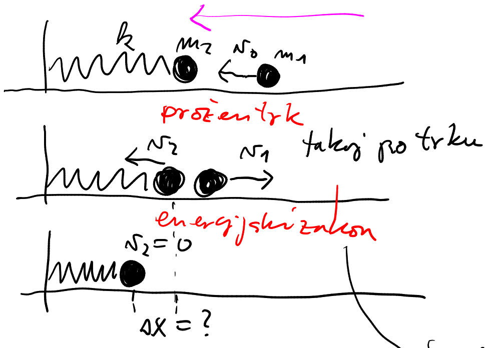
5. 10



$m_1 = 50\,g$

$v_0 = 0{,}8\,m/s$

$m_2 = 180\,g$

$k = 20\,N/m$

$\Delta x = ?$

prožen trk: ohranjata se gibalna količina in energija

$$\frac{m_1 v_0^2}{\cancel{2}} = \frac{m_1 v_1^2}{\cancel{2}} + \frac{m_2 v_2^2}{\cancel{2}}$$

$$m_1 v_0 = m_2 v_2 + m_1 v_1$$

$v_2 = ?$

$$v_1 = v_0 - \frac{m_2}{m_1} v_2$$

$$m_1 v_0^2 = m_1 \left(v_0 - \frac{m_2}{m_1} v_2\right)^2 + m_2 v_2^2$$

$$\cancel{m_1 v_0^2} = \cancel{m_1 v_0^2} - 2 \cancel{m_2} v_0 \cancel{v_2} + \frac{m_2^{\cancel{2}}}{m_1} v_2^{\cancel{2}} + \cancel{m_2} v_2^{\cancel{2}}$$

$$0 = -2 v_0 + \frac{m_2}{m_1} v_2 + v_2$$

$$v_2 = \frac{2 v_0}{1 + \frac{m_2}{m_1}} \qquad \boxed{v_2 = \frac{2 m_1}{m_1 + m_2} v_0}$$

2. kroglica:

$$\Delta W_k + \Delta W_{pr} = 0$$

$$-\frac{1}{2} m_2 v_2^2 + \frac{1}{2} k \Delta x^2 = 0$$

$$\Delta x = \sqrt{\frac{m_2}{k}}\, v_2$$

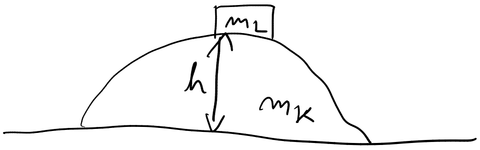
$$\Delta x = \sqrt{\frac{m_2}{k}}\, \frac{2 m_1}{m_1 + m_2} v_0 =$$

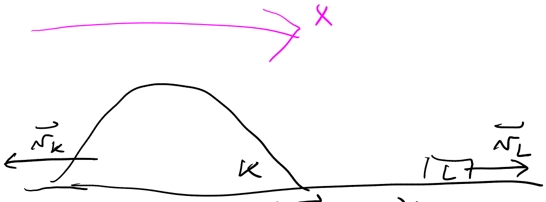
$$= \sqrt{\frac{0{,}18\,kg}{20\,N/m}}\, \frac{2 \cdot 0{,}05\,\cancel{kg}}{0{,}23\,\cancel{kg}} \cdot 0{,}8\,m/s =$$

$$= \underline{0{,}033\,m}$$

5.12

začetek





$\Delta v = |\vec{v}_L - \vec{v}_K|$

$m_K = 3\,kg$
$m_L = 1\,kg$
$\Delta v = 4\,m/s$

---

h

$$\left[m_L \frac{v_L^2}{2} + \frac{m_K v_K^2}{2}\right] + \left[-m_L g h\right] = 0$$

$$\Delta W_k + \Delta W_p = 0$$

$$\Delta \vec{G} = 0$$

x: $m_L v_L + m_K v_K = 0$

$$\hookrightarrow v_K = -\frac{m_L}{m_K} v_L$$

$$\cancel{m_L} \frac{v_L^2}{2} + \frac{m_L^{\cancel{2}}}{m_K} \frac{v_L^2}{2} = \cancel{m_L} g h$$

$$v_L^2 = \frac{2gh}{1 + \frac{m_L}{m_K}} \Rightarrow \sqrt{\frac{2gh\,m_K}{m_L + m_K}} = v_L$$

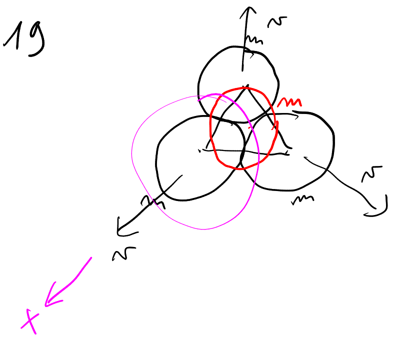
$\Delta v = v_L - v_K$

$$\Delta v = \sqrt{\frac{2gh\,m_K}{m_L + m_K}}\left(1 + \frac{m_L}{m_K}\right) = \sqrt{\frac{2gh\,\cancel{m_K}}{m_L + \cancel{m_K}}}\sqrt{\frac{(m_K + m_L)^{\cancel{2}}}{(m_K)^{\cancel{2}}}} =$$

$$= \sqrt{\frac{2gh\,(m_K + m_L)}{m_K}} \longrightarrow h = \frac{m_K}{m_L + m_K}\,\frac{\Delta v^2}{2g} = \frac{3}{4}\,\frac{\cdot 16\,m^{\cancel{2}}\,\cancel{s^2}}{\cancel{s^2} \cdot 2 \cdot 10\,\cancel{m}} =$$

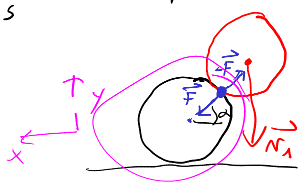
$$= 0{,}6\,m$$

5.19

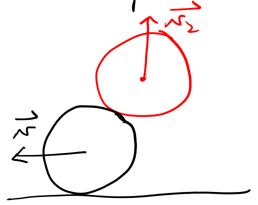


$v = 2\,m/s$

pred trkom



po trku



$$\frac{m v_1^2}{2} = \frac{m v_2^2}{2} + 3\,\frac{m v^2}{2} \longrightarrow \underline{v_1^2 = v_2^2 + 3v^2}$$

zakon o gibalni količini za črno kroglico

$$\Delta \vec{G} = \int \vec{F}\,dt$$

$$x:\quad m v - 0 = \int F \cos\alpha\, dt$$

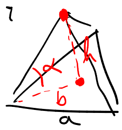
$-\!\!\parallel\!\!-$ za rdečo kroglico

$$y:\quad m v_2 - m v_1 = 3\int F \sin\alpha\, dt$$

ker so 3 črne kroglice

$$: \quad \underline{\frac{v_2 - v_1}{v} = 3\,tg\,\alpha = 3\sqrt{2}}$$

tetraeder:



$$tg\,\alpha = \frac{h}{b} \qquad b = \frac{a\sqrt{3}}{2}\,\frac{2}{3} = \frac{a}{\sqrt{3}}$$

$$h^2 = a^2 - b^2 = a^2 - \frac{a^2}{3} = \frac{2}{3}a^2 \qquad h = \sqrt{\frac{2}{3}}\,a$$

$$tg\,\alpha = \frac{\sqrt{\frac{2}{3}}\,a}{\frac{a}{\sqrt{3}}} = \sqrt{2}$$

$$v_1^2 = v_2^2 + 3v^2 \qquad v_1 = ?$$

$$\frac{v_2 - v_1}{v} = 3\sqrt{2}$$

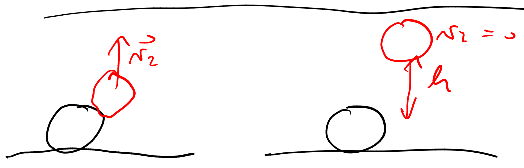
$$v_2 = 3\sqrt{2}\,v + v_1$$

$$v_1^2 = (3\sqrt{2}\,v + v_1)^2 + 3v^2$$

$$\cancel{v_1^2} = 18v^2 + 6\sqrt{2}\,v v_1 + \cancel{v_1^2} + 3v^2$$

$$21 v^{\cancel{2}} + 6\sqrt{2}\,\cancel{v} v_1 = 0$$

$$v_1 = -\frac{21}{6\sqrt{2}}\,v = -\frac{7}{2\sqrt{2}}\cdot 2\,m/s = \underline{-5\ m/s}$$



$$\Delta W_k + \Delta W_p = 0$$

$$-\frac{1}{2} m v_2^2 + m g h = 0 \qquad h = \frac{v_2^2}{2g}$$

$$h = \frac{v_1^2 - 3v^2}{2g} = \frac{25\,\frac{m^2}{s^2} - 3\cdot 4\,\frac{m^2}{s^2}}{2\cdot 10\,m/s^2} = \underline{0{,}65\ m}$$

3.33

$\nu_0 = 0.3\,Hz$
$M = -100\,Nm$
$J_0 = 500\,kg\,m^2$
$m_1 = 25\,kg$
$m_2 = 30\,kg$
$m_3 = 35\,kg$
$m_4 = 40\,kg$
$R = 1{,}5\,m$

$N = ?$

$F = ma \longrightarrow M = J\alpha$

$\omega_k^2 = \omega_z^2 + 2\alpha\varphi$

$0 = (2\pi\nu_0)^2 + 2\alpha \cdot 2\pi N$

$N = -\frac{\pi \nu_0^2}{\alpha}$

$\alpha = \frac{M}{J}$ $\quad J = \sum_n J_n = J_0 + m_1R^2 + m_2R^2 + m_3R^2 + m_4R^2$

$m = m_1 + m_2 + m_3 + m_4 = 130\,kg$

$\alpha = \frac{M}{J_0 + mR^2}$

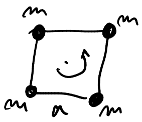
$$N = -\frac{\pi \nu_0^2 (J_0 + mR^2)}{M} = \cancel{-}\frac{\pi \cdot 0.3 \cdot (500\,kgm^2 + 130\,kg \cdot 1{,}5^2 m^2)}{s^2 \cdot (\cancel{-}100\,Nm)} =$$

$= \underline{2.25}$

3.35

$$J = \sum_i J_i = \sum_i m_i R_i^2$$

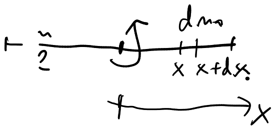
$$J = \int dJ = \int dm\, R^2$$

a) 

$$J = 4m\left(\frac{a\sqrt{2}}{2}\right)^2 = 4m\frac{a^2}{2} = 2ma^2$$

b) 

$$J = \int dm\, R^2 = R^2\int dm = R^2 \cdot m = mR^2$$

c) 

$$J = \int dm\, R^2 = \int \overbrace{\frac{dx}{l}m}^{dm} \cdot \overbrace{x^2}^{R^2} = \frac{m}{l}\int_{-l/2}^{l/2} x^2 dx = \frac{m}{l}\frac{x^3}{3}\Big|_{-l/2}^{l/2} =$$

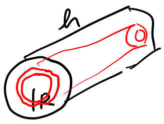
$$= \frac{m}{3l}\left[\left(\frac{l}{2}\right)^3 - \left(-\frac{l}{2}\right)^3\right] = \frac{ml^3}{3l\,8}\cdot 2 = \frac{ml^2}{12}$$

d) 

→ ploščina kolobarja med r in $r+dr$

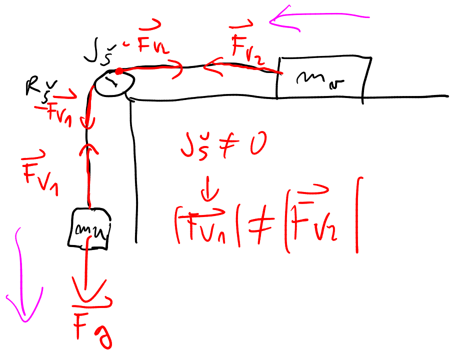
$$J = \int dm\, R^2 = \int \underbrace{\frac{dS}{\pi R^2}m}_{dm} \cdot r^2 = \int_0^R \frac{2\pi r\, dr}{\pi R^2} m r^2 = \frac{2m}{R^2}\int_0^R r^3 dr = \frac{2m}{R^2}\frac{R^4}{4} = \frac{mR^2}{2}$$

$$dS = \pi(r+dr)^2 - \pi r^2 = \cancel{\pi r^2} + 2\pi r\, dr + \underbrace{\cancel{(dr)^2}}_{\text{zanemarimo}} - \cancel{\pi r^2} = 2\pi r\, dr$$

e) 

$$J = \int dm\, R^2 = \int \frac{dV}{\pi R^2 h} m r^2 = \int \frac{2\pi r\, dr\, \cancel{h}}{\pi R^2 \cancel{h}} m r^2 = \frac{mR^2}{2}$$

3.36



$J_{š} \neq 0$

$\Downarrow$

$|\vec{F}_{v_1}| \neq |\vec{F}_{v_2}|$

$a = \cancel{a}_T = \alpha R$

$J_{š} =$ a) $\frac{m_{š} R^2}{2}$ poln valj

b) $m_{š} R^2$ obroč

c) 0 brez mase

$m_v = 100\,g$

$m_u = 10\,g$

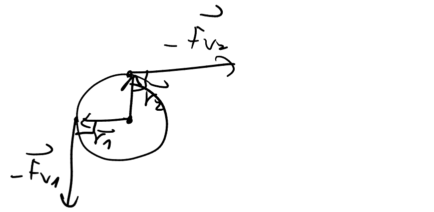
$m_{š} = 14\,g$

$R_{š} = 2\,cm$

$a = ?$

$\vec{M} = J\vec{\alpha}$

$\vec{M} = \vec{r} \times \vec{F}$



$\vec{M}_1 = \vec{r}_1 \times (-\vec{F}_{v_1})$

$|\vec{M}_1| = R \cdot F_{v_1} \qquad |\vec{M}_2| = R\, F_{v_2}$

uteži: $m_u g - F_{v_1} = m_u a$

vozilo: $F_{v_2} = m_v a$

škripec: $|\vec{M}_1| - |\vec{M}_2| = J_{š}\, \alpha$

$(F_{v_1} - F_{v_2}) R = J_{š} \cdot \frac{a}{R}$

$F_{v_1} = m_u (g - a)$

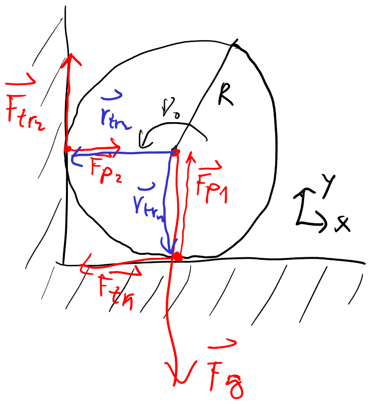
$F_{v_2} = m_v a$

$m_u g - m_u a - m_v a = \frac{J_{š}}{R^2} a$

$J_{š} =$ a) $\frac{1}{2} m R^2$ b) $m R^2$ c) 0

$$a = \frac{m_u g}{m_u + m_v + \frac{J_{š}}{R^2}} = \frac{10\,g \cdot 9.81\,m/s^2}{10\,g + 100\,g + 14\,g \cdot \begin{cases} 1/2 \\ 1 \\ 0 \end{cases}} = \begin{cases} a)\ 0{,}84 \\ b)\ 0{,}79 \\ c)\ 0{,}89 \end{cases} m/s^2$$

3.38



$\nu_0 = 10,1\ Hz$

$R = 10\ cm$

$k_{tr} = 0,1$

$N = ?$

$M = J\alpha$

$\omega_k^2 = \omega_z^2 + 2\alpha\varphi$

$0 = (2\pi\nu_0)^2 + 2\alpha\cdot(2\pi N)$

$N = -\frac{\pi\nu_0^2}{\alpha}$

$J = \frac{1}{2}mR^2$

$$\vec{M} = \sum \vec{r}\times\vec{F} = \underbrace{\vec{0}\times\vec{F}_g}_{\vec{0}} + \underbrace{\vec{r}_{p_1}\times\vec{F}_{p_1}}_{\vec{0}} + \underbrace{\vec{r}_{p_2}\times\vec{F}_{p_2}}_{0} + \vec{r}_{tr_1}\times\vec{F}_{tr_1} + \vec{r}_{tr_2}\times\vec{F}_{tr_2}$$

$\vec{r}_{p_1} \parallel \vec{F}_{p_1}$ $\quad$ $\vec{r}_{p_2} \parallel \vec{F}_{p_2}$ $\quad$ $\vec{r}_{tr_1} \perp \vec{F}_{tr_1}$ $\otimes$ $\quad$ $\vec{r}_{tr_2} \perp \vec{F}_{tr_2}$ $\otimes$

$|\vec{r}_{tr_1}| = |\vec{r}_{tr_2}| = R$

$$\boxed{|\vec{M}| = R(F_{tr_1} + F_{tr_2})}$$

2 N.Z. $\quad \sum\vec{F} = m\vec{a}^*$

$\vec{F}_g + \vec{F}_{p_1} + \vec{F}_{p_2} + \vec{F}_{tr_1} + \vec{F}_{tr_2} = 0$

$x:\ 0 + 0 + F_{p_2} - F_{tr_1} + 0 = 0$

$y:\ -mg + F_{p_1} + 0 + 0 + F_{tr_2} = 0$

$F_{p_2} - F_{tr_1} = 0$

$-mg + F_{p_1} + F_{tr_2} = 0$

$F_{tr_1} = k_{tr}F_{p_1}$

$F_{tr_2} = k_{tr}F_{p_2}$

$\frac{F_{tr_2}}{k_{tr}} = F_{tr_1}$

$-mg + \frac{F_{tr_1}}{k_{tr}} + F_{tr_2} = 0$

$-mg + \frac{F_{tr_2}}{k_{tr}^2} + F_{tr_2} = 0$

$F_{tr_2} = \frac{mg}{1+\frac{1}{k_{tr}^2}} = \frac{mg\,k_{tr}^2}{1+k_{tr}^2}$

$F_{tr_1} = \frac{mg\,k_{tr}}{1+k_{tr}^2}$

$|\vec{M}| = \frac{mgR\,k_{tr}(1+k_{tr})}{1+k_{tr}^2}$

$M = -|\vec{M}| = J\alpha = \frac{1}{2}mR^2\alpha$

$-\frac{\cancel{m}g\cancel{R}\,k_{tr}(1+k_{tr})}{1+k_{tr}^2} = \frac{1}{2}\cancel{m}R^{\cancel{2}}\alpha$

$\alpha = -\frac{2g}{R}\,\frac{k_{tr}(1+k_{tr})}{1+k_{tr}^2}$

$N = -\frac{\pi\nu_0^2}{\alpha} = \frac{\pi\nu_0^2R(1+k_{tr}^2)}{2g\,k_{tr}(1+k_{tr})} = \frac{\pi\cdot(10,1)^2\cdot 0,1\,m\,s^2\cdot(1+0,1^2)}{2\cdot s^2\cdot 9.81\,m\cdot 0,1\cdot(1+0.1)} = \underline{15}$