① a) $|\psi\rangle = \sqrt{\frac{1}{3}}|1\rangle + i\sqrt{\frac{2}{3}}|2\rangle$

$$\langle H\rangle = \langle\psi|H|\psi\rangle = \langle\psi|\left(\sqrt{\tfrac{1}{3}}E_1|1\rangle + i\sqrt{\tfrac{2}{3}}E_2|2\rangle\right) =$$

1/4⁻

$$= \left|\sqrt{\tfrac{1}{3}}\right|^2 E_1 + \left|i\sqrt{\tfrac{2}{3}}\right|^2 E_2 = \tfrac{1}{3}E_1 + \tfrac{2}{3}E_2 = \frac{3\hbar^2\pi^2}{2ma^2} \quad \left(E_n = \frac{\hbar^2\pi^2 n^2}{2ma^2}\right)$$

b) $|\psi,t\rangle = \sqrt{\frac{1}{3}}e^{-i\omega_1 t}|1\rangle + i\sqrt{\frac{2}{3}}e^{-i\omega_2 t}|2\rangle\,; \quad \omega_n \equiv \frac{E_n}{\hbar}$ 1/4

c) $\langle H,t\rangle = \left|\sqrt{\frac{1}{3}}e^{-i\omega_1 t}\right|^2 E_1 + \left|i\sqrt{\frac{2}{3}}e^{-i\omega_2 t}\right|^2 E_2 = \frac{1}{3}E_1 + \frac{2}{3}E_2 = \langle H,0\rangle$

Lahko tudi s pomočjo Ehrenfestovega izreka: 1/4⁻

$$\frac{d}{dt}\langle H,t\rangle = \frac{i}{\hbar}\langle [H,H],t\rangle = 0 \longrightarrow \langle H,t\rangle = \langle H,0\rangle$$

d) $|\psi,t\rangle = c_1(t)|1\rangle + c_2(t)|2\rangle$

$$\langle p,t\rangle = |c_1(t)|^2\langle 1|p|1\rangle + c_1^*(t)c_2(t)\langle 1|p|2\rangle + c_2^*(t)c_1(t)\underbrace{\langle 2|p|1\rangle}_{\langle 1|p|2\rangle^*} + |c_2(t)|^2\langle 2|p|2\rangle$$

$$\langle 1|p|1\rangle = \int_{-a/2}^{a/2} dx\, \underbrace{\psi_1^*(x)}_{\text{soda}}[-i\hbar]\underbrace{\psi_1'(x)}_{\text{liha}} = 0$$

$$\langle 2|p|2\rangle = \int_{-a/2}^{a/2} dx\, \underbrace{\psi_2^*(x)}_{\text{liha}}[-i\hbar]\underbrace{\psi_2'(x)}_{\text{soda}} = 0$$

$$\langle p,t\rangle = 2\,\mathrm{Re}\left\{c_1^*(t)c_2(t)\langle 1|p|2\rangle\right\} =$$

$$= 2\,\mathrm{Re}\left\{\sqrt{\tfrac{1}{3}}e^{i\omega_1 t}\cdot i\sqrt{\tfrac{2}{3}}e^{-i\omega_2 t}\int_{-a/2}^{a/2}\underbrace{\psi_1^*(x)}_{\text{soda}}[-i\hbar]\underbrace{\psi_2'(x)}_{\text{soda}}dx\right\} =$$

$$= \frac{4\sqrt{2}\hbar}{3}\mathrm{Re}\left\{e^{-i(\omega_2-\omega_1)t}\int_0^{a/2}\psi_1^*(x)\psi_2'(x)dx\right\} =$$

$$= \frac{16\sqrt{2}\hbar\pi}{3a^2}\mathrm{Re}\left\{e^{-i(\omega_2-\omega_1)t}\int_0^{a/2}\cos\frac{\pi x}{a}\cos\frac{2\pi x}{a}dx\right\} =$$

$$= \frac{16\sqrt{2}\hbar\pi}{3a^2}\cos[(\omega_2-\omega_1)t]\int_0^{\pi/2}(1-2\sin^2 u)\cos u\,du\,\frac{a}{\pi} =$$

$$= \frac{16\sqrt{2}\hbar}{3a}\cos[(\omega_2-\omega_1)t]\int_0^1(1-2v^2)dv =$$

$$= \frac{16\sqrt{2}\hbar}{9a}\cos[(\omega_2-\omega_1)t]$$ 1/2⁻

① e) $\frac{d\langle p, t\rangle}{dt} \overset{?}{=} \langle -\frac{dV}{dx}, t\rangle$

$$\frac{d\langle p,t\rangle}{dt} = -\frac{16\sqrt{2}\hbar}{9a}(\omega_2-\omega_1)\sin[(\omega_2-\omega_1)t] = -\frac{8\sqrt{2}\pi^2\hbar^2}{3ma^2}\sin[(\omega_2-\omega_1)t]$$

$$\langle -\frac{dV}{dx}, t\rangle = \lim_{V_0\to\infty}\langle V_0\delta(x+\frac{a}{2}) - V_0\delta(x-\frac{a}{2}), t\rangle = \lim_{V_0\to\infty} V_0\left(|\psi(-\tfrac{a}{2},t)|^2 - |\psi(\tfrac{a}{2},t)|^2\right)$$

$$\psi(-\tfrac{a}{2},t) = c_1(t)\psi_1(-\tfrac{a}{2}) + c_2(t)\psi_2(-\tfrac{a}{2}) = c_1(t)\psi_1(\tfrac{a}{2}) - c_2(t)\psi_2(\tfrac{a}{2})$$

$$\psi(\tfrac{a}{2},t) = c_1(t)\psi_1(\tfrac{a}{2}) + c_2(t)\psi_2(\tfrac{a}{2})$$

$$\langle -\frac{dV}{dx}, t\rangle = \lim_{V_0\to\infty} V_0\cdot(-4)\mathrm{Re}\{c_1^*(t)c_2(t)\psi_1^*(\tfrac{a}{2})\psi_2(\tfrac{a}{2})\} =$$

$$= \lim_{V_0\to\infty} V_0\cdot(-4)\mathrm{Re}\left\{\sqrt{\tfrac{1}{3}}e^{i\omega_1 t}\, i\sqrt{\tfrac{2}{3}}e^{-i\omega_2 t}\psi_1(\tfrac{a}{2})\psi_2(\tfrac{a}{2})\right\} =$$

$$= -\frac{4\sqrt{2}}{3}\sin[(\omega_2-\omega_1)t]\lim_{V_0\to\infty} V_0\,\psi_1(\tfrac{a}{2})\psi_2(\tfrac{a}{2})$$

$\psi_1(x)$ in $\psi_2(x)$ sta osnovno in 1. vzbujeno stanje končne potencialne jame z globino V_0.

⊛ $\psi_1(x)$ je soda, $\operatorname{tg}\frac{u}{2} = \sqrt{(\frac{u_0}{u})^2 - 1}$; $u = ka$, $u_0^2 = \frac{2mV_0a^2}{\hbar^2}$

$\psi_1(|x|<\frac{a}{2}) = A\cos kx$ $\qquad u = \pi - \varepsilon$ za velik V_0

$\psi_1(x>\frac{a}{2}) = Be^{-\kappa(x-\frac{a}{2})}$ $\qquad \operatorname{tg}\frac{u}{2} \approx \frac{2}{\varepsilon} \approx \sqrt{(\frac{u_0}{\pi})^2-1} \approx \frac{u_0}{\pi} \Rightarrow \varepsilon \approx \frac{2\pi}{u_0}$

$\psi_1(\frac{a}{2}) = B = A\cos\frac{u}{2} \approx A\frac{\pi}{u_0}$

Ker je $\frac{\pi}{u_0} \propto \frac{1}{\sqrt{V_0}}$, lahko uporabim normalizacijsko konstanto za ∞ potencialno jamo $A = \sqrt{\frac{2}{a}}$, saj bi popravki izginili pri $\lim_{V_0\to\infty}$.

$\psi_1(\frac{a}{2}) \approx \sqrt{\frac{2}{a}}\frac{\pi}{u_0}$

⊛ $\psi_2(x)$ je liha, ..., $\psi_2(\frac{a}{2}) \approx \sqrt{\frac{2}{a}}\cdot\frac{2\pi}{u_0}$

$$\langle -\frac{dV}{dx}, t\rangle = -\frac{4\sqrt{2}}{3}\sin[(\omega_2-\omega_1)t]\lim_{V_0\to\infty} V_0\frac{4\pi^2}{a u_0^2} =$$

$$= -\frac{4\sqrt{2}}{3}\sin[(\omega_2-\omega_1)t]\frac{4\pi^2\hbar^2}{a\cdot 2ma^2} = -\frac{8\sqrt{2}\pi^2\hbar^2}{3ma^3}\sin[(\omega_2-\omega_1)t]$$ ✓

1/2

$\Sigma = 1\,1/2$

② a) $H_0|0\rangle_0 = \frac{1}{2}\hbar\omega_0|0\rangle_0$; $\omega_0 = \sqrt{\frac{k_0}{m}}$

$a|0\rangle_0 = 0|0\rangle_0$ osnovno stanje H_0 je koherentno stanje H_0 z lastno vrednostjo $z=0$

$\langle x\rangle = \sqrt{2}\, x_{00}\, \mathrm{Re}\, z = 0$; $x_{00} = \sqrt{\frac{\hbar}{m\omega_0}}$

$\langle p\rangle = \sqrt{2}\, p_{00}\, \mathrm{Im}\, z = 0$; $p_{00} = \hbar/x_{00}$ 1/4 +

$\delta^2 x = \langle x^2\rangle = \frac{x_{00}^2}{2}$

$\delta^2 p = \langle p^2\rangle = \frac{p_{00}^2}{2}$

b) $[H,x] = \left[\frac{p^2}{2m}, x\right] = -\frac{i\hbar}{m}p \longrightarrow \frac{d}{dt}x(t) = \frac{p(t)}{m}$

$[H,p] = \left[\frac{kx^2}{2}, p\right] = i\hbar k x \longrightarrow \frac{d}{dt}p(t) = -k\,x(t)$

$\frac{d^2}{dt^2}x(t) = \frac{1}{m}\frac{d}{dt}p(t) = -\frac{k}{m}x(t) = -\omega^2 x(t)$; $\omega = \sqrt{\frac{k}{m}}$

$x(t) = A\cos\omega t + B\sin\omega t$ & začetna pogoja $x(0) = x$,

$p(0) = m\frac{d}{dt}x(t)\Big|_{t=0} = p$

$x(t) = x\cos\omega t + \frac{p}{m\omega}\sin\omega t$ 1/4 +

Alternativna izpeljava:

$x(t) = \frac{x_0}{\sqrt{2}}\left(a(t) + a^\dagger(t)\right) = \frac{x_0}{\sqrt{2}}\left(a e^{-i\omega t} + a^\dagger e^{i\omega t}\right) =$; $x_0 = \sqrt{\frac{\hbar}{m\omega}}$

$= \frac{x_0}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left(\frac{x}{x_0} + i\frac{p}{p_0}\right)e^{-i\omega t} + \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} - i\frac{p}{p_0}\right)e^{i\omega t}\right) = x\cos\omega t + \frac{p}{m\omega}\sin\omega t$

c) $\langle x, t\rangle = \langle\psi,0|x(t)|\psi,0\rangle = \langle x\rangle\cos\omega t + \frac{\langle p\rangle}{m\omega}\sin\omega t = 0$

$\delta^2 x(t) = \langle x^2, t\rangle = \langle x^2\rangle\cos^2\omega t + \frac{\langle xp + px\rangle}{m\omega}\cos\omega t\sin\omega t + \frac{\langle p^2\rangle}{m^2\omega^2}\sin^2\omega t$

$\langle xp + px\rangle = 2\,\mathrm{Re}\langle xp\rangle = 2\,\mathrm{Re}\left\{\int_{-\infty}^{\infty}dx\, \psi_{00}^*(x)\, x\left(-i\hbar\frac{d}{dx}\right)\psi_{00}(x)\right\} = 0$, saj je $\psi_{00}(x)$ realna

$\delta^2 x(t) = \delta^2 x\cos^2\omega t + \frac{\delta^2 p}{m^2\omega^2}\sin^2\omega t =$

$= \delta^2 x\left[\cos^2\omega t + \frac{\delta^2 p}{m^2\omega^2\delta^2 x}\sin^2\omega t\right] =$; $\frac{\delta^2 p}{m^2\omega^2\delta^2 x} = \frac{\hbar^2}{2m^2\omega^2\delta^2 x} = \frac{x_0^4}{x_{00}^4} = \frac{\omega_0^2}{\omega^2} = \frac{k_0}{k}$

$= \delta^2 x\left[\cos^2\omega t + \frac{k_0}{k}\sin^2\omega t\right] =$

$= \delta^2 x\left[1 + \left(\frac{k_0}{k} - 1\right)\sin^2\omega t\right]$ 1/4 +

$\Sigma = 1$