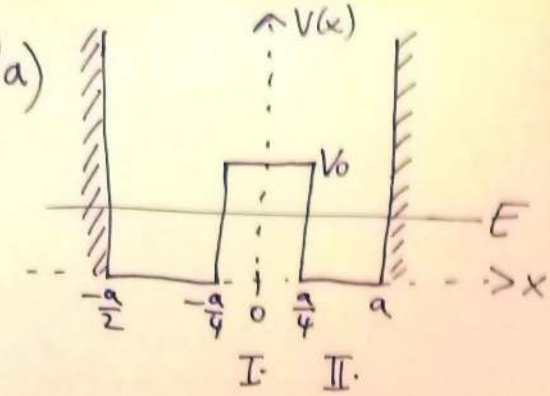
1 a)




sode: $\psi_I(x) = A\,ch(\kappa x)$ $\quad \kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$

$\psi_{II}(x) = B \sin k\left(\frac{a}{2} - x\right)$ $\quad k = \sqrt{\frac{2mE}{\hbar^2}}$

lihe: $\psi_I(x) = A\,sh(\kappa x)$

$\psi_{II}(x) = B \sin k\left(\frac{a}{2} - x\right)$ 1/4

b) robna pogoja: $\psi_I\left(\frac{a}{4}\right) = \psi_{II}\left(\frac{a}{4}\right)$

$\psi_I'\left(\frac{a}{4}\right) = \psi_{II}'\left(\frac{a}{4}\right)$

sode: $A\,ch\frac{\kappa a}{4} = B \sin\frac{ka}{4}$

$\kappa A\,sh\frac{\kappa a}{4} = -kB\cos\frac{ka}{4}$

$: \quad \kappa\, th\frac{\kappa a}{4} = -k\, ctg\frac{ka}{4}$

lihe: $A\,sh\frac{\kappa a}{4} = B\sin\frac{ka}{4}$

$\kappa A\,ch\frac{\kappa a}{4} = -Bk\cos\frac{ka}{4}$

$: \quad \kappa\, cth\frac{\kappa a}{4} = -k\, ctg\frac{ka}{4}$ 1/4

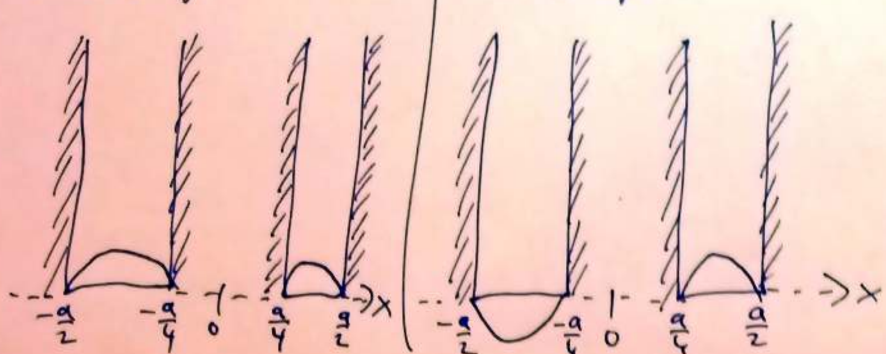
c) $V_0 \to \infty \Rightarrow \kappa a \to \infty \Rightarrow \kappa a\, th\frac{\kappa a}{4} \to \infty$, $\kappa a\, cth\frac{\kappa a}{4} \to \infty$

$ctg\frac{ka}{4} = -\infty$ tako za soda kot tudi za liha stanja

$\frac{ka}{4} = \pi$, $k = \frac{4\pi}{a}$, $E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2\pi^2}{2m(a/4)^2}$ 2x degenerirano

energija neskončne pot. jame s širino a/4

sodo stanje: | liho stanje:




$\psi(x) = \frac{\psi_-(x) + \psi_+(x)}{\sqrt{2}}$ $\qquad \psi(x) = \frac{-\psi_-(x) + \psi_+(x)}{\sqrt{2}}$

$\psi_-(x) = \sqrt{\frac{2}{a/4}} \sin\left[\frac{\pi}{a/4}\left(x + \frac{a}{2}\right)\right]$ za $-\frac{a}{2} < x < -\frac{a}{4}$, sicer $\phi$ — val. funkcija osnovnega stanja leve jame

$\psi_+(x) = \sqrt{\frac{2}{a/4}} \sin\left[\frac{\pi}{a/4}\left(x - \frac{a}{4}\right)\right]$ za $\frac{a}{4} < x < \frac{a}{2}$, sicer $\phi$ — val. funkcija osn. stanja desne jame

1/4

① $\varkappa a \,\mathrm{th}\,\frac{\varkappa a}{4} \doteq \varkappa a\left(1-2e^{-\frac{\varkappa a}{2}}\right) \doteq \varkappa_0 a\left(1-2e^{-\frac{\varkappa_0 a}{2}}\right)$

d) $\varkappa a \,\mathrm{cth}\,\frac{\varkappa a}{4} \doteq \varkappa a\left(1+2e^{-\frac{\varkappa a}{2}}\right) \doteq \varkappa_0 a\left(1+2e^{-\frac{\varkappa_0 a}{2}}\right)$

$-ka \,\mathrm{ctg}\,\frac{ka}{4} \doteq -4\pi \frac{-1}{\varrho a/4} = \frac{16\pi}{\varrho a}$

$ka = 4\pi - \varrho a$

$(ka)^2 + (\varkappa a)^2 = \frac{2mV_0 a^2}{\hbar^2} = (\varkappa_0 a)^2$

$(\varkappa a)^2 = (\varkappa_0 a)^2 - (ka)^2 \doteq (\varkappa_0 a)^2$

sodo stanje: $\varkappa_0 a\left(1-2e^{-\frac{\varkappa_0 a}{2}}\right) = \frac{16\pi}{\varrho_s a} \rightarrow \varrho_s a \doteq \frac{16\pi}{\varkappa_0 a}\left(1+2e^{-\frac{\varkappa_0 a}{2}}\right)$

liho stanje: $\varkappa_0 a\left(1+2e^{-\frac{\varkappa_0 a}{2}}\right) = \frac{16\pi}{\varrho_\ell a} \rightarrow \varrho_\ell a \doteq \frac{16\pi}{\varkappa_0 a}\left(1-2e^{-\frac{\varkappa_0 a}{2}}\right)$

$E_\ell - E_s = \frac{\hbar^2}{2ma^2}\left[(4\pi-\varrho_\ell a)^2 - (4\pi-\varrho_s a)^2\right] \doteq$

$\doteq \frac{\hbar^2}{2ma^2} 8\pi(\varrho_s a - \varrho_\ell a) = \frac{\hbar^2}{2ma^2} 8\pi \cdot \frac{16\pi}{\varkappa_0 a} \cdot 4e^{-\frac{\varkappa_0 a}{2}} =$

$= \underline{\underline{\frac{\hbar^2\pi^2}{2m(a/4)^2} \cdot \frac{32e^{-\frac{\varkappa_0 a}{2}}}{\varkappa_0 a}}}$

1/4

② a) $H|n_x n_y\rangle = \left[\hbar\omega_x\left(n_x + \frac{1}{2}\right) + \hbar\omega_y\left(n_y + \frac{1}{2}\right)\right]|n_x n_y\rangle$

$$\omega_x = \sqrt{\frac{k}{m}} \equiv \omega$$

$$\omega_y = \sqrt{\frac{4k}{m}} = 2\omega$$

$$E_{n_x n_y} = \hbar\omega\left(n_x + 2n_y + \frac{3}{2}\right)$$

$E_{30} = E_{11} = \frac{9}{2}\hbar\omega$ 2x degeneriran, baza: $|30\rangle, |11\rangle$ 1/2⁻

b) $H' = \lambda x^2 y = \lambda\left[\frac{x_0}{\sqrt{2}}(a_x + a_x^+)\right]^2 \frac{y_0}{\sqrt{2}}(a_y + a_y^+)$

$$x_0 = \sqrt{\frac{\hbar}{m\omega_x}} = \sqrt{\frac{\hbar}{m\omega}}$$

$$y_0 = \sqrt{\frac{\hbar}{m\omega_y}} = \frac{x_0}{\sqrt{2}}$$

$$H' = \frac{\lambda x_0^3}{4}(a_x + a_x^+)^2(a_y + a_y^+)$$

$$H'|30\rangle = 0|30\rangle + \frac{\lambda x_0^3}{4} a_x^2 a_y |30\rangle + \dots =$$

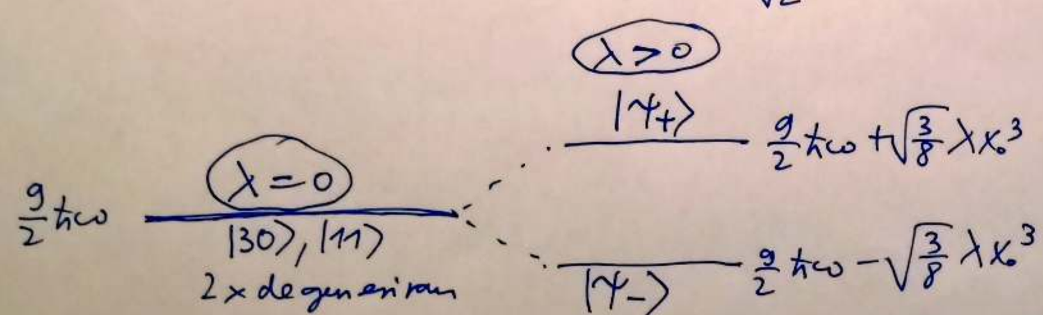
$$= 0|30\rangle + \frac{\lambda x_0^3}{4}\sqrt{3}\cdot\sqrt{2}\cdot\sqrt{1}|11\rangle + \dots =$$

$$= \sqrt{\frac{3}{8}}\lambda x_0^3 |11\rangle + \dots \quad \Rightarrow \langle 11|H'|30\rangle = \sqrt{\frac{3}{8}}\lambda x_0^3 = \langle 30|H'|11\rangle$$

$$\begin{bmatrix} -E^{(1)}, & \sqrt{\frac{3}{8}}\lambda x_0^3 \\ \sqrt{\frac{3}{8}}\lambda x_0^3, & -E^{(1)} \end{bmatrix}\begin{bmatrix} c_{30} \\ c_{11} \end{bmatrix} = 0$$

$\hookrightarrow E^{(1)} = \pm\sqrt{\frac{3}{8}}\lambda x_0^3 \qquad |\psi_+\rangle = \frac{|30\rangle + |11\rangle}{\sqrt{2}}$

$$|\psi_-\rangle = \frac{|30\rangle - |11\rangle}{\sqrt{2}}$$




1/2⁺