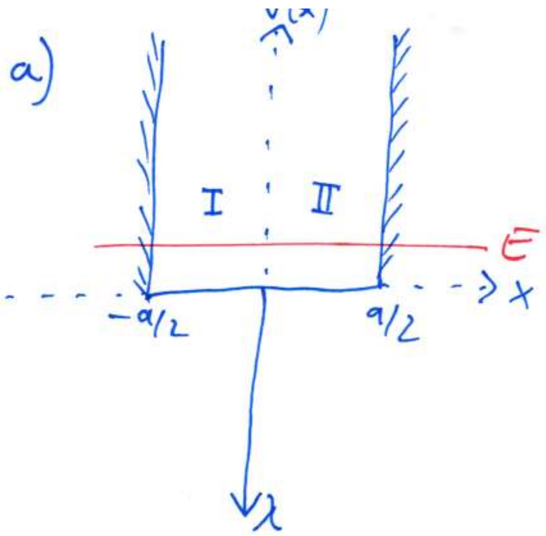
1) a)



$V(-x)=V(x) \Rightarrow$ lahko iščem sode in lihe lastne funkcije

Osnovno stanje je sodo.

$\psi_{II}(x) = A\cos kx + B\sin kx \;;\; k=\sqrt{\frac{2mE}{\hbar^2}}$

robni pogoj: $\psi_{II}(\frac{a}{2})=0$, torej

$\psi_{II}(x) = C\sin k(\frac{a}{2}-x)$

$\psi_{I}(x) = \psi_{II}(-x)$

robna pogoja pri $x=0$:

$\psi_I(0)=\psi_{II}(0)$ ✓

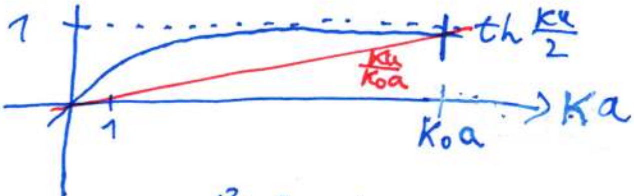
$\psi_{II}'(0)-\psi_I'(0) = -2K_0\psi(0) \;;\; K_0=\frac{m\lambda}{\hbar^2} \rightarrow -2Ck\cos\frac{ka}{2} = -2K_0 C\sin k\frac{a}{2}$

$$\boxed{\operatorname{tg}\frac{ka}{2}=\frac{k}{K_0}}$$ 1/2

b) Pričakujem $E<0$, torej $k=\sqrt{\frac{-2m(-E)}{\hbar^2}}=iK \;;\; K=\sqrt{\frac{2m(-E)}{\hbar^2}}$

$\operatorname{tg}\frac{ka}{2}=\operatorname{tg}\frac{iKa}{2}=\frac{iK}{K_0} \rightarrow \operatorname{th}\frac{Ka}{2}=\frac{K}{K_0}$

Za $\lambda \gg \frac{\hbar^2}{ma}$ je $K_0 a \gg 1$. Transcendentno enačbo rešimo grafično:



Rešitev je $K\approx K_0$, torej $E=-\frac{\hbar^2K_0^2}{2m}$, kar je energija vezanega stanja v potencialu funkciji delta v odsotnosti potencialne jame. ++

c) Pri $E=0$ je $\psi_{II}(x)=\tilde{C}(\frac{a}{2}-x)$, saj je tam $\psi_{II}''(x)=0$.

$\psi_{II}'(0)-\psi_I'(0)=-2K_0\psi(0) \rightarrow -2\tilde{C}=-2K_0\tilde{C}\frac{a}{2} \rightarrow \frac{K_0a}{2}=1 \rightarrow \lambda=\frac{2\hbar^2}{ma}$

d) $\int_{-a/2}^{a/2}|\psi(x)|^2dx = 1 = 2\int_0^{a/2}\tilde{C}^2(\frac{a}{2}-x)^2dx \rightarrow \tilde{C}=\frac{\sqrt{12}}{a^{3/2}}$ ++

$\langle x\rangle=0$, saj je $\psi(x)$ soda.

$\langle x^2\rangle = 2\int_0^{a/2}dx\, x^2|\psi(x)|^2 = \frac{a^2}{40} \rightarrow \Delta x=\frac{a}{\sqrt{40}}$ ++

② Za koherentno stanje $a|z\rangle = z|z\rangle$ 1D harmonskega oscilatorja $H = \frac{p^2}{2m} + \frac{1}{2}kx^2$ velja:

$$\left.\begin{aligned}
\langle x\rangle &= \sqrt{2}\,x_0 \operatorname{Re} z \;;\; x_0 = \sqrt{\tfrac{\hbar}{m\omega}} \\
\langle p\rangle &= \sqrt{2}\,p_0 \operatorname{Im} z \;;\; p_0 = \hbar/x_0 \\
\langle x^2\rangle &= x_0^2\left(2\operatorname{Re}^2 z + 1/2\right) \\
\langle p^2\rangle &= p_0^2\left(2\operatorname{Im}^2 z + 1/2\right)
\end{aligned}\right\}\text{ iz vaj}$$

$$\langle xp\rangle = \left\langle x_0\frac{a+a^+}{\sqrt{2}}\,p_0\frac{a-a^+}{\sqrt{2}\,i}\right\rangle = \frac{\hbar}{2i}\langle a^2 + \overbrace{a^+a - aa^+}^{-[a,a^+]=-1} - a^{+2}\rangle =$$

$$= \frac{\hbar}{2i}\left(z^2 - 1 - z^{*2}\right) = \hbar \operatorname{Im} z^2 - \frac{\hbar}{2i}$$

$$\langle px\rangle = \langle xp - [x,p]\rangle = \langle xp\rangle - i\hbar = \hbar\operatorname{Im} z^2 + \frac{\hbar}{2i}$$

a) $z_x = 2i,\ z_y = 3$

$\langle x\rangle = 0 \quad \langle p_x\rangle = 2\sqrt{2}\,p_0 \quad \langle y\rangle = 3\sqrt{2}\,x_0 \quad \langle p_y\rangle = 0$ 1/4

b) $\langle L_z\rangle = \langle x p_y - y p_x\rangle = \langle x\rangle\langle p_y\rangle - \langle y\rangle\langle p_x\rangle = -12\hbar$ 1/4

$$\langle L_z^2\rangle = \langle (xp_y - yp_x)^2\rangle = \langle xp_yxp_y - xp_yyp_x - yp_xxp_y + yp_xyp_x\rangle =$$

$$= \langle x^2\rangle\langle p_y^2\rangle - \langle xp_x\rangle\langle p_y y\rangle - \langle p_x x\rangle\langle y p_y\rangle + \langle p_x^2\rangle\langle y^2\rangle =$$

$$= \frac{x_0^2}{2}\cdot\frac{p_0^2}{2} - \left(-\frac{\hbar}{2i}\right)\frac{\hbar}{2i} - \frac{\hbar}{2i}\left(-\frac{\hbar}{2i}\right) + \frac{17p_0^2}{2}\,\frac{37x_0^2}{2} =$$

$$= 157\,\hbar^2$$

$\Delta L_z = \sqrt{\langle L_z^2\rangle - \langle L_z\rangle^2} = \sqrt{13}\,\hbar$ 1/4

c) $\langle L_z, t\rangle = \langle L_z(t), 0\rangle$

$\langle L_z^2, t\rangle = \langle [L_z(t)]^2, 0\rangle$

$\dot{L}_z(t) = \frac{i}{\hbar}\underbrace{[H, L_z](t)}_{\text{vaje}} = 0 \to L_z(t) = L_z$

$\Rightarrow \langle L_z, t\rangle = \langle L_z, 0\rangle$

$\Delta L_z(t) = \Delta L_z(0)$ 1/4