1 a) $E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$; $\psi_n(x) = \sqrt{\frac{2}{a}} \cdot \begin{cases} \cos\frac{n\pi x}{a} & \text{za } n \text{ lih} \\ \sin\frac{n\pi x}{a} & \text{za } n \text{ sod} \end{cases}$; $n \in \mathbb{N}$ 1/4

b) $E_n^{(1)} = \langle n | H' | n \rangle = \int_{-a/2}^{a/2} dx\, \psi_n^*(x)\, \lambda \sin\frac{\pi x}{a}\, \psi_n(x) = 0$, saj je integrand lih, ker je $\psi_n(x)$ soda za lih n in liha za sod n. 1/4

c) $$E_1^{(2)} = \sum_{n>1} \frac{|\langle n|H'|1\rangle|^2}{E_1 - E_n}$$

$$\langle n|H'|1\rangle = \int_{-a/2}^{a/2} dx\, \psi_n^*(x)\, \lambda \sin\frac{\pi x}{a} \sqrt{\frac{2}{a}} \cos\frac{\pi x}{a} =$$

$$= \frac{\lambda}{2} \int_{-a/2}^{a/2} dx\, \psi_n^*(x) \underbrace{\sqrt{\frac{2}{a}} \sin\frac{2\pi x}{a}}_{\psi_2(x)} = \frac{\lambda}{2} \delta_{n,2}$$

$$E_1^{(2)} = \frac{\left(\frac{\lambda}{2}\right)^2}{\frac{\hbar^2\pi^2}{2ma^2}(1^2 - 2^2)} = -\frac{\lambda^2 m a^2}{6\pi^2\hbar^2}$$ 1/4

d) V prvem redu teorije motnje je valovna funkcija osnovnega stanja

$$|1\rangle + \sum_{n>1} \frac{\langle n|H'|1\rangle}{E_1 - E_n} |n\rangle = |1\rangle - \frac{\lambda m a^2}{3\pi^2\hbar^2} |2\rangle$$

Verjetnost, da delec najdemo v območju $x<0$, je

$$\int_{-a/2}^{0} dx\, \left|\psi_1(x) - \frac{\lambda m a^2}{3\pi^2\hbar^2} \psi_2(x)\right|^2 =$$

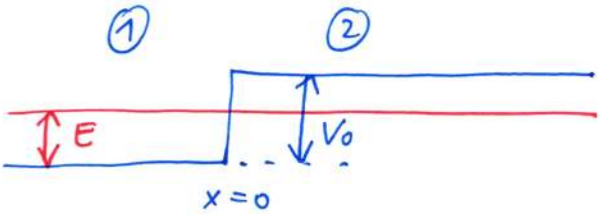
$$= \int_{-a/2}^{0} dx |\psi_1(x)|^2 - \frac{2\lambda m a^2}{3\pi^2\hbar^2} \int_{-a/2}^{0} \psi_1(x)\psi_2(x)\, dx + \mathcal{O}(\lambda^2) =$$

$$= \frac{1}{2} - \frac{2\lambda m a^2}{3\pi^2\hbar^2} \int_{-a/2}^{0} \sqrt{\frac{2}{a}} \cos\frac{\pi x}{a} \sqrt{\frac{2}{a}} \sin\frac{2\pi x}{a}\, dx + \mathcal{O}(\lambda^2) =$$

$$= \frac{1}{2} - \frac{8\lambda m a}{3\pi^2\hbar^2} \int_{-a/2}^{0} \cos^2\frac{\pi x}{a} \sin\frac{\pi x}{a}\, dx + \mathcal{O}(\lambda^2) = \frac{1}{2} + \frac{8\lambda m a^2}{9\pi^3\hbar^2} + \mathcal{O}(\lambda^2)$$ 1/4

Verjetnost se poveča za $\frac{8\lambda m a^2}{9\pi^3\hbar^2}$.

2 a)



$$k=\sqrt{\frac{2mE}{\hbar^2}} \qquad \kappa=\sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

$$\psi_1(x)=e^{ikx}+re^{-ikx} \qquad \psi_2(x)=te^{-\kappa x}$$

robna pogoja: $\psi_1(0)=\psi_2(0) \rightarrow 1+r=t$

$\psi_1'(0)=\psi_2'(0) \rightarrow ik(1-r)=-\kappa t$

$$r=\frac{k-i\kappa}{k+i\kappa}=\frac{\sqrt{E}-i\sqrt{V_0-E}}{\sqrt{E}+i\sqrt{V_0-E}}$$ 1/4

b) $H=\left[\frac{p^2}{2m}+V_0\theta(x)\right]I+V_0\theta(x)\frac{2\lambda}{\hbar}S_z$

$[H,S_z]=0$ ker $[I,S_z]=[S_z,S_z]=0$, pri čemer je I identiteta v spinskem delu Hilbertovega prostora. 1/4 −

c) $|\uparrow_x\rangle=\cos\frac{\vartheta}{2}|\uparrow\rangle+\sin\frac{\vartheta}{2}e^{i\varphi}|\downarrow\rangle=\frac{1}{\sqrt{2}}|\uparrow\rangle+\frac{1}{\sqrt{2}}|\downarrow\rangle$ 1/4 −

$\vartheta=\frac{\pi}{2},\ \varphi=0$

d) zaradi b) lahko najdemo lastne funkcije H, ki so hkrati tudi lastne funkcije S_z:

$\psi_{1\uparrow}(x)=e^{ikx}+r_\uparrow e^{-ikx}$

$\psi_{1\downarrow}(x)=e^{ikx}+r_\downarrow e^{-ikx}$

$V_{0\uparrow}=V_0(1+\lambda)=\frac{4}{3}V_0$

$V_{0\downarrow}=V_0(1-\lambda)=\frac{2}{3}V_0$

$E=\frac{V_0}{3}$

$$r_\uparrow=\frac{\sqrt{\frac{V_0}{3}}-i\sqrt{V_0}}{\sqrt{\frac{V_0}{3}}+i\sqrt{V_0}}=\frac{1-i\sqrt{3}}{1+i\sqrt{3}}=-\frac{1}{2}-i\frac{\sqrt{3}}{2}=e^{-i\frac{2\pi}{3}}$$

$$r_\downarrow=\frac{\sqrt{\frac{V_0}{3}}-i\sqrt{\frac{V_0}{3}}}{\sqrt{\frac{V_0}{3}}+i\sqrt{\frac{V_0}{3}}}=\frac{1-i}{1+i}=-i$$

Zaradi c) je $\psi_1(x)=\frac{1}{\sqrt{2}}\psi_{1\uparrow}(x)+\frac{1}{\sqrt{2}}\psi_{1\downarrow}(x)=$

$=e^{ikx}|\uparrow_x\rangle+e^{-ikx}\left[\frac{1}{\sqrt{2}}e^{-i\frac{2\pi}{3}}|\uparrow\rangle-\frac{i}{\sqrt{2}}|\downarrow\rangle\right]=$

$$=e^{ikx}|\uparrow_x\rangle+e^{-ikx}e^{-i\frac{2\pi}{3}}\underbrace{\left[\frac{1}{\sqrt{2}}|\uparrow\rangle+\frac{1}{\sqrt{2}}e^{i\frac{\pi}{6}}|\downarrow\rangle\right]}_{\text{spin odbitega delca kaže v smer } \vartheta=\frac{\pi}{2},\ \varphi=\frac{\pi}{6}.}$$ 1/2 −