① a) $\langle z|-z\rangle = \left(e^{-\frac{|z|^2}{2}}\sum_m \frac{(z^*)^m}{\sqrt{m!}}\langle m|\right)\left(e^{-\frac{|z|^2}{2}}\sum_m \frac{(-z)^m}{\sqrt{m!}}|m\rangle\right) =$

$$= e^{-|z|^2}\sum_m \frac{(z^*)^m}{\sqrt{m!}}\frac{(-z)^m}{\sqrt{m!}} = e^{-|z|^2}\sum_m \frac{(-|z|^2)^m}{m!} = e^{-2|z|^2}$$ 1/4

b) $|\psi\rangle = A(|z\rangle + |-z\rangle)$

$$\langle\psi|\psi\rangle = |A|^2\left(\langle z|z\rangle + \langle z|-z\rangle + \langle -z|z\rangle + \langle -z|-z\rangle\right) =$$

$$= |A|^2\left(1 + e^{-2|z|^2} + e^{-2|z|^2} + 1\right) =$$

$$= |A|^2\left(2 + 2e^{-2|z|^2}\right) = 1$$

$$|\psi\rangle = \frac{|z\rangle + |-z\rangle}{\sqrt{2(1+e^{-2|z|^2})}}$$ 1/4

c) $\langle H\rangle = \frac{1}{2}\hbar\omega + \hbar\omega\langle a^+a\rangle = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{2(1+e^{-2|z|^2})}\left(\langle z| + \langle -z|\right)a^+a\left(|z\rangle + |-z\rangle\right)$

$$= \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{2(1+e^{-2|z|^2})}\left(z^*z + \langle a(-z)|az\rangle + \langle az|a(-z)\rangle + (-z)^*(-z)\right)$$

$$= \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{2(1+e^{-2|z|^2})}\left(|z|^2 + (-z)^*z\langle -z|z\rangle + z^*(-z)\langle z|-z\rangle + |z|^2\right) =$$

$$= \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{2(1+e^{-2|z|^2})}\left(2|z|^2 - 2|z|^2e^{-2|z|^2}\right) =$$

$$= \frac{1}{2}\hbar\omega + \hbar\omega|z|^2\frac{1-e^{-2|z|^2}}{1+e^{-2|z|^2}} = \frac{1}{2}\hbar\omega + \hbar\omega|z|^2\,\mathrm{th}|z|^2$$ 1/4

d) $|\psi\rangle = \frac{1}{\sqrt{2(1+e^{-2|z|^2})}}\left[e^{-\frac{|z|^2}{2}}\left(|0\rangle + z|1\rangle + \frac{z^2}{\sqrt{2}}|2\rangle + \dots\right) + \right.$

$$\left. + e^{-\frac{|z|^2}{2}}\left(|0\rangle + (-z)|1\rangle + \frac{(-z)^2}{\sqrt{2}}|2\rangle + \dots\right)\right] =$$

$$= \frac{e^{-\frac{|z|^2}{2}}}{\sqrt{2(1+e^{-2|z|^2})}}\left[2|0\rangle + \sqrt{2}\,z^2|2\rangle + \dots\right]$$

$$p_{E=\frac{\hbar\omega}{2}} = \frac{2e^{-|z|^2}}{1+e^{-2|z|^2}} = \frac{1}{\mathrm{ch}|z|^2}$$

$$p_{E=\frac{3}{2}\hbar\omega} = 0$$

$$p_{E>2\hbar\omega} = 1 - p_{E=\frac{1}{2}\hbar\omega} - p_{E=\frac{3}{2}\hbar\omega} = 1 - \frac{1}{\mathrm{ch}|z|^2}$$ 1/4

② a) $H_0 |n_x n_y n_z\rangle = \left[\hbar\omega(n_x+\frac{1}{2}) + \hbar\omega(n_y+\frac{1}{2}) + \hbar\omega(n_z+\frac{1}{2})\right] |n_x n_y n_z\rangle =$

$= \hbar\omega(n_x+n_y+n_z+\frac{3}{2}) |n_x n_y n_z\rangle$

osnovno stanje $|000\rangle$ z energijo $\frac{3}{2}\hbar\omega$ ni degenerirano
1. vzbujeno stanje z energijo $\frac{5}{2}\hbar\omega$ je 3× degenerirano:
$|100\rangle, |010\rangle, |001\rangle$ ++

b) $\psi_{000}(\vec{r}) = f(r) = \underline{e^{i0\varphi}} f(r)$

$\psi_{100}(\vec{r}) = x\, g(r) = r \sin\vartheta \cos\varphi\, g(r)$
$\psi_{010}(\vec{r}) = y\, g(r) = r \sin\vartheta \sin\varphi\, g(r)$
$\Big\}\ \psi_{100}(\vec{r}) \pm i\,\psi_{010}(\vec{r}) = \underline{e^{\pm i\varphi}} r \sin\vartheta\, g(r)$

$\psi_{001}(\vec{r}) = z\, g(r) = r\cos\vartheta\, g(r) = \underline{e^{i0\varphi}} r\cos\vartheta\, g(r)$

$L_z|000\rangle = 0\hbar|000\rangle$

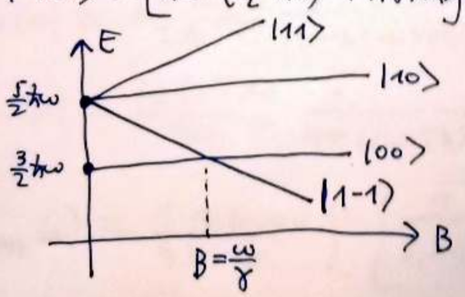
$L_z|001\rangle = 0\hbar|001\rangle$

$L_z \frac{|100\rangle + i|010\rangle}{\sqrt{2}} = 1\hbar \frac{|100\rangle + i|010\rangle}{\sqrt{2}}$

$L_z \frac{|100\rangle - i|010\rangle}{\sqrt{2}} = -1\hbar \frac{|100\rangle - i|010\rangle}{\sqrt{2}}$ ++

nova notacija:
$|00\rangle \equiv |000\rangle$
$|10\rangle \equiv |001\rangle$
$|11\rangle \equiv \frac{|100\rangle + i|010\rangle}{\sqrt{2}}$
$|1\,{-1}\rangle \equiv \frac{|100\rangle - i|010\rangle}{\sqrt{2}}$
$n = n_x+n_y+n_z$, m

c) $H_1|nm\rangle = \left[\hbar\omega(\frac{3}{2}+n) + \hbar\gamma m B\right]|nm\rangle$



(za $\gamma > 0$)

++

d) $H_2 = H_1 - e\mathcal{E}_0 e^{-\frac{t}{\tau}} z$

$C_{1m}(\infty) = -\frac{i}{\hbar}\int_0^\infty dt \langle 1m, t| -e\mathcal{E}_0 e^{-\frac{t}{\tau}} z |00, t\rangle =$

$= -\frac{i}{\hbar}\int_0^\infty dt\, e^{\frac{i}{\hbar}(E_{1m}-E_{00})t} e^{-\frac{t}{\tau}} (-e\mathcal{E}_0) \langle 1m|z|00\rangle =$

$= \frac{ie\mathcal{E}_0}{\hbar} \frac{1}{\frac{1}{\tau} - \frac{i}{\hbar}(\hbar\omega + \hbar\gamma B m)} \langle 1m|z|00\rangle =$

$= \frac{ie\mathcal{E}_0\tau}{\hbar} \frac{1}{1 - i(\omega + \gamma B m)\tau} \langle 1m|z|00\rangle$

② d) ...

$$\left.\begin{aligned}\langle 100|z|000\rangle &= 0\\ \langle 010|z|000\rangle &= 0\end{aligned}\right\} \ \langle 11|z|00\rangle = 0,\ \langle 1\,{-1}|z|00\rangle = 0$$

$$\langle 001|z|000\rangle = \langle 1|\frac{x_0}{\sqrt{2}}(a+a^\dagger)|0\rangle = \frac{x_0}{\sqrt{2}} \Rightarrow \langle 10|z|00\rangle = \frac{x_0}{\sqrt{2}}$$

$$x_0 = \sqrt{\frac{\hbar}{m\omega}}$$

$$C_{11}(\infty) = C_{1-1}(\infty) = 0$$

$$C_{10}(\infty) = \frac{ie\varepsilon_0 x_0 \tau}{\sqrt{2}\hbar}\ \frac{1}{1-i\omega\tau}$$

++

$$P_{n=1}(\infty) = |C_{11}(\infty)|^2 + |C_{10}(\infty)|^2 + |C_{1-1}(\infty)|^2 = \frac{1}{2}\left(\frac{e\varepsilon_0 x_0\tau}{\hbar}\right)^2 \frac{1}{1+(\omega\tau)^2}$$

e) $H_2 = H_1 - e\varepsilon_0 e^{-\frac{t}{\tau}} x$

$$C_{1m}(\infty) = \frac{ie\varepsilon_0\tau}{\hbar}\ \frac{1}{1-i(\omega+\gamma B m)\tau}\ \langle 1m|x|00\rangle$$

$$\langle 11|x|00\rangle = \frac{1}{\sqrt{2}}\Big(\underbrace{\langle 100|x|000\rangle}_{=\frac{x_0}{\sqrt{2}}} + i\underbrace{\langle 010|x|000\rangle}_{=0}\Big) = \frac{x_0}{2}$$

$$\langle 1\,{-1}|x|00\rangle = \dots\dots\dots = \frac{x_0}{\sqrt{2}}$$

$$\langle 10|x|00\rangle = 0$$

$$C_{10}(\infty) = 0$$

$$C_{11}(\infty) = \frac{ie\varepsilon_0\tau x_0}{2\hbar}\ \frac{1}{1-i(\omega+\gamma B)\tau}$$

$$C_{1-1}(\infty) = \frac{ie\varepsilon_0\tau x_0}{2\hbar}\ \frac{1}{1-i(\omega-\gamma B)\tau}$$

++

$$P_{n=1}(\infty) = \frac{1}{4}\left(\frac{e\varepsilon_0 x_0\tau}{\hbar}\right)^2\left[\frac{1}{1+(\omega+\gamma B)^2\tau^2} + \frac{1}{1+(\omega-\gamma B)^2\tau^2}\right]$$

f) $P_{n\geq 2}(\infty) = ?$

$$\langle n_x n_y n_z|z|000\rangle = \langle n_x|0\rangle\langle n_y|0\rangle\langle n_z|z|0\rangle =$$
$$= \delta_{n_x,0}\,\delta_{n_y,0}\,\frac{x_0}{\sqrt{2}}\,\delta_{n_z,1} \Rightarrow$$ v stanja z $n = n_x+n_y+n_z > 1$ ni prehodov!

enako za $\langle n_x n_y n_z|x|000\rangle$

++

$$\Rightarrow P_{n\geq 2}(\infty) = 0$$