{dr}$$

$$\frac{d}{dr} W_{pot} = \frac{d}{dr} \int_0^r F_g(r') d\vec{r}\,' = F_g(r)$$

$$F_g(r) = \frac{d}{dr} W_{pot}(r)$$

$$F_g(r) = \begin{cases} r < R; & 0 \\ r > R; & \frac{GmM}{r^2} \end{cases} = m\, g(r)$$

$$g(r) = \begin{cases} r < R; & 0 \\ r > R; & \frac{GM}{r^2} \end{cases}$$