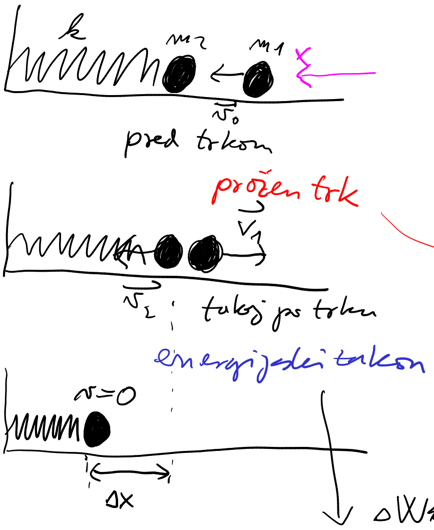
5.10



$m_1 = 50\,g$

$m_2 = 180\,g$

$v_0 = 0{,}8\,m/s$

$k = 20\,N/m$

$\Delta x = ?$

$\Delta W_k = 0$

$W_k^{pred} = W_k^{po}$

$\Delta \vec{G} = 0$

$\vec{G}^{pred} = \vec{G}^{po}$

$$\left.\begin{aligned} \cancel{\tfrac{1}{2}} m_1 v_0^2 &= \cancel{\tfrac{1}{2}} m_1 v_1^2 + \cancel{\tfrac{1}{2}} m_2 v_2^2 \\ m_1 v_0 &= m_1 v_1 + m_2 v_2 \end{aligned}\right\} v_2 = ?$$

$$v_1 = v_0 - \frac{m_2}{m_1} v_2$$

$$m_1 v_0^2 = m_1 \left(v_0 - \frac{m_2}{m_1} v_2\right)^2 + m_2 v_2^2$$

$$\cancel{m_1 v_0^2} = \cancel{m_1 v_0^2} - 2 \cancel{m_2} v_0 \cancel{v_2} + \frac{m_2^{\cancel{2}}}{m_1} v_2^{\cancel{2}} + \cancel{m_2} v_2^{\cancel{2}}$$

$$0 = -2 v_0 + \frac{m_2}{m_1} v_2 + v_2$$

$$v_2 = \frac{2 v_0}{1 + \frac{m_2}{m_1}} = \frac{2 m_1 v_0}{m_1 + m_2}$$

$$\Delta W_k + W_{pr} = 0$$

$$-\frac{1}{2} m_2 v_2^2 + \frac{1}{2} k \Delta x^2 = 0$$

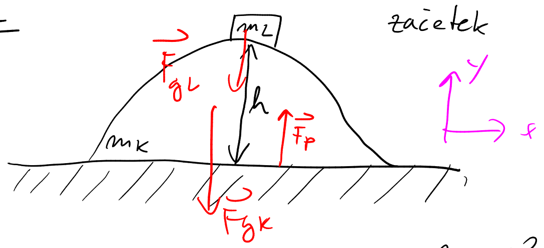
$$\Delta x = \sqrt{\frac{m_2}{k}}\, v_2$$

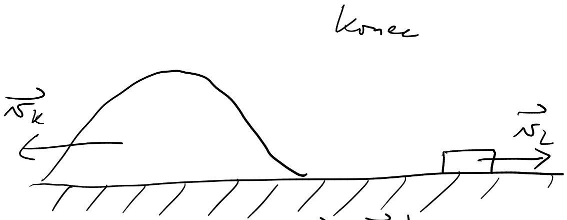
$$\Delta x = \sqrt{\frac{m_2}{k}} \cdot \frac{2 m_1 v_0}{m_1 + m_2} =$$

$$= \sqrt{\frac{0{,}18\,kg}{20\,N/m}} \cdot \frac{2 \cdot 50\,\cancel{g}}{230\,\cancel{g}} \cdot 0{,}8\,m/s =$$

$$= 0{,}033\,m$$

5.12





$$\Delta v = |\vec{v}_L - \vec{v}_K|$$

$m_K = 3\,kg$

$m_L = 1\,kg$

$\Delta v = 4\,m/s$

$h = ?$

$$\Delta W = \Delta W_k + \Delta W_p = 0$$

$$\Delta \vec{G} = \int \vec{F} \cdot dt \longrightarrow x: \Delta G_x = \int F_x\, dt = 0$$

$$\frac{1}{2} m_K v_K^2 + \frac{1}{2} m_L v_L^2 - m_L g h = 0$$

$$(m_L v_L + m_K v_K) - 0 = 0$$

$$v_K = -\frac{m_L}{m_K} v_L$$

$$\Delta v = v_L - v_K$$

$$\Delta v = v_L\left(1 + \frac{m_L}{m_K}\right)$$

$$\frac{1}{2} m_K \left(-\frac{m_L}{m_K} v_L\right)^2 + \frac{1}{2} m_L v_L^2 = m_L g h$$

$$\frac{1}{2} \frac{m_L^{\cancel{2}}}{m_K} v_L^2 + \frac{1}{2} \cancel{m_L} v_L^2 = \cancel{m_L} g h$$

$$\frac{1}{2}\left(\frac{m_L}{m_K} + 1\right) v_L^2 = g h \longrightarrow v_L = \sqrt{\frac{2gh}{1 + \frac{m_L}{m_K}}} = \sqrt{\frac{2 m_K g h}{m_L + m_K}}$$

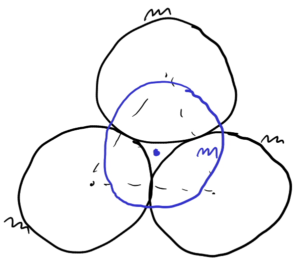
$$\Delta v = \sqrt{\frac{2 \cancel{m_K} g h}{\cancel{m_L + m_K}}} \cdot \sqrt{\frac{(m_L + m_K)^{\cancel{2}}}{(m_K)^{\cancel{2}}}} = \sqrt{\frac{2(m_L + m_K) g h}{m_K}}$$

$$h = \frac{m_K \Delta v^2}{2(m_K + m_L) g} = \frac{3\,\cancel{kg} \cdot 16\,m^{\cancel{2}}\,\cancel{s^2}}{\cancel{s^2} \cdot 2 \cdot 4\,\cancel{kg} \cdot 10\,\cancel{m}} = \underline{\underline{0{,}6\,m}}$$

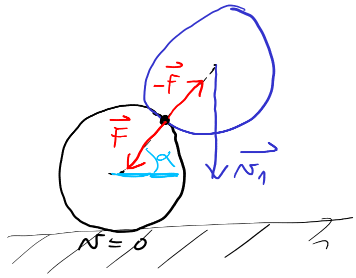
5.19

$v_2 = ?$

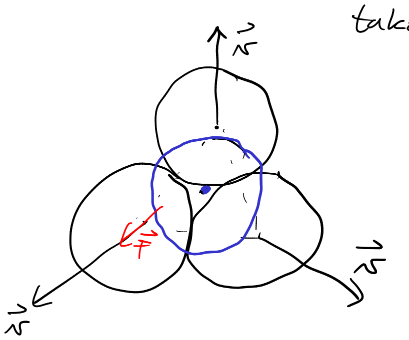
$v = 2\ m/s$

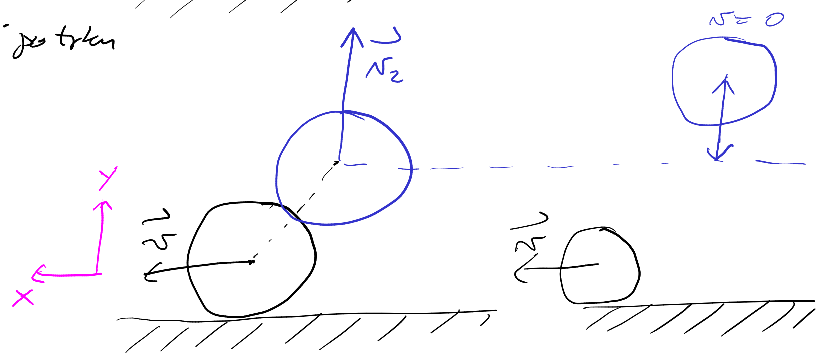


tik pred trkom



takoj po trku





$$W_k^{pred} = W_k^{po} \longrightarrow \frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 + 3 \cdot \frac{1}{2} m v^2$$

$$\boxed{v_1^2 = v_2^2 + 3v^2}$$

črna kroglica: x: $G_x^{po} - G_x^{pred} = \int F_x\, dt$

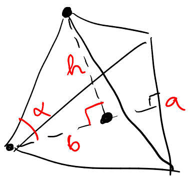
$$mv - 0 = \int F \cos\alpha\, dt = \cos\alpha \int F\, dt$$

modra kroglica: y: $G_y^{po} - G_y^{pred} = \int F_y\, dt$

$$mv_2 - mv_1 = 3\int F \sin\alpha\, dt = 3\sin\alpha \int F\, dt$$

:

$$\boxed{\frac{v_2 - v_1}{v} = 3\, tg\,\alpha} = 3\sqrt{2}$$



$tg\,\alpha = \frac{h}{b}$

$b = \frac{a\sqrt{3}}{2} \cdot \frac{2}{3} = \frac{a}{\sqrt{3}}$

$b^2 + h^2 = a^2$

$h = \sqrt{a^2 - b^2} = \sqrt{a^2 - \frac{a^2}{3}} = \sqrt{\frac{2a^2}{3}} = a\sqrt{\frac{2}{3}}$

$$tg\,\alpha = \frac{a\sqrt{\frac{2}{3}}}{a/\sqrt{3}} = \sqrt{2}$$

$$v_1^2 = v_2^2 + 3v^2$$

$$v_2 = v_1 + 3\sqrt{2}\, v$$

$$v_1^2 = (v_1 + 3\sqrt{2}\, v)^2 + 3v^2 = v_1^2 + 6\sqrt{2}\, v_1 v + 18 v^2 + 3v^2$$

$$0 = 6\sqrt{2}\, \cancel{v}\, v_1 + 21 v^{\cancel{2}}$$

$$v_1 = -\frac{21}{6\sqrt{2}} v = -\frac{7}{2\sqrt{2}} v = -\frac{7}{2\sqrt{2}} \cdot 2\ m/s = \underline{-4{,}9\ m/s}$$

$h = ?$ $\Delta W_k + \Delta W_p = 0$ (za modro kroglico)

$$-\frac{1}{2} \cancel{m} v_2^2 + \cancel{m} g h = 0$$

$$h = \frac{v_2^2}{2g}$$

$$v_2^2 = v_1^2 - 3v^2$$

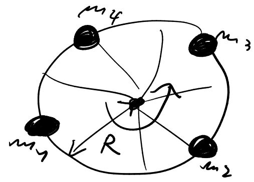
$$h = \frac{v_1^2 - 3v^2}{2g} = \frac{(-4{,}9\ m/s)^2 - 3\,(2\ m/s)^2}{2 \cdot 9{,}81\ m/s^2} = \underline{0{,}6\ m}$$

3.33

$F = ma$

$M = J\alpha \rightarrow$ kotni pospešek

navor, vztrajnostni moment



$\nu_0 = 0{,}3\ \text{Hz}$

$M = -100\ \text{N/m}$

$J_0 = 500\ \text{kg}\,\text{m}^2$

$m_{1,2,3,4} = 25, 30, 35, 40\ \text{kg}$

$R = 1{,}5\ \text{m}$

$N = ?$

$$\omega_k^2 = \omega_z^2 + 2\alpha\varphi$$

$$0 = (2\pi\nu_0)^2 + 2\alpha(2\pi N)$$

$$N = -\frac{\pi\nu_0^2}{\alpha}$$

$$M = (J_0 + m_1R^2 + m_2R^2 + m_3R^2 + m_4R^2)\alpha$$

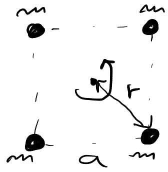
$$\alpha = \frac{M}{J_0 + (m_1 + m_2 + m_3 + m_4)R^2}$$

$$N = -\frac{\pi\nu_0^2\left(J_0 + \left(\sum_i m_i\right)R^2\right)}{M} = \frac{\pi\, 0{,}3^2\left(500\ \cancel{\text{kg}\,\text{m}^2} + 130\ \cancel{\text{kg}} \cdot 1{,}5^2\ \cancel{\text{m}^2}\right)}{\cancel{\text{s}^2} \cdot 100\ \cancel{\text{Nm}}} =$$

$$= 2{,}24$$

3.35

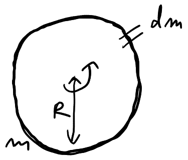
a)



$$J = \sum_i J_i = \sum_i m_i r_i^2 = 4 \cdot m \cdot \underbrace{\left(\frac{a\sqrt{2}}{2}\right)}_{=r}^2 = 2ma^2$$

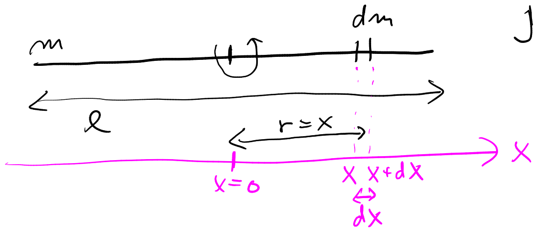
$$J = \int dJ = \int dm\, r^2$$

b)



$$J = \int dm \cdot R^2 = R^2 \int dm = R^2 m$$

c)

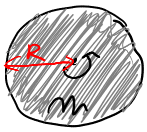


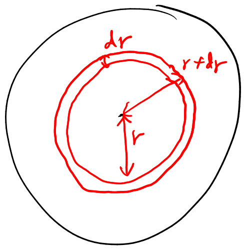
$$J = \int dm\, r^2 = \int_{-l/2}^{l/2} \underbrace{\frac{dx}{l} m}_{dm} \cdot x^2 = \frac{m}{l} \frac{x^3}{3}\Big|_{-l/2}^{l/2} =$$

$$= \frac{m}{3l}\left(\left(\frac{l}{2}\right)^3 - \left(-\frac{l}{2}\right)^3\right) = \frac{m}{3l} 2 \cdot \frac{l^3}{8} =$$

$$= \frac{1}{12} m l^2$$

d)





ploščina kolobarja

$$J = \int dm\, r^2 = \int \underbrace{\frac{dS}{\pi R^2} m}_{dm}\, r^2$$

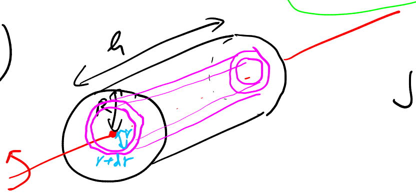
ploščina kroga

$$dS = \pi (r+dr)^2 - \pi r^2 =$$

$$= \cancel{\pi r^2} + 2\pi r\, dr + \underbrace{\cancel{dr^2}}_{\text{zanemarim}} - \cancel{\pi r^2} \qquad dS = 2\pi r\, dr$$

$$J = \int_0^R \frac{2\pi r\, dr}{\pi R^2} m\, r^2 = \frac{2m}{R^2} \int_0^R r^3 dr = \frac{2m}{R^2} \frac{r^4}{4}\Big|_0^R = \frac{2m}{R^2} \frac{R^4}{4} = m \frac{R^2}{2}$$

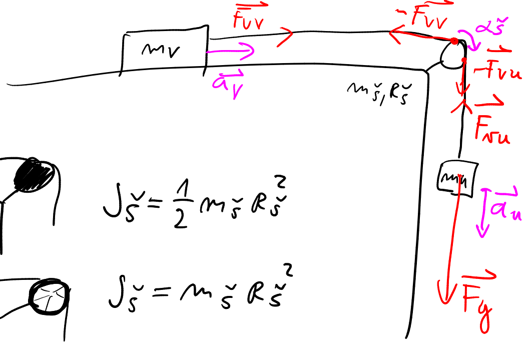
e)



$$J = \int dm\, r^2 = \int \underbrace{\frac{dV}{\pi R^2 h} m}_{dm} \cdot r^2 = \int_0^R \frac{2\pi r\, dr\, \cancel{h}}{\pi R^2 \cancel{h}} m\, r^2 = \frac{1}{2} m R^2$$

$$dV = 2\pi r\, dr \cdot h$$

3,36



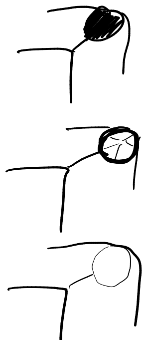
$m_v = 100g$

$m_{\check s} = 14g$

$R_{\check s} = 2cm$

$m_u = 10g$

$a = ?$



a) $J_{\check s} = \frac{1}{2} m_{\check s} R_{\check s}^2$

b) $J_{\check s} = m_{\check s} R_{\check s}^2$

c) $J_{\check s} = 0$

$m_{\check s} = 0$

$|\vec F_{vv}| \neq |\vec F_{vu}|$

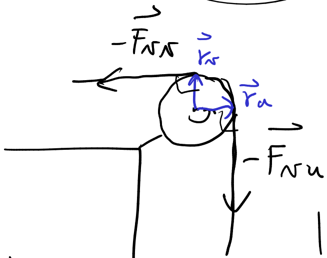
$a = |\vec a_u| = |\vec a_v| = a_T = \alpha_{\check s} R_{\check s}$

utež: 2NZ $\boxed{m_u g - F_{vu} = m_u a}$

vozilo: 2NZ $\boxed{F_{vv} = m_v a}$

škripec: $\sum \vec M = J \vec \alpha$

$\vec M = \vec r \times \vec F$



$\vec M = \vec r_u \times (-\vec F_{vu}) + \vec r_v \times (-\vec F_{vv})$

$|\vec M| = R_{\check s} F_{vu} - R_{\check s} F_{vv}$

$$R_{\check s}(F_{vu} - F_{vv}) = J_{\check s} \cdot \alpha_{\check s}$$

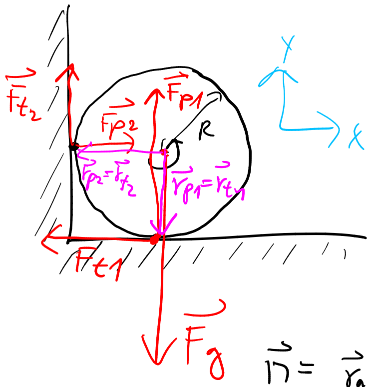
$$R_{\check s}(m_u(g-a) - m_v a) = J_{\check s} \cdot \frac{a}{R_{\check s}}$$

$$a\left(m_u + m_v + \frac{J_{\check s}}{R_{\check s}^2}\right) = m_u g$$

$$a = \frac{m_u g}{m_u + m_v + \frac{J_{\check s}}{R_{\check s}^2}} = \frac{m_u g}{m_u + m_v + m_{\check s} \cdot \begin{cases} 1/2 \\ 1 \\ 0 \end{cases}}$$

$$a = 9{,}81 \frac{m}{s^2} \cdot \frac{10}{10 + 100 + \begin{cases} 7 \\ 14 \\ 0 \end{cases}} = \begin{cases} a)\ 0{,}84 \\ b)\ 0{,}79 \\ c)\ 0{,}89 \end{cases} m/s^2$$

3.38



$\nu_0 = 10{,}1\ \mathrm{Hz}$ $k_t = 0{,}1$

$R = 10\ \mathrm{cm}$

$N = ?$

$$\boxed{N = -\frac{\pi \nu_0^2}{\alpha}}$$

$$\Sigma \vec{M} = J\vec{\alpha} \qquad J = \frac{1}{2} m R^2$$

$$\vec{M} = \underbrace{\vec{r}_g \times \vec{F}_g}_{=0} + \underbrace{\vec{r}_{p1} \times \vec{F}_{p1}}_{=0} + \vec{r}_{t_1} \times \vec{F}_{t_1} + \underbrace{\vec{r}_{p2} \times \vec{F}_{p2}}_{=0} + \vec{r}_{t_2} \times \vec{F}_{t_2}$$

$$\vec{r}_g = 0 \qquad \vec{r}_{p1} \parallel \vec{F}_{p1} \qquad \vec{r}_{p2} \parallel \vec{F}_{p2}$$

$$M = -R(F_{t_1} + F_{t_2}) = \frac{1}{2} m R^2 \cdot \alpha \qquad \boxed{\alpha = \frac{-2(F_{t_1} + F_{t_2})}{mR}}$$

$$\alpha = \frac{-2g}{R} \frac{k_t + k_t^2}{1 + k_t^2}$$

2NZ: $\Sigma \vec{F} = m\vec{a}^* = \vec{0}$

$$\vec{F}_g + \vec{F}_{p1} + \vec{F}_{t_1} + \vec{F}_{p2} + \vec{F}_{t_2} = 0$$

x: $-F_{t_1} + F_{p_2} = 0$ $F_{t_1} = k_t F_{p1}$

y: $-mg + F_{p1} + F_{t_2} = 0$ $F_{t_2} = k_t F_{p2}$

$$-F_{t_1} + \frac{F_{t_2}}{k_t} = 0 \longrightarrow F_{t_1} = \frac{F_{t_2}}{k_t}$$

$$\frac{F_{t_1}}{k_t} + F_{t_2} = mg \rightarrow \frac{F_{t_2}}{k_t^2} + F_{t_2} = mg \rightarrow F_{t_2} = \frac{mg}{1 + \frac{1}{k_t^2}} = \frac{k_t^2 mg}{1 + k_t^2}$$

$$F_{t_1} = \frac{k_t mg}{1 + k_t^2}$$

$$N = -\frac{\pi \nu_0^2}{\alpha} = \frac{\pi \nu_0^2 R}{2g} \frac{1 + k_t^2}{k_t + k_t^2} = \frac{\pi \cdot 10{,}1^2 \cdot 0{,}1\ \cancel{m}\ \cancel{s^2}}{2\ \cancel{s^2} \cdot 9{,}81\ \cancel{m}} \cdot \frac{1{,}01}{0{,}11} = \underline{15}$$