① a) kvadratna BM

$\vec{a}_1 = a(1,0) \quad \vec{A}_1 = \frac{2\pi}{a}(1,0) \quad \vec{K} = \frac{2\pi}{a}(m_1, m_2)$

$\vec{a}_2 = a(0,1) \quad \vec{A}_2 = \frac{2\pi}{a}(0,1) \quad |\vec{K}| = \frac{2\pi}{a}\sqrt{m_1^2+m_2^2}$

$d = \frac{2\pi}{|K|} \Rightarrow \frac{a}{\sqrt{m_1^2+m_2^2}}$

$d_1 = a, \quad m_1 m_2 = 10$

$d_2 = \frac{a}{\sqrt{2}}, \quad m_1 m_2 = 11$

$d_3 = a/2, \quad m_1 m_2 = 20$

$2d \sin\frac{\vartheta}{2} = \lambda$

$\frac{\sin\frac{\vartheta_1}{2}}{\sin\frac{\vartheta_2}{2}} = \frac{d_2}{d_1} = \frac{1}{\sqrt{2}} = 0.707$

uzorak A	uzorak B	uzorak C
0.707	0.577	0.500

$2d_1 \sin\frac{\vartheta_1}{2} = \lambda \Rightarrow \underline{a} = \frac{\lambda}{2\sin\frac{\vartheta_1}{2}} = \underline{1.7\ \text{Å}}$

$\sin\frac{\vartheta_3}{2} = \frac{\lambda}{2d_3} = \frac{\lambda}{a} > 1$ ✓

1/4+

b) trikotna BM

$\vec{a}_1 = a(1,0) \quad \vec{A}_1 = \frac{4\pi}{\sqrt{3}a}\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \quad \vec{K} = \frac{4\pi}{\sqrt{3}a}\left(\frac{\sqrt{3}}{2}m_1, -\frac{1}{2}m_1 + m_2\right)$

$\vec{a}_2 = a\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \quad \vec{A}_2 = \frac{4\pi}{\sqrt{3}a}(0,1) \quad |\vec{K}| = \frac{4\pi}{\sqrt{3}a}\sqrt{m_1^2 - m_1 m_2 + m_2^2}$

$d = \frac{\sqrt{3}a}{2}\frac{1}{\sqrt{m_1^2 - m_1 m_2 + m_2^2}}$

$d_1 = \frac{\sqrt{3}}{2}a, \quad m_1 m_2 = 10$

$d_2 = \frac{a}{2}, \quad m_1 m_2 = 1\,{-1}$

$d_3 = \frac{\sqrt{3}a}{4}, \quad m_1 m_2 = 20$

$\frac{\sin\frac{\vartheta_1}{2}}{\sin\frac{\vartheta_2}{2}} = \frac{d_2}{d_1} = \frac{1}{\sqrt{3}} = 0.577$

$\Downarrow$

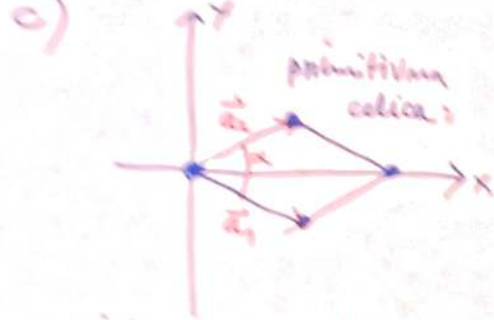
uzorak B

$\underline{a} = \frac{2}{\sqrt{3}}d_1 = \frac{2}{\sqrt{3}}\frac{\lambda}{2\sin\frac{\vartheta_1}{2}} = \frac{\lambda}{\sqrt{3}\sin\frac{\vartheta_1}{2}} = \underline{2.2\ \text{Å}}$

$\sin\frac{\vartheta_3}{2} = \frac{\lambda}{2d_3} = \frac{\lambda}{\frac{\sqrt{3}a}{2}} = \frac{2\lambda}{\sqrt{3}a} > 1$ ✓

1/4+

c)



primitivna celica:

$$\vec a_1 = a\left(\cos\frac{\alpha}{2}, -\sin\frac{\alpha}{2}\right)$$
$$\vec a_2 = a\left(\cos\frac{\alpha}{2}, \sin\frac{\alpha}{2}\right)$$

$$\begin{pmatrix}\vec a_1\\ \vec a_2\end{pmatrix}(\vec b_1, \vec b_2) = 2\pi I \longrightarrow (\vec b_1, \vec b_2) = 2\pi\begin{pmatrix}\vec a_1\\ \vec a_2\end{pmatrix}^{-1} =$$
$$= 2\pi\begin{pmatrix} a\cos\frac{\alpha}{2}, & -a\sin\frac{\alpha}{2}\\ a\cos\frac{\alpha}{2}, & a\sin\frac{\alpha}{2}\end{pmatrix}^{-1} = \frac{2\pi}{a^2\sin\alpha}\begin{pmatrix} a\sin\frac{\alpha}{2}, & a\sin\frac{\alpha}{2}\\ -a\cos\frac{\alpha}{2}, & a\cos\frac{\alpha}{2}\end{pmatrix}$$

naj bo $\alpha \leq 90°$!

$$\vec b_1 = \frac{2\pi}{a\sin\alpha}\left(\sin\frac{\alpha}{2}, -\cos\frac{\alpha}{2}\right)$$
$$\vec b_2 = \frac{2\pi}{a\sin\alpha}\left(\sin\frac{\alpha}{2}, \cos\frac{\alpha}{2}\right)$$
$$\vec K = \frac{2\pi}{a\sin\alpha}\left((m_1+m_2)\sin\frac{\alpha}{2}, (-m_1+m_2)\cos\frac{\alpha}{2}\right)$$
$$|\vec K| = \frac{2\pi}{a\sin\alpha}\sqrt{m_1^2 - 2\cos\alpha\, m_1 m_2 + m_2^2}$$
$$d = \frac{a\sin\alpha}{\sqrt{m_1^2 - 2\cos\alpha\, m_1 m_2 + m_2^2}}$$

1/4

recipročna mreža:

$m_1 m_2$	$d_{m_1 m_2}$
10	$a\sin\alpha$
11	$a\cos\frac{\alpha}{2}$
20	$\frac{1}{2}a\sin\alpha$
22	$\frac{1}{2}a\cos\frac{\alpha}{2}$
21	$\frac{a\sin\alpha}{\sqrt{5-4\cos\alpha}}$
1-1	$a\sin\frac{\alpha}{2}$

največji d (torej d_1) je $\begin{cases} d_{10} & \text{za } \alpha > 60° \\ d_{11} & \text{za } \alpha < 60° \end{cases}$ ($\alpha = 60°$ bi bila trikotna BM)

$\frac{\sin\frac{\vartheta_1}{2}}{\sin\frac{\vartheta_2}{2}} = 0.500$ za vzorec C, torej je $\frac{d_2}{d_1} = \frac{1}{2}$

1.) če je $\alpha > 60°$, je $d_1 = d_{10} = a\sin\alpha$

$d_2 = d_{20} = \frac{1}{2}a\sin\alpha$

$d_{11} = a\cos\frac{\alpha}{2} \lessgtr d_{20} = \frac{1}{2}a\sin\alpha$ X

torej je 2.) $\alpha < 60°$

$d_1 = d_{11} = a\cos\frac{\alpha}{2}$

$d_2 = d_{22} = \frac{1}{2}a\cos\frac{\alpha}{2}$

2a) $d_{10} = a\sin\alpha = d_2$

$\sin\frac{\alpha}{2} = \frac{1}{4}$

$d_3 = d_{21}$

X

2b) $d_{10} = a\sin\alpha = d_3$

$$\left.\begin{aligned} d_1 &= a\cos\frac{\alpha}{2} = \frac{\lambda}{2\sin\frac{\vartheta_1}{2}} \\ d_3 &= a\sin\alpha = \frac{\lambda}{2\sin\frac{\vartheta_3}{2}}\end{aligned}\right\} \sin\frac{\alpha}{2} = \frac{1}{2}\frac{\sin\frac{\vartheta_1}{2}}{\sin\frac{\vartheta_3}{2}}$$

$\Rightarrow \underline{\underline{\alpha = 25°}}$

$\underline{\underline{a}} = \frac{\lambda}{2\sin\frac{\vartheta_1}{2}\cos\frac{\alpha}{2}} = \underline{\underline{2.5\,\text{Å}}}$

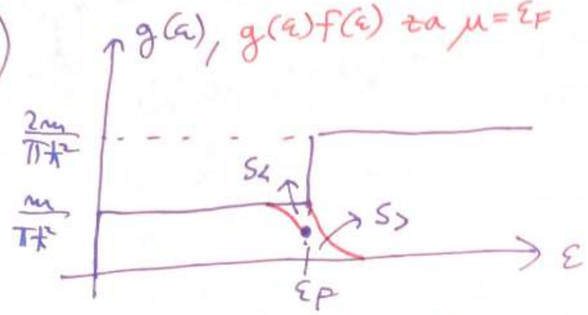
1/4

(2) a) $g(\varepsilon) = \frac{m}{\pi\hbar^2}\theta(\varepsilon)$ +

b) $\frac{N}{S} = \int_0^{\varepsilon_F} g(\varepsilon)d\varepsilon = \frac{m}{\pi\hbar^2}\varepsilon_F \Rightarrow \varepsilon_F = \frac{\pi\hbar^2}{m}\frac{N}{S} = \underline{2.4\,eV}$ 1/4

c) $g(\varepsilon) = \frac{m}{\pi\hbar^2}\left(\theta(\varepsilon) + \theta(\varepsilon-\varepsilon_F)\right)$ ++

d)



$S_> > S_<$

$\Downarrow$

kemijski potencial pri $0 < k_BT \ll \varepsilon_F$ je pod ε_F ++

e) $N_2 = \int_{\varepsilon_F}^{\infty} \frac{m}{\pi\hbar^2} f(\varepsilon)d\varepsilon = \int_{\varepsilon_F}^{\infty} \frac{m}{\pi\hbar^2}\frac{d\varepsilon}{e^{\beta(\varepsilon-\mu)}+1} \to$ $\quad u = e^{\beta(\varepsilon-\mu)}+1$

$= \int_{e^{\beta(\varepsilon_F-\mu)}+1}^{\infty} \frac{m}{\pi\hbar^2\beta}\frac{du}{u(u-1)} = \frac{m}{\pi\hbar^2\beta}\ln\frac{u-1}{u}\Big|_{e^{\beta(\varepsilon_F-\mu)}+1}^{\infty} =$

$= \frac{m}{\pi\hbar^2\beta}\ln\left(1+e^{-\beta(\varepsilon_F-\mu)}\right)$ 1/4

potrebujem $\mu(T)$!

$N(T=0\,K) = N(T)$

$\int_0^{\varepsilon_F} \frac{\cancel{m}}{\cancel{\pi\hbar^2}}d\varepsilon = \int_0^{\infty}\frac{\cancel{m}}{\cancel{\pi\hbar^2}}f(\varepsilon)d\varepsilon + \int_{\varepsilon_F}^{\infty}\frac{\cancel{m}}{\cancel{\pi\hbar^2}}f(\varepsilon)d\varepsilon$

$\varepsilon_F = \int_0^{\infty}\frac{d\varepsilon}{e^{\beta(\varepsilon-\mu)}+1} + \int_{\varepsilon_F}^{\infty}\frac{d\varepsilon}{e^{\beta(\varepsilon-\mu)}+1}$ $\quad u = e^{\beta(\varepsilon-\mu)}+1$

$\varepsilon_F = \frac{1}{\beta}\ln\left(1+e^{\beta\mu}\right) + \frac{1}{\beta}\ln\left(1+e^{-\beta(\varepsilon_F-\mu)}\right)$

$\delta \equiv \varepsilon_F - \mu$

$e^{\beta\varepsilon_F} = \left(1+e^{\beta\mu}\right)\left(1+e^{-\beta(\varepsilon_F-\mu)}\right)$

$e^{\beta\varepsilon_F} = \left(1+e^{\beta(\varepsilon_F-\delta)}\right)\left(1+e^{-\beta\delta}\right)$

$e^{\beta\varepsilon_F} = 1 + e^{\beta(\varepsilon_F-\delta)} + e^{-\beta\delta} + e^{\beta(\varepsilon_F-2\delta)} \quad / \cdot e^{-\beta\varepsilon_F}$

$1 = \cancel{e^{-\beta\varepsilon_F}} + e^{-\beta\delta} + \cancel{e^{-\beta\varepsilon_F}e^{-\beta\delta}} + e^{-2\beta\delta}$ $\quad$ zanemarim, saj je $\beta\varepsilon_F \gg 1$

$e^{-\beta\delta} \doteq \frac{-1+\sqrt{5}}{2}$

$\underline{N_2} = \frac{m}{\pi\hbar^2\beta}\ln\left(1+e^{-\beta\delta}\right) = \frac{m}{\pi\hbar^2}k_BT\ln\left(\frac{1+\sqrt{5}}{2}\right)$ 1/4