1) a) $a|0\rangle = 0|0\rangle$ osnovno stanje h.o. je koherentno stanje, $z=0$. Iz vaj:

$\langle x\rangle = \sqrt{2}\, x_0 \,\mathrm{Re}\, z = 0$

$\langle p\rangle = \sqrt{2}\, p_0 \,\mathrm{Im}\, z = 0$

$\sigma_x = \frac{x_0}{\sqrt{2}}$; $x_0 = \sqrt{\frac{\hbar}{m\omega}}$; $\omega = \sqrt{\frac{k}{m}}$

$\sigma_p = \frac{p_0}{\sqrt{2}}$; $p_0 = \frac{\hbar}{x_0}$ 1/4

b) $\dot{x}(t) = \frac{i}{\hbar}[H,x](t) = \frac{i}{\hbar}\left[\frac{p^2}{2m}, x\right](t) = \frac{p(t)}{m}$

$\dot{p}(t) = \frac{i}{\hbar}(H,p)(t) = \frac{i}{\hbar}\left[-\frac{1}{2}kx^2, p\right](t) = kx(t)$

$\ddot{x}(t) = \frac{k}{m}x(t)$

$x(t) = A\,\mathrm{ch}\,\omega t + B\,\mathrm{sh}(\omega t)$; $x(0) = x \Rightarrow A = x$

$p(t) = m\dot{x}(t) = m\omega A\,\mathrm{sh}\,\omega t + m\omega B\,\mathrm{ch}\,\omega t$; $p(0) = p \Rightarrow m\omega B = p$

$x(t) = x\,\mathrm{ch}\,\omega t + \frac{p}{m\omega}\mathrm{sh}\,\omega t$

$p(t) = p\,\mathrm{ch}\,\omega t + m\omega x\,\mathrm{sh}\,\omega t$ 1/2⁻

c) $\langle x,t\rangle = \langle x\rangle \mathrm{ch}\,\omega t + \frac{\langle p\rangle}{m\omega}\mathrm{sh}\,\omega t = 0$

$\langle p,t\rangle = \langle p\rangle \mathrm{ch}\,\omega t + \langle x\rangle m\omega\,\mathrm{sh}\,\omega t = 0$

$\sigma_x^2(t) = \langle x^2,t\rangle = \langle x^2\rangle \mathrm{ch}^2\omega t + \frac{\langle xp+px\rangle}{m\omega}\mathrm{ch}\,\omega t\,\mathrm{sh}\,\omega t + \frac{\langle p^2\rangle}{m^2\omega^2}\mathrm{sh}^2\omega t$

$\sigma_p^2(t) = \langle p^2,t\rangle = \langle p^2\rangle \mathrm{ch}^2\omega t + \langle xp+px\rangle m\omega\,\mathrm{ch}\,\omega t\,\mathrm{sh}\,\omega t + m^2\omega^2\langle x^2\rangle \mathrm{sh}^2\omega t$

$\langle xp+px\rangle = 2\,\mathrm{Re}\langle xp\rangle = 2\,\mathrm{Re}\langle -i\hbar x\frac{d}{dx}\rangle = 2\hbar\,\mathrm{Im}\langle x\frac{d}{dx}\rangle = 0$, ker je valovna funkcija osnovnega stanja h.o. realna

$\langle x^2\rangle = \sigma_x^2 = \frac{x_0^2}{2}$

$\langle p^2\rangle = \sigma_p^2 = \frac{p_0^2}{2}$

$\sigma_x^2(t) = \frac{x_0^2}{2}\mathrm{ch}^2\omega t + \frac{p_0^2}{2m^2\omega^2}\mathrm{sh}^2\omega t = \frac{x_0^2}{2}(\mathrm{ch}^2\omega t + \mathrm{sh}^2\omega t) = \frac{x_0^2}{2}\mathrm{ch}\,2\omega t$

$\sigma_p^2(t) = \frac{p_0^2}{2}\mathrm{ch}^2\omega t + m^2\omega^2\frac{x_0^2}{2}\mathrm{sh}^2\omega t = \frac{p_0^2}{2}(\mathrm{ch}^2\omega t + \mathrm{sh}^2\omega t) = \frac{p_0^2}{2}\mathrm{ch}\,2\omega t$

$$\left.\begin{aligned}\sigma_x(t) &= \frac{x_0}{\sqrt{2}}\sqrt{\mathrm{ch}\,2\omega t}\\ \sigma_p(t) &= \frac{p_0}{\sqrt{2}}\sqrt{\mathrm{ch}\,2\omega t}\end{aligned}\right\}\quad \sigma_x(t)\,\sigma_p(t) = \frac{\hbar}{2}\mathrm{ch}\,2\omega t$$

1/4⁺

(2) a) $HP = \gamma\left(|a\rangle\langle a| + |b\rangle\langle b|\right) + \gamma'\left(|a\rangle\langle c| + |c\rangle\langle b| + |b\rangle\langle c| + |c\rangle\langle a|\right)$

$PH = \gamma\left(|b\rangle\langle b| + |a\rangle\langle a|\right) + \gamma'\left(|b\rangle\langle c| + |c\rangle\langle a| + |a\rangle\langle c| + |c\rangle\langle b|\right)$

$HP = PH \Rightarrow [H, P] = 0$ 1/4

b) $P|c\rangle = 1|c\rangle$

$P\left(\alpha|a\rangle + \beta|b\rangle\right) = \lambda\left(\alpha|a\rangle + \beta|b\rangle\right)$

$\alpha|b\rangle + \beta|a\rangle = \lambda\alpha|a\rangle + \lambda\beta|b\rangle$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \lambda\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

$P\dfrac{|a\rangle + |b\rangle}{\sqrt{2}} = 1\,\dfrac{|a\rangle + |b\rangle}{\sqrt{2}}$

$P\dfrac{|a\rangle - |b\rangle}{\sqrt{2}} = -1\,\dfrac{|a\rangle - |b\rangle}{\sqrt{2}}$ 1/4

c) $|+\rangle \equiv \dfrac{|a\rangle + |b\rangle}{\sqrt{2}}$, $|-\rangle \equiv \dfrac{|a\rangle - |b\rangle}{\sqrt{2}} \Rightarrow |a\rangle = \dfrac{|+\rangle + |-\rangle}{\sqrt{2}}$, $|b\rangle = \dfrac{|+\rangle - |-\rangle}{\sqrt{2}}$

$|a\rangle\langle b| = \dfrac{|+\rangle + |-\rangle}{\sqrt{2}}\,\dfrac{\langle +| - \langle -|}{\sqrt{2}} = \frac{1}{2}\left(|+\rangle\langle +| + |-\rangle\langle +| - |+\rangle\langle -| - |-\rangle\langle -|\right)$

$|b\rangle\langle a| = \dfrac{|+\rangle - |-\rangle}{\sqrt{2}}\,\dfrac{\langle +| + \langle -|}{\sqrt{2}} = \frac{1}{2}\left(|+\rangle\langle +| - |-\rangle\langle +| + |+\rangle\langle -| - |-\rangle\langle -|\right)$

$|a\rangle\langle b| + |b\rangle\langle a| = |+\rangle\langle +| - |-\rangle\langle -|$

$|a\rangle\langle c| + |b\rangle\langle c| = \sqrt{2}\,|+\rangle\langle c|$

$|c\rangle\langle a| + |c\rangle\langle b| = |c\rangle\cdot\sqrt{2}\,\langle +|$

$H = \gamma\left(|+\rangle\langle +| - |-\rangle\langle -|\right) + 2\gamma'\left(|+\rangle\langle c| + |c\rangle\langle +|\right)$

v bazi $|+\rangle, |-\rangle, |c\rangle$ je $H = \begin{bmatrix} \gamma, & 0 & \sqrt{2}\gamma' \\ 0, & -\gamma, & 0 \\ \sqrt{2}\gamma', & 0, & 0 \end{bmatrix}$ 1/4

d) $|\psi\rangle = \alpha|+\rangle + \beta|-\rangle + \gamma|c\rangle$

$H|\psi\rangle = E|\psi\rangle \Rightarrow \begin{bmatrix} \gamma, & 0, & \sqrt{2}\gamma' \\ 0, & -\gamma, & 0 \\ \sqrt{2}\gamma', & 0, & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = E\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$

$\det\begin{bmatrix} \gamma - E, & 0, & \sqrt{2}\gamma' \\ 0, & -\gamma - E, & 0 \\ \sqrt{2}\gamma', & 0, & -E \end{bmatrix} = 0$

$E_1 = -\gamma$

$(\gamma - E)(-E) - 2\gamma'^2 = 0$

$E^2 - \gamma E - 2\gamma'^2 = 0$

$E_{2,3} = \dfrac{\gamma \pm \sqrt{\gamma^2 + 8\gamma'^2}}{2}$ 1/4