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Condensed matter physics

Pauli paramagnetism at low temperatures

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Show that if T is small compared with the Fermi temperature, the temperature-dependent correction to the Pauli susceptibility is given by

$$\chi(T) = \chi(0)\left(1 - \frac{{(k_{BT)}}^2\pi^2}{6}\frac{g'^2(\varepsilon_F)}{g(\varepsilon_F)} + \frac{g''(\mu){(k_{BT)}}^2\pi^2}{6g(\varepsilon_F)}\right)$$

Where g, g‘ and g‘‘ are the density of levels and it derivatives at the Fermi energy. Show that for free electrons this reduces to

$$\chi(T) = \chi(0)\left(1 + \frac{{(k_{BT)}}^2\pi^2}{12\varepsilon_F^2}\right).$$

Susceptibility is determined as: χ(t)=$\frac{dM}{dH}$

Magnetization is equal to

$$M = \mu_B^2 B_0 \int_{-\infty}^{\infty} g'(\varepsilon)f(\varepsilon)d\varepsilon$$

Where *f(ε)* is Fermi function and it is expressed as *f(ε)*=$\frac{1}{e^{(\beta-\mu)}+1}$, μ_B Bohr magnetron*, g'(ε)* density of levels function.

Integral in magnetization equation is calculated as follows:

$$\int_{-\infty}^{\infty} g'(\varepsilon)f(\varepsilon)d\varepsilon = [g(\infty)f(\infty) - g(0)f(0)] - \int_{-\infty}^{\infty} g(\varepsilon)f'(\varepsilon)d\varepsilon = \int_{-\infty}^{\infty} g(\varepsilon)(-f'(\varepsilon))d\varepsilon$$

At low temperatures $\mu = \varepsilon_F$, so $g(\varepsilon) = g(\mu)$.We will write density of levels as Teilor series:

$$\int_{-\infty}^{\infty}\left\{g(\mu) + g'(\mu)\varepsilon + \frac{1}{2}g''(\mu)(\varepsilon-\mu)^2 + ..\right\}\left(-f'(\varepsilon)\right)d\varepsilon$$
$$= \int_{-\infty}^{\infty}\left\{g(\mu) + g'(\mu)\varepsilon + \frac{1}{2}g''(\mu)(\varepsilon)^2 + ..\right\}\left(-f'(\varepsilon+\mu)\right)d\varepsilon$$
$$= g(\mu)\int_{-\infty}^{\infty}\left(-f'(\varepsilon)\right)d\varepsilon$$
$$+ g'(\mu)\int_{-\infty}^{\infty}\varepsilon\left(-f'(\varepsilon+\mu)\right)d\varepsilon + \frac{g''(\mu)}{2}\int_{-\infty}^{\infty}\varepsilon^2\left(-f'(\varepsilon+\mu)\right)d\varepsilon + ..$$

Integrals can be calculated separately:

(i) $\int_{-\infty}^{\infty}\big(-f'(\varepsilon)\big)d\varepsilon = -f(\infty) - f(-\infty) = 1$

(ii) $\int_{-\infty}^{\infty}\varepsilon\big(-f'(\varepsilon+\mu)\big)d\varepsilon = 0$

(iii) $\int_{-\infty}^{\infty}\varepsilon^2\big(-f'(\varepsilon+\mu)\big)d\varepsilon = \int_{-\infty}^{\infty}\frac{\varepsilon^2 d\varepsilon}{4k_BTch^2\frac{\varepsilon}{2k_BT}} = [\frac{\varepsilon}{k_{..B}T} = x] = \int_{-\infty}^{\infty}\frac{(k_BT)^3x^2dx}{4k_BTch^2\frac{x}{2}} = [\,ch\frac{x}{2} = \frac{e^{x/2}+e^{-x/2}}{2}] = (k_B\,T)^2\int_{-\infty}^{\infty}\frac{x^2e^x dx}{(e^x+1)^2} = \frac{{(k_{BT)}}^2\pi^2}{3}$

Final expression: $\int_{-\infty}^{\infty}g'(\varepsilon)f(\varepsilon)d\varepsilon = g(\mu) + \frac{g''(\mu){(k_{BT)}}^2\pi^2}{6}$

$g(\mu)$ can be determine from first member of Teilor series:

$$g(\mu) = g(\varepsilon_F) + (\mu - \varepsilon_F)g'(\varepsilon_F)$$

$$\mu = \varepsilon_F - \frac{{(k_{BT)}}^2\pi^2}{6}\frac{g'(\varepsilon_F)}{g(\varepsilon_F)}$$

$$g(\mu) = g(\varepsilon_F) + \left(\varepsilon_F - \frac{{(k_{BT)}}^2\pi^2}{6}\frac{g'(\varepsilon_F)}{g(\varepsilon_F)} - \varepsilon_F\right)g'(\varepsilon_F) = \left(g(\varepsilon_F) - \frac{{(k_{BT)}}^2\pi^2}{6}\frac{g'^2(\varepsilon_F)}{g(\varepsilon_F)}\right)$$

All results can be written to magnetization expression:

$$M = \mu_B^2 H\left(g(\varepsilon_F) - \frac{{(k_{BT)}}^2\pi^2}{6}\frac{g'^2(\varepsilon_F)}{g(\varepsilon_F)} + \frac{g''(\mu){(k_{BT)}}^2\pi^2}{6}\right)$$

$\mu_B^2\, g(\varepsilon_F) = \chi(0)$, because of this relation equation needs to be divided and multiply by $g(\varepsilon_F)$.

Temperature is small, there for $g''(\mu) = g''(\varepsilon_F)$.

$$M = \chi(0)H\left(1 - \frac{{(k_{BT)}}^2\pi^2}{6}\frac{g'^2(\varepsilon_F)}{g(\varepsilon_F)} + \frac{g''(\mu){(k_{BT)}}^2\pi^2}{6g(\varepsilon_F)}\right)$$

Susceptibility expression:

$$\boldsymbol{\chi(T) = \chi(0)\left(1 - \frac{{(k_{BT)}}^2\pi^2}{6}\frac{g'^2(\varepsilon_F)}{g(\varepsilon_F)} + \frac{g''(\mu){(k_{BT)}}^2\pi^2}{6g(\varepsilon_F)}\right)}.$$

In free electron case we need to remember that density of levels has square root dependence of energy $g(\varepsilon) = A\sqrt{\varepsilon}$, where A is constant.

$$g'(\varepsilon) = \frac{A}{2\sqrt{\varepsilon}}$$

$$g''(\varepsilon) = \frac{-A}{4\varepsilon^{3/2}}$$

These results we put into susceptibility equation and get susceptibility value for free electron:

$$\boldsymbol{\chi(T) = \chi(0)\left(1 + \frac{{(k_{BT)}}^2\pi^2}{12\varepsilon_F^2}\right)}.$$