1) $H = \frac{p_1^2}{2m_1} - \frac{\lambda}{\hbar^2}\vec{S}_1\cdot\vec{S}_2\,\delta(x_1) \qquad s_1 = s_2 = 1/2$

a) $\vec{S} = \vec{S}_1 + \vec{S}_2 \qquad s = 1$ ali $\emptyset$

$$-\frac{\lambda}{\hbar^2}\vec{S}_1\cdot\vec{S}_2 = -\frac{\lambda}{2\hbar^2}\left(S^2 - S_1^2 - S_2^2\right) = -\frac{\lambda}{2\hbar^2}\left(S^2 - \frac{3\hbar^2}{2}\right) \Rightarrow \begin{cases} -\frac{\lambda}{4} & ; s = 1 \\ \frac{3}{4}\lambda & ; s = 0 \end{cases}$$

(i) $\underline{s = 1}$, $M = 1, 0, -1$: $\langle x_1 | \Psi_{1M}\rangle = \psi_1(x_1)\,|1M\rangle$

$$\left[\frac{p_1^2}{2m_1} - \frac{\lambda}{4}\delta(x_1)\right]\psi_1(x_1) = E\,\psi_1(x_1)$$

$$\psi_1(x_1) = \begin{cases} e^{ikx_1} + r_1 e^{-ikx_1} & ; x_1 < 0 \\ t_1 e^{ikx_1} & ; x_1 > 0 \end{cases} \qquad k = \sqrt{\frac{2m_1E}{\hbar^2}}$$

(ii) $\underline{s = 0}$ $\langle x_1 | \Psi_{00}\rangle = \psi_0(x_1)\,|00\rangle$

1/2⁻

$$\left[\frac{p_1^2}{2m_1} + \frac{3\lambda}{4}\delta(x_1)\right]\psi_0(x_1) = E\,\psi_0(x_1)$$

$$\psi_0(x_1) = \begin{cases} e^{ikx_1} + r_0 e^{-ikx_1} & ; x_1 < 0 \\ t_0 e^{ikx_1} & ; x_1 > 0 \end{cases} \qquad k = \sqrt{\frac{2m_1E}{\hbar^2}}$$

b) $|\uparrow\rangle|\downarrow\rangle = \frac{|10\rangle + |00\rangle}{\sqrt{2}}$

$$\langle x_1|\Psi\rangle = \frac{1}{\sqrt{2}}\langle x_1|\Psi_{10}\rangle + \frac{1}{\sqrt{2}}\langle x_1|\Psi_{00}\rangle$$

1/4

c) $x_1 > 0$: ~~$\frac{1}{\sqrt{2}}e^{ikx_1}t_1 + \frac{1}{\sqrt{2}}e^{ikx_1}t_0$~~ =

$$\frac{1}{\sqrt{2}}t_1 e^{ikx_1}|10\rangle + \frac{1}{\sqrt{2}}t_0 e^{ikx_1}|00\rangle =$$

$$= \frac{1}{2}(t_1 - t_0)e^{ikx_1}|\downarrow\rangle|\uparrow\rangle + \dots$$

$$|10\rangle = \frac{|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}} \qquad |00\rangle = \frac{|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}}$$

$$P_{(ii)} = \left|\frac{1}{2}(t_1 - t_0)\right|^2 =$$

iz vaj: $t_1 = \frac{k}{k + iK_1} \qquad K_1 = -\frac{m\lambda}{4\hbar} = -\frac{1}{4}K$

$t_0 = \frac{k}{k + iK_2} \qquad K_2 = \frac{3m\lambda}{4\hbar} = \frac{3}{4}K$

$K = \frac{m\lambda}{\hbar^2}$

$$= \frac{1}{4}\left|\frac{k}{k - \frac{i}{4}K} - \frac{k}{k + i\frac{3}{4}K}\right|^2 =$$

$$= \frac{k^2}{4}\left|\frac{k + i\frac{3}{4}K - k + \frac{i}{4}K}{(k - \frac{i}{4}K)(k + i\frac{3}{4}K)}\right|^2 =$$

$$= \frac{k^2K^2}{4\left(k^2 + \frac{K^2}{16}\right)\left(k^2 + \frac{9K^2}{16}\right)} = \frac{EE_0}{4\left(E + \frac{1}{16}E_0\right)\left(E + \frac{9}{16}E_0\right)}$$

$$E_0 = \frac{\hbar^2K^2}{2m} = \frac{m\lambda^2}{2\hbar^2}$$

c) ...

$$p_{(ii)}(E) = \frac{E E_0}{4\left(E+\frac{E_0}{16}\right)\left(E+\frac{9E_0}{16}\right)}$$

$$\frac{d p_{(ii)}(E)}{dE} = 0 = \frac{\cancel{E_0}}{\cancel{4}\left(E+\frac{E_0}{16}\right)\left(E+\frac{9E_0}{16}\right)} - \frac{E\cancel{E_0}\left(E+\frac{9}{16}E_0+E+\frac{1}{16}E_0\right)}{\cancel{4}\left(E+\frac{E_0}{16}\right)^2\left(E+\frac{9E_0}{16}\right)^2}$$

$$\left(E+\frac{E_0}{16}\right)\left(E+\frac{9E_0}{16}\right) = E\left(2E+\frac{5}{8}E_0\right)$$

$$E^2+\cancel{\frac{5}{8}EE_0}+\frac{9}{256}E_0^2 = 2E^2+\cancel{\frac{5}{8}EE_0}$$

$$E = \frac{3}{16}E_0 \Rightarrow \boxed{E = \frac{3\lambda^2 m}{32\hbar^2}}$$

1/4+

a) $H = \lambda\left(|1\rangle\langle 1| + |2\rangle\langle 2| + 2|2\rangle\langle 1| + 2|1\rangle\langle 2|\right)$

$|\psi\rangle = \alpha|1\rangle + \beta|2\rangle$

$H|\psi\rangle = E|\psi\rangle \longrightarrow \begin{pmatrix} \lambda & 2\lambda \\ 2\lambda & \lambda \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$$(\lambda - E)^2 - (2\lambda)^2 = 0$$

$$E = \lambda \pm 2\lambda = \begin{cases} 3\lambda \\ -\lambda \end{cases}$$

$$H\left(\frac{|1\rangle + |2\rangle}{\sqrt{2}}\right) = \overset{E_a}{\overset{\Uparrow}{3\lambda}}\left(\frac{|1\rangle + |2\rangle}{\sqrt{2}}\right) \Rightarrow |a\rangle$$

$$H\left(\frac{|1\rangle - |2\rangle}{\sqrt{2}}\right) = \overset{E_b}{\overset{\Uparrow}{-\lambda}}\left(\frac{|1\rangle - |2\rangle}{\sqrt{2}}\right) \Rightarrow |b\rangle$$

1/4⁻

b) $|\psi\rangle = \alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle + \delta|4\rangle$

$$\left(\begin{array}{cc|cc} \lambda & 2\lambda & \multicolumn{2}{c}{\emptyset} \\ 2\lambda & \lambda & & \\ \hline \multicolumn{2}{c|}{\emptyset} & -\lambda & 2\lambda \\ & & 2\lambda & -\lambda \end{array}\right)\begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = E\begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}$$

$E = \lambda \pm 2\lambda,\ -\lambda \pm 2\lambda$

blok-diagonalna matrika!

Poleg lastnih energij [redacted] in lastnih funkcij iz točke a) imamo še:

$$H\left(\frac{|3\rangle + |4\rangle}{\sqrt{2}}\right) = \overset{E_c}{\overset{\Uparrow}{\lambda}}\left(\frac{|3\rangle + |4\rangle}{\sqrt{2}}\right) \Rightarrow |c\rangle$$

$$H\left(\frac{|3\rangle - |4\rangle}{\sqrt{2}}\right) = \overset{E_d}{\overset{\Uparrow}{-3\lambda}}\left(\frac{|3\rangle - |4\rangle}{\sqrt{2}}\right) \Rightarrow |d\rangle$$

1/4⁻

c) $H' = \Delta\left(|1\rangle\langle 3| + |2\rangle\langle 4|\right) + \Delta^*\left(|3\rangle\langle 1| + |4\rangle\langle 2|\right)$

$$\left.\begin{aligned} H'|a\rangle &= \Delta^*\left(\frac{|3\rangle + |4\rangle}{\sqrt{2}}\right) = \Delta^*|c\rangle \\ H'|b\rangle &= \Delta^*\left(\frac{|3\rangle - |4\rangle}{\sqrt{2}}\right) = \Delta^*|d\rangle \\ H'|c\rangle &= \Delta|a\rangle \\ H'|d\rangle &= \Delta|b\rangle \end{aligned}\right\} \Rightarrow \text{v 1. redu ni popravka}$$

v 2. redu:

$$\tilde{E}_a = 3\lambda + \frac{|\langle c|\Delta^*|c\rangle|^2}{E_a - E_c} = 3\lambda + \frac{|\Delta|^2}{2\lambda}$$

$$\tilde{E}_b = -\lambda + \frac{|\langle d|\Delta^*|d\rangle|^2}{E_b - E_d} = -\lambda + \frac{|\Delta|^2}{2\lambda}$$

$$\tilde{E}_c = \lambda + \frac{|\langle a|\Delta|a\rangle|^2}{E_c - E_a} = \lambda - \frac{|\Delta|^2}{2\lambda}$$

$$\tilde{E}_d = -3\lambda + \frac{|\langle b|\Delta|b\rangle|^2}{E_d - E_b} = -3\lambda - \frac{|\Delta|^2}{2\lambda}$$

1/4⁺

d) $\{UK, H_3\} = ?$

$$UKH_3 + H_3UK = \left[UKH_3(UK)^{-1} + H_3\right]UK =$$
$$= (UKH_3KU^{-1} + H_3)UK =$$
$$= (UKH_3KU + H_3)UK \qquad \left(UU^\dagger = I \Rightarrow U^{-1} = U^\dagger = U\right)$$

$$KH_3K = H_2 + \Delta^*(|1\rangle\langle 3| + |2\rangle\langle 4|) + \Delta(|3\rangle\langle 1| + |4\rangle\langle 2|) \qquad \begin{pmatrix}\lambda \in \mathbb{R} \\ \Delta \in \mathbb{C}\end{pmatrix}$$

$$UKH_3KU = \lambda\Big[-|4\rangle(-\langle 4|) + |3\rangle\langle 3| + 2|3\rangle(-\langle 4|) + 2(-|4\rangle)\langle 3| -$$
$$- |2\rangle\langle 2| - (-|1\rangle)(-\langle 1|) + 2(-|1\rangle\langle 2|) + 2|2\rangle(-\langle 1|)\Big] +$$
$$+ \Delta^*\left(-|4\rangle\langle 2| + |3\rangle(-\langle 1|)\right) + \Delta\left(|2\rangle(-\langle 4|) + (-|1\rangle)\langle 3|\right) =$$

$$= \lambda\Big[|4\rangle\langle 4| + |3\rangle\langle 3| - 2|3\rangle\langle 4| - 2|4\rangle\langle 3| -$$
$$- |2\rangle\langle 2| - |1\rangle\langle 1| - 2|1\rangle\langle 2| - 2|2\rangle\langle 1|\Big] -$$
$$- \Delta^*(|4\rangle\langle 2| + |3\rangle\langle 1|) - \Delta(|2\rangle\langle 4| + |1\rangle\langle 3|) = -H_3$$

$$\Rightarrow \{UK, H_3\} = 0 \qquad 1/4^-$$

e) $H_3|\psi\rangle = E|\psi\rangle$

$$H_3 UK|\psi\rangle = -UKH_3|\psi\rangle = -UKE|\psi\rangle = -E\,UK|\psi\rangle$$

$\Rightarrow$ Lastne energije nastopajo v parih $\pm E$.

$1/4^-$