① a) $S_1 = \frac{3}{2}$, $S_2 = 1$

$$S^2|\psi,0\rangle = \hbar^2 S(S+1)|\psi,0\rangle \Rightarrow S = \frac{1}{2}$$
$$S_z|\psi,0\rangle = \hbar S_z|\psi,0\rangle \Rightarrow S_z = \frac{1}{2}$$

S pomočjo tabel Clebsch-Gordanovih koeficientov:

$$|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{2}}\,|\tfrac{3}{2}\rangle|-1\rangle - \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\rangle|0\rangle + \sqrt{\tfrac{1}{6}}\,|-\tfrac{1}{2}\rangle|1\rangle$$ +++

b) $H = J S_{1z} S_{2z}$

$$H|S_{1z}\rangle|S_{2z}\rangle = J\left(S_{1z}|S_{1z}\rangle\right)\left(S_{2z}|S_{2z}\rangle\right) = J\hbar^2 S_{1z}S_{2z}|S_{1z}\rangle|S_{2z}\rangle$$

E

- $\frac{3}{2}J\hbar^2$: $|\tfrac{3}{2}\rangle|1\rangle$, $|-\tfrac{3}{2}\rangle|-1\rangle$
- $\frac{1}{2}J\hbar^2$: $|\tfrac{1}{2}\rangle|1\rangle$, $|-\tfrac{1}{2}\rangle|-1\rangle$
- 0: $|\tfrac{3}{2}\rangle|0\rangle$, $|\tfrac{1}{2}\rangle|0\rangle$, $|-\tfrac{1}{2}\rangle|0\rangle$, $|-\tfrac{3}{2}\rangle|0\rangle$
- $-\frac{1}{2}J\hbar^2$: $|-\tfrac{1}{2}\rangle|1\rangle$, $|\tfrac{1}{2}\rangle|-1\rangle$
- $-\frac{3}{2}J\hbar^2$: $|-\tfrac{3}{2}\rangle|1\rangle$, $|\tfrac{3}{2}\rangle|-1\rangle$

++

c) $|\psi,t\rangle = \sqrt{\tfrac{1}{2}}\,e^{i\frac{3}{2}J\hbar t}|\tfrac{3}{2}\rangle|-1\rangle - \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\rangle|0\rangle + \sqrt{\tfrac{1}{6}}\,e^{i\frac{1}{2}J\hbar t}|-\tfrac{1}{2}\rangle|1\rangle$ ++

d) $|\psi,\tfrac{\pi}{J\hbar}\rangle = -i\sqrt{\tfrac{1}{2}}\,|\tfrac{3}{2}\rangle|-1\rangle - \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\rangle|0\rangle + i\sqrt{\tfrac{1}{6}}\,|-\tfrac{1}{2}\rangle|1\rangle$

$$S^2|S S_z\rangle = \hbar^2 S(S+1)|S S_z\rangle = \frac{35}{4}\hbar^2|S S_z\rangle \Rightarrow S = \frac{5}{2}$$

S pomočjo tabel Clebsh-Gordanovih koeficientov prepišem valovno funkcijo v bazo z dobrim skupnim spinom:

$$|\psi,\tfrac{\pi}{J\hbar}\rangle = -i\sqrt{\tfrac{1}{2}}\left[\sqrt{\tfrac{1}{10}}\,|\tfrac{5}{2}\,\tfrac{1}{2}\rangle + \dots\right] - \sqrt{\tfrac{1}{3}}\left[\sqrt{\tfrac{3}{5}}\,|\tfrac{5}{2}\,\tfrac{1}{2}\rangle + \dots\right] + i\sqrt{\tfrac{1}{6}}\left[\sqrt{\tfrac{3}{10}}\,|\tfrac{5}{2}\,\tfrac{1}{2}\rangle + \dots\right] =$$
$$= \left(-i\sqrt{\tfrac{1}{20}} - \sqrt{\tfrac{1}{5}} + i\sqrt{\tfrac{1}{20}}\right)|\tfrac{5}{2}\,\tfrac{1}{2}\rangle + \dots = -\sqrt{\tfrac{1}{5}}\,|\tfrac{5}{2}\,\tfrac{1}{2}\rangle + \dots$$

$$P_{\frac{35}{4}\hbar^2} = \frac{1}{5}$$ +++

e) Ker je $[H, S_z] = 0$, je $|\psi,t\rangle$ lastna funkcija operatorja S_z:

$$S_z|\psi,t\rangle = S_z e^{-i\frac{H}{\hbar}t}|\psi,0\rangle = e^{-i\frac{H}{\hbar}t}S_z|\psi,0\rangle = e^{-i\frac{H}{\hbar}t}\cdot\frac{\hbar}{2}|\psi,0\rangle =$$
$$= \frac{\hbar}{2}|\psi,t\rangle$$

Pri meritvi S_z se torej valovna funkcija ne spremeni. Verjetnost, da pri meritvi S^2 izmerimo $\frac{35}{4}\hbar^2$, ostane enaka kot pri točki (d). ++

② a) $H = H_0 + H'$

$$H'|0\rangle = \lambda p x p |0\rangle = \lambda p x \frac{p_0}{\sqrt{2}i}(a - a^+)|0\rangle =$$

$$= \lambda p x \frac{p_0}{\sqrt{2}i}(-|1\rangle) = \frac{-\lambda p_0}{\sqrt{2}i} p \frac{x_0}{\sqrt{2}}(a + a^+)|1\rangle =$$

$$= \frac{-\lambda p_0}{\sqrt{2}i} p \frac{x_0}{\sqrt{2}}\left(|0\rangle + \sqrt{2}|2\rangle\right) = \frac{-\lambda p_0 x_0}{2i} \frac{p_0}{\sqrt{2}i}(a - a^+)\left(|0\rangle + \sqrt{2}|2\rangle\right) =$$

$$= \frac{\lambda p_0^2 x_0}{2\sqrt{2}}\left(2|1\rangle - |1\rangle - \sqrt{6}|3\rangle\right) = \frac{\lambda p_0^2 x_0}{2\sqrt{2}}\left(|1\rangle - \sqrt{6}|3\rangle\right)$$

$\langle 0|H'|0\rangle = 0$; v 1. redu teorije motnje ni popravka. +++

V 2. redu:

$$\Delta E = \sum_{m\neq 0} \frac{|\langle m|H'|0\rangle|^2}{E_0 - E_m} = \frac{|\langle 1|H'|0\rangle|^2}{-\hbar\omega} + \frac{|\langle 3|H'|0\rangle|^2}{-3\hbar\omega} =$$

$$= \left(\frac{\lambda p_0^2 x_0}{2\sqrt{2}}\right)^2 \left[\frac{1}{-\hbar\omega} + \frac{6}{-3\hbar\omega}\right] = -\frac{3\lambda^2 p_0^4 x_0^2}{8\hbar\omega} = -\frac{3}{8}\lambda^2 m \hbar^2$$

++++++

b) $$H = p\frac{1}{2m(1+\alpha x)}p + \frac{1}{2}k x^2 =$$

$$= p\frac{1}{2m}\left(1 - \alpha x + (\alpha x)^2 + \dots\right)p + \frac{1}{2}k x^2 =$$

$$= H_0 \underbrace{- \frac{\alpha}{2m} p x p + \frac{\alpha^2}{2m} p x^2 p + \mathcal{O}(\alpha^3)}_{H'}$$

V 1. redu teorije motnje:

$$\langle 0|H'|0\rangle = \langle 0|\frac{\alpha^2}{2m} p x^2 p|0\rangle + \mathcal{O}(\alpha^3)$$

$$\langle 0|p x^2 p|0\rangle = \langle x p 0 | x p 0\rangle = \left|\frac{\hbar}{2i}\right|^2\left((-1)^2 + (-\sqrt{2})^2\right) = \frac{3}{4}\hbar^2$$

$$x p|0\rangle = \frac{x_0 p_0}{2i}(a + a^+)(a - a^+)|0\rangle = \frac{\hbar}{2i}\left[-|0\rangle - \sqrt{2}|2\rangle\right]$$

$$\langle 0|H'|0\rangle = \frac{3\alpha^2\hbar^2}{8m} + \mathcal{O}(\alpha^3)$$

V 2. redu teorije motnje [iz točke (a)]:

$$\sum_{m\neq 0}\frac{|\langle m|H'|0\rangle|^2}{E_0 - E_m} = -\frac{3}{8}\left(-\frac{\alpha}{2m}\right)^2 m\hbar^2 + \mathcal{O}(\alpha^3) = -\frac{3\alpha^2\hbar^2}{32m} + \mathcal{O}(\alpha^3)$$

$$\Delta E(\alpha) = \left(\frac{3}{8} - \frac{3}{32}\right)\frac{\alpha^2\hbar^2}{m} + \mathcal{O}(\alpha^3) = \frac{9}{32}\frac{\alpha^2\hbar^2}{m} + \mathcal{O}(\alpha^3)$$ ++++