1. $H = \underbrace{\frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2}_{H_0} + \underbrace{\lambda \hbar\omega \left(\frac{x}{x_0}\right)^3 e^{-\frac{t}{\tau}} \theta(t)}_{H'(t)}$

a) $P_n(\infty) = |c_n(\infty)|^2$

$$c_n(\infty) = c_n(0) - \frac{i}{\hbar} \int_0^\infty dt \langle n,t | H'(t) | 0,t \rangle$$

$$c_n(0) = \delta_{n,0}$$

$$c_n(\infty) = \delta_{n0} - \frac{i}{\hbar} \lambda \hbar\omega \langle n | \left(\frac{x}{x_0}\right)^3 | 0 \rangle \underbrace{\int_0^\infty dt\, e^{in\omega t} e^{-\frac{t}{\tau}}}_{\frac{-1}{in\omega - \frac{1}{\tau}}}$$

iz vaj: $\langle n | \left(\frac{x}{x_0}\right)^3 | m \rangle = \frac{\sqrt{(m+1)(m+2)(m+3)}}{2\sqrt{2}} \delta_{n,m+3} + \frac{3(m+1)^{3/2}}{2\sqrt{2}} \delta_{n,m+1} + \frac{3 m^{3/2}}{2\sqrt{2}} \delta_{n,m-1} + \frac{\sqrt{m(m-1)(m-2)}}{2\sqrt{2}} \delta_{n,m-3}$ (*)

$$\langle n | \left(\frac{x}{x_0}\right)^3 | 0 \rangle = \frac{\sqrt{6}}{2\sqrt{2}} \delta_{n,3} + \frac{3}{2\sqrt{2}} \delta_{n,1}$$

$$c_1(\infty) = \frac{i\lambda\omega\tau}{i\omega\tau - 1} \cdot \frac{3}{2\sqrt{2}} \rightarrow P_1(\infty) = \frac{9}{8}\lambda^2 \frac{\omega^2\tau^2}{1+\omega^2\tau^2} \xrightarrow{\omega\tau=1} \underline{\underline{\frac{9}{16}\lambda^2}}$$

$$c_3(\infty) = \frac{i\lambda\omega\tau}{3i\omega\tau - 1} \cdot \frac{\sqrt{6}}{2\sqrt{2}} \rightarrow P_3(\infty) = \frac{3}{4}\lambda^2 \frac{\omega^2\tau^2}{1+9\omega^2\tau^2} \xrightarrow{\omega\tau=1} \underline{\underline{\frac{3}{40}\lambda^2}}$$

$$P_0(\infty) = 1 - P_1(\infty) - P_3(\infty) = \underline{\underline{1 - \frac{51}{80}\lambda^2}}$$

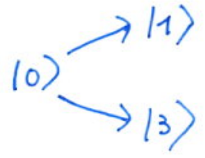
Z verjetnostjo $P_1(\infty)$ izmerimo energijo $\frac{3}{2}\hbar\omega$, z verjetnostjo $P_3(\infty)$ energijo $\frac{7}{2}\hbar\omega$ in z verjetnostjo $P_0(\infty)$ energijo $\frac{1}{2}\hbar\omega$. 3/4

b) V 1. redu časovno odvisne teorije motnje so neničelni:

$$c_0^{(1)}(t) = 1$$

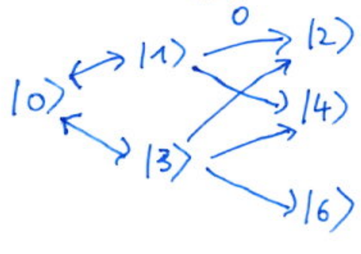
$$c_1^{(1)}(t) = -\frac{i}{\hbar} \int_0^t dt' \langle 1,t' | H'(t') | 0,t' \rangle$$

$$c_3^{(1)}(t) = -\frac{i}{\hbar} \int_0^t dt' \langle 3,t' | H'(t') | 0,t' \rangle$$

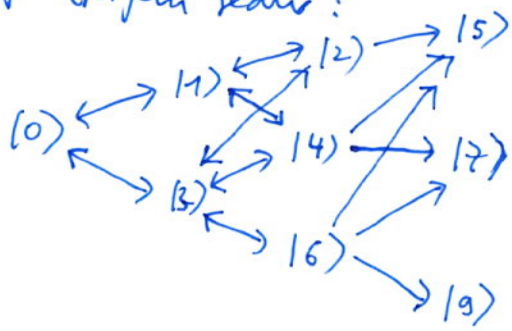


V 2. redu so mogoči tudi prehodi v stanja $|2\rangle$, $|4\rangle$ in $|6\rangle$ (*):

$$c_n^{(2)}(t) = \delta_{n0} - \frac{i}{\hbar} \int_0^t dt' \langle n,t' | H'(t') | m,t' \rangle c_m^{(1)}(t')$$



① b) ... V tretjem redu:



V stanje $|5\rangle$ z energijo $\frac{11}{2}\hbar\omega$ lahko pridemo na več načinov:

$|0\rangle \to |1\rangle \to |2\rangle \to |5\rangle$

$|0\rangle \to |1\rangle \to |4\rangle \to |5\rangle$

$|0\rangle \to |3\rangle \to |6\rangle \to |5\rangle$

$|0\rangle \to |3\rangle \to |4\rangle \to |5\rangle$

$|0\rangle \to |3\rangle \to |2\rangle \to |5\rangle$

$$C_5^{(3)}(\infty) = -\frac{i}{\hbar}\int_0^\infty dt \langle 5,t|H'(t)|2,t\rangle C_2^{(2)}(t) + \dots$$

$$= -\frac{i}{\hbar}\int_0^\infty dt \langle 5,t|H'(t)|2,t\rangle \left[-\frac{i}{\hbar}\int_0^t dt' \langle 2,t'|H'(t')|1,t'\rangle C_1^{(1)}(t')\right] + \dots =$$

$$= -\frac{i}{\hbar}\int_0^\infty dt \langle 5,t|H'(t)|2,t\rangle \left[-\frac{i}{\hbar}\int_0^t dt' \langle 2,t'|H'(t')|1,t'\rangle \left\{-\frac{i}{\hbar}\int_0^{t'} dt'' \langle 1,t''|H'(t'')|0,t''\rangle \cdot 1\right\}\right] + \dots$$

Razpisan je samo prispevek prvega načina. Če se amplitude vseh petih prispevkov ne seštejejo v nič, je torej neničelna verjetnost za prehod v stanje z energijo $\frac{11}{2}\hbar\omega$ pojavi v tretjem redu teorije motnje.

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② $H = J\vec{S}_1 \cdot \vec{S}_2$; $S_1 = S_2 = 1$

a) $\vec{S} = \vec{S}_1 + \vec{S}_2$

$$H = J\,\frac{S^2 - S_1^2 - S_2^2}{2} \Rightarrow H|SM\rangle = \frac{J\hbar^2}{2}\left(S(S+1) - 1\cdot 2 - 1\cdot 2\right)|SM\rangle =$$
$$\doteq \frac{J\hbar^2}{2}S(S+1) - 2J\hbar^2|SM\rangle$$

$E_2 = J\hbar^2$; $|22\rangle, |21\rangle, |20\rangle, |2-1\rangle, |2-2\rangle$

$E_1 = -J\hbar^2$; $|11\rangle, |10\rangle, |1-1\rangle$

$E_0 = -2J\hbar^2$; $|00\rangle$

} baza z dobrim skupnim spinom

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b) $S_{1z}|\psi_1\rangle = \hbar|\psi_1\rangle \longrightarrow |\psi_1\rangle = |1\rangle$

$S_{1x}|\psi_2\rangle = \hbar|\psi_2\rangle \longrightarrow |\psi_2\rangle = \frac{1}{2}|1\rangle + \frac{\sqrt{2}}{2}|0\rangle + \frac{1}{2}|-1\rangle$ ← iz vaj

$|\psi,0\rangle = |\psi_1\rangle|\psi_2\rangle = \frac{1}{2}|1\rangle|1\rangle + \frac{\sqrt{2}}{2}|1\rangle|0\rangle + \frac{1}{2}|1\rangle|-1\rangle$ ← v produktni bazi

Prepišano v bazo z dobrim skupnim spinom:

$$|\psi,0\rangle = \frac{1}{2}\left(|22\rangle\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{\sqrt{2}}|21\rangle + \frac{1}{\sqrt{2}}|11\rangle\right) + \frac{1}{2}\left(\frac{1}{\sqrt{6}}|20\rangle + \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{3}}|00\rangle\right)$$

$$|\psi,t\rangle = \left(\frac{1}{2}|22\rangle + \frac{1}{2}|21\rangle + \frac{1}{2\sqrt{6}}|20\rangle\right)e^{-iJ\hbar t} + \left(\frac{1}{2}|11\rangle + \frac{1}{2\sqrt{2}}|10\rangle\right)e^{iJ\hbar t} +$$
$$+ \frac{1}{2\sqrt{3}}|00\rangle e^{2iJ\hbar t}$$

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c) Prepišemo nazaj v produktno bazo in obdržimo samo stanja $|1\rangle|1\rangle$, $|1\rangle|0\rangle$ in $|1\rangle|-1\rangle$ $\left(S_{1z}|1\rangle|M_2\rangle = \hbar|1\rangle|M_2\rangle\right)$:

$$|\psi,t\rangle = \left(\frac{1}{2}|1\rangle|1\rangle + \frac{1}{2}\left[\frac{1}{\sqrt{2}}|1\rangle|0\rangle + \dots\right] + \frac{1}{2\sqrt{6}}\left[\frac{1}{\sqrt{6}}|1\rangle|-1\rangle + \dots\right]\right)e^{-iJ\hbar t} +$$
$$+ \left(\frac{1}{2}\left[\frac{1}{\sqrt{2}}|1\rangle|0\rangle + \dots\right] + \frac{1}{2\sqrt{2}}\left[\frac{1}{\sqrt{2}}|1\rangle|-1\rangle + \dots\right]\right)e^{iJ\hbar t} +$$
$$+ \frac{1}{2\sqrt{3}}\left[\frac{1}{\sqrt{3}}|1\rangle|-1\rangle + \dots\right]e^{2iJ\hbar t} =$$
$$= \frac{1}{2}|1\rangle|1\rangle e^{-iJ\hbar t} + \left(\frac{1}{2\sqrt{2}}e^{-iJ\hbar t} + \frac{1}{2\sqrt{2}}e^{iJ\hbar t}\right)|1\rangle|0\rangle +$$
$$+ \left(\frac{1}{12}e^{-iJ\hbar t} + \frac{1}{4}e^{iJ\hbar t} + \frac{1}{6}e^{2iJ\hbar t}\right)|1\rangle|-1\rangle + \dots \xrightarrow{J\hbar t = \frac{\pi}{2}}$$
$$\longrightarrow -\frac{i}{2}|1\rangle|1\rangle + 0|1\rangle|0\rangle + \left(-\frac{i}{12} + \frac{i}{4} - \frac{1}{6}\right)|1\rangle|-1\rangle + \dots$$

$$P_{S_{1z}=\hbar} = \left|-\frac{i}{2}\right|^2 + \left|\frac{i-1}{6}\right|^2 = \frac{1}{4} + \frac{1}{18} = \underline{\underline{\frac{11}{36}}}$$

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