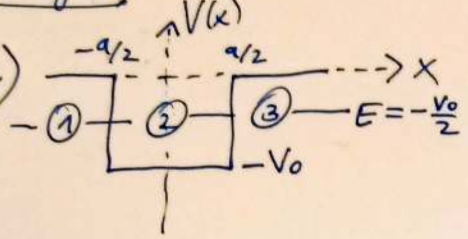
KM 1. kolokvij 2020/21

1. naloga

a) 


vaje: v.f. osnovnega stanja je soda

$$\psi_2(x) = A\cos(kx)$$
$$\psi_3(x) = B e^{-\kappa x}$$

R.P. $\psi_2(\frac{a}{2}) = \psi_3(\frac{a}{2})$

$\psi_2'(\frac{a}{2}) = \psi_3'(\frac{a}{2})$ 1/4+

$$k = \sqrt{\frac{2m(E+V_0)}{\hbar^2}} = \sqrt{\frac{mV_0}{\hbar^2}}$$
$$\kappa = \sqrt{\frac{-2mE}{\hbar^2}} = \sqrt{\frac{mV_0}{\hbar^2}} = k$$

b) vaje: $\mathrm{tg}\frac{u}{2} = \sqrt{\left(\frac{u_0}{u}\right)^2 - 1}$ (osnovno stanje ustreza prvi pozitivni rešitvi)

$u = ka \longrightarrow u^2 = \frac{mV_0a^2}{\hbar^2} = \frac{1}{2}u_0^2 \rightarrow \mathrm{tg}\frac{u}{2} = 1$

$u_0^2 = \frac{2mV_0a^2}{\hbar^2}$

osnovno stanje: $u = \frac{\pi}{2}$ 1/4

$$\Rightarrow u_0^2 = \boxed{\frac{\pi^2}{2} = \frac{2mV_0a^2}{\hbar^2}}$$

c) $u_0 = \frac{\pi}{\sqrt{2}} < \pi \Rightarrow$ vezano stanje je samo eno (vaje) 1/4-

d) iz R.P.: $A\cos\left(\frac{ka}{2}\right) = Be^{-k\frac{a}{2}} \Rightarrow B = A\cos\left(k\frac{a}{2}\right)e^{+\frac{ka}{2}}$ ; $ka = \frac{\pi}{2}$

$$\psi(x) = \begin{cases} A\cos\left(\frac{\pi x}{2a}\right) \\ A\cos\left(\frac{\pi}{4}\right)e^{-\frac{\pi}{2a}\left(x-\frac{a}{2}\right)} \end{cases}$$ 1/4

$$P\left(|x| \le \frac{a}{2}\right) = \frac{\int_0^{a/2}\cos^2\left(\frac{\pi x}{2a}\right)dx}{\int_0^{a/2}\cos^2\left(\frac{\pi x}{2a}\right)dx + \cos^2\left(\frac{\pi}{4}\right)\int_{a/2}^{\infty}e^{-\frac{\pi}{a}\left(x-\frac{a}{2}\right)}dx} = \frac{\pi+2}{\pi+4} = 72\%$$

$\Sigma = 1$

## 2. naloga

vaje

a) $\langle x\rangle = \langle \psi_x|\langle \psi_y| x |\psi_x\rangle|\psi_y\rangle = \langle \psi_x | x | \psi_x\rangle = \sqrt{2}\, x_0 \,\mathrm{Re}\{\alpha\} = \sqrt{2}\, x_0 \alpha$

$\langle y\rangle = \sqrt{2}\, x_0 \,\mathrm{Re}\{i\beta\} = 0$

$\langle p_x\rangle = \sqrt{2}\, p_0 \,\mathrm{Im}\{\alpha\} = 0$ 1/4 +

$\langle p_y\rangle = \sqrt{2}\, p_0 \,\mathrm{Im}\{i\beta\} = \sqrt{2}\, p_0 \beta$

b) $\Delta y = \frac{x_0}{\sqrt{2}}$ (vaje) +

c) $L_z = -i\hbar\,(a_x^\dagger a_y - a_x a_y^\dagger)$ (vaje)

$\langle L_z\rangle = -i\hbar\left(\alpha^* \beta - \alpha (i\beta)^*\right) = -i\hbar\,(i\alpha\beta + i\alpha\beta) = 2\hbar\alpha\beta$

$\langle L_z^2\rangle = -\hbar^2 \langle a_x^\dagger a_y a_x^\dagger a_y - a_x^\dagger a_y a_x a_y^\dagger - a_x a_y^\dagger a_x^\dagger a_y + a_x a_y^\dagger a_x a_y^\dagger \rangle =$

v normalni vrstni red ↗

$= -\hbar^2 \langle (a_x^\dagger)^2 a_y^2 - 2 a_x^\dagger a_y^\dagger a_y a_x + (a_y^\dagger)^2 a_x^2 - a_y^\dagger a_y - a_x^\dagger a_x \rangle =$

$= -\hbar^2 \left( (\alpha^*)^2 (i\beta)^2 - 2\alpha^* (i\beta)^* i\beta\alpha + [(i\beta)^*]^2 \alpha^2 - (i\beta)^* i\beta - \alpha^*\alpha \right) =$

$= -\hbar^2 \left( -\alpha^2\beta^2 - 2\alpha^2\beta^2 - \alpha^2\beta^2 - \beta^2 - \alpha^2 \right) =$

$= \hbar^2 \left( 4\alpha^2\beta^2 + \alpha^2 + \beta^2 \right)$ 1/4

$\Delta L_z = \hbar\sqrt{\alpha^2 + \beta^2}$

d) $\Delta L_z \cdot \Delta y \geq \frac{1}{2} \left| \langle [L_z, y] \rangle \right| = \frac{\hbar}{2} |\langle x\rangle| \quad \Leftarrow \quad [L_z, y] = -i\hbar x$

$\hbar\sqrt{\alpha^2+\beta^2} \cdot \frac{x_0}{\sqrt{2}} \geq \frac{\hbar}{2} \cdot \sqrt{2}\, x_0 |\alpha|$ 1/4 −

$\sqrt{\alpha^2+\beta^2} \geq |\alpha|$ ✓

e) $\langle L_z, t\rangle = \langle L_z(t)\rangle = \langle L_z, 0\rangle \qquad \dot{L}_z(t) = \frac{i}{\hbar}[H, L_z](t) = 0$

$\langle L_z^2, t\rangle = \langle [L_z(t)]^2 \rangle = \langle L_z^2, 0\rangle \qquad [H, L_z] = 0$ (na vajah)

$\Downarrow \qquad\qquad \Downarrow$

$\Delta L_z(t) = \Delta L_z(0) \qquad L_z(t) = L_z$ 1/4 −

$\Sigma = 1$