① (a) $H = J\vec{S}_1\cdot\vec{S}_2 = J\,\dfrac{S^2 - S_1^2 - S_2^2}{2} = J\left(\dfrac{S^2}{2} - \dfrac{11}{8}\hbar^2\right)$

$|S_1 - S_2| \le S \le S_1 + S_2$

$$S = \begin{cases} 3/2: & E = \frac{1}{2}J\hbar^2, \quad |SM\rangle = |\tfrac{3}{2}\,\tfrac{3}{2}\rangle, |\tfrac{3}{2}\,\tfrac{1}{2}\rangle, |\tfrac{3}{2}\,{-\tfrac{1}{2}}\rangle, |\tfrac{3}{2}\,{-\tfrac{3}{2}}\rangle \\ 1/2: & E = -J\hbar^2, \quad |SM\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle, |\tfrac{1}{2}\,{-\tfrac{1}{2}}\rangle \end{cases}$$ ++++

(b) i. $|\psi\rangle = |1\rangle|\tfrac{1}{2}\rangle = |\tfrac{3}{2}\,\tfrac{3}{2}\rangle \rightarrow$ meritev energije da rezultat $\frac{1}{2}J\hbar^2$. ++

ii. $|\psi\rangle = |1\rangle\left(\cos\tfrac{\vartheta}{2}|\tfrac{1}{2}\rangle + \sin\tfrac{\vartheta}{2}\, e^{i\varphi}|{-\tfrac{1}{2}}\rangle\right) = |1\rangle\left(\tfrac{1}{\sqrt{2}}|\tfrac{1}{2}\rangle + \tfrac{1}{\sqrt{2}}|{-\tfrac{1}{2}}\rangle\right) =$

$\vartheta = \frac{\pi}{2}, \ \varphi = 0$

$= \frac{1}{\sqrt{2}}|\tfrac{3}{2}\,\tfrac{3}{2}\rangle + \frac{1}{\sqrt{2}}\left[\sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\right] =$

$= \frac{1}{\sqrt{2}}|\tfrac{3}{2}\,\tfrac{3}{2}\rangle + \frac{1}{\sqrt{6}}|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \frac{1}{\sqrt{3}}|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$ +++

↳ meritev energije da rezultat $\frac{1}{2}J\hbar^2$ z verjetnostjo $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 = \frac{2}{3}$ in rezultat $-J\hbar^2$ z verjetnostjo $\left(\frac{1}{\sqrt{3}}\right)^2 = 1/3$.

iii. enako kot pri točki i., saj se pri transformaciji $xyz \rightarrow zxy$ $H$ ne spremeni, $|\psi\rangle$ pa se transformira v tistega iz točke i. +++

② (a) $H = -\gamma B_0 S_z$

$$E = \begin{cases} -\gamma B_0\hbar\,; & |SM\rangle = |11\rangle \\ 0\,; & |SM\rangle = |10\rangle \\ \gamma B_0\hbar\,; & |SM\rangle = |1\,{-1}\rangle \end{cases}$$ +++

(b) $c_M(t) = c_M(0) - \frac{i}{\hbar}\int_0^t dt'\,\langle M,t'|\,-\gamma\vec{S}\cdot\vec{B}_1(t')\,|11,t'\rangle$

$c_M(0) = \delta_{M1}$

$-\gamma\vec{S}\cdot\vec{B}_1(t') = -\gamma B_1\left(S_x\cos\omega t' + S_y\sin\omega t'\right) =$

$= -\frac{\gamma B_1}{2}\left[(S_+ + S_-)\cos\omega t' + \frac{S_+ - S_-}{i}\sin\omega t'\right] =$

$= -\frac{\gamma B_1}{2}\left[S_+ e^{-i\omega t'} + S_- e^{i\omega t'}\right]$

$-\gamma\vec{S}\cdot\vec{B}_1(t)|11\rangle = -\frac{\gamma B_1}{2}\sqrt{2}\,\hbar\, e^{i\omega t}|10\rangle$

$c_1(t) = 1$

$c_{-1}(t) = 0$

$c_0(t) = -\frac{i}{\cancel{\hbar}}\int_0^t dt'\, e^{i\gamma B_0 t'}\left(-\frac{\gamma B_1}{2}\sqrt{2}\,\cancel{\hbar}\, e^{i\omega t'}\right) =$

$= \frac{i\gamma B_1}{\sqrt{2}}\int_0^t dt'\, e^{i(\gamma B_0 + \omega)t'} = \frac{i\gamma B_1}{\sqrt{2}}\,\frac{1}{i(\gamma B_0 + \omega)}\, e^{i(\gamma B_0 + \omega)t'}\Big|_0^t = \frac{\gamma B_1}{\sqrt{2}(\gamma B_0 + \omega)}\left[e^{i(\gamma B_0 + \omega)t} - 1\right]$

$|\psi,t\rangle = |11\rangle + \frac{\gamma B_1}{\sqrt{2}(\gamma B_0 + \omega)}\left[e^{i(\gamma B_0 + \omega)t} - 1\right]|10\rangle$ +++ +++

② (b) $|\psi,t\rangle = |11\rangle + \frac{\gamma B_1}{\sqrt{2}(\gamma B_0+\omega)}\left[e^{i(\gamma B_0+\omega)t}-1\right]|10\rangle$

$\frac{\gamma B_1}{\gamma B_0+\omega} \ll 1$ sicer uporaba 1. reda teorije motnje ni upravičena!

(c) $C_1(t) = 1 + \mathcal{O}(B_1^2)$

$C_0(t) = \frac{\gamma B_1}{\sqrt{2}(\gamma B_0+\omega)}\left[e^{i(\gamma B_0+\omega)t}-1\right] + \mathcal{O}(B_1^2)$

$C_{-1}(t) = \mathcal{O}(B_1^2)$

$|C_1(t)|^2 = 1 + \mathcal{O}(B_1^2)$

$|C_0(t)|^2 = 2\left(\frac{\gamma B_1}{\gamma B_0+\omega}\right)^2 \sin^2\frac{\gamma B_0+\omega}{2}t + \mathcal{O}(B_1^3)$

$|C_{-1}(t)|^2 = \mathcal{O}(B_1^4)$

V okviru natančnosti $\mathcal{O}(B_1^2)$ pri meritvi $S_z$ izmerimo rezultat 0 z verjetnostjo $2\left(\frac{\gamma B_1}{\gamma B_0+\omega}\right)^2 \sin^2\frac{\gamma B_0+\omega}{2}t$ in rezultat $\hbar$ z verjetnostjo $1-2\left(\frac{\gamma B_1}{\gamma B_0+\omega}\right)^2 \sin^2\frac{\gamma B_0+\omega}{2}t$.

+++