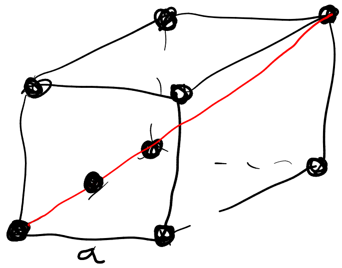
1. Z rentgensko svetlobo z valovno dolžino $\lambda = 2$ Å izmerimo difraktogram na praškastem vzorcu enoatomnega kristala z navadno kubično mrežo z bazo. V kubični primitivni osnovni celici kristala sta dva enaka atoma: v ogljišču ter na neznanem mestu na telesni diagonali celice. Braggova vrhova opazimo pri sipalnih kotih 83.6° in 141.1°.

   (a) Določi mrežno razdaljo.

   (b) Koliko Braggovih vrhov opazimo pri vsakem od teh sipalnih kotov pri metodi rotacije kristala?

   (c) Določi položaj atoma na telesni diagonali, če so pri metodi rotacije kristala intenzitete vrhov pri sipalnem kotu 141.1° v razmerju 1:4. Predpostavi, da je atomski strukturni faktor konstanten.



$\vec{a}_1 = a(1,0,0)$ $\quad \vec{A}_1 = \frac{2\pi}{a}(1,0,0)$

$\vec{a}_2 = a(0,1,0)$ $\quad \vec{A}_2 = \frac{2\pi}{a}(0,1,0)$

$\vec{a}_3 = a(0,0,1)$ $\quad \vec{A}_3 = \frac{2\pi}{a}(0,0,1)$

$\vec{r}_1 = 0$

$\vec{r}_2 = (x,x,x)$ $\quad x = ?$

$\vec{K} = \frac{2\pi}{a}(m_1, m_2, m_3)$

$$S_{\vec{K}} = \sum_m e^{-i\vec{K}\cdot\vec{r}_m} = 1 + e^{-i\vec{K}\cdot\vec{r}_2} = 1 + e^{-i2\pi\frac{x}{a}(m_1+m_2+m_3)}$$

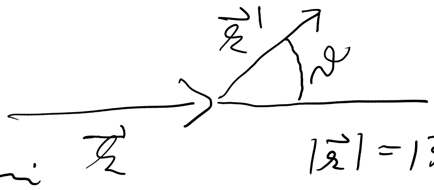
$$I \propto |S_{\vec{K}}|^2$$

BCC: $x = \frac{a}{2}$ $\quad S_{\vec{K}} = 1 + e^{-i\pi(m_1+m_2+m_3)}$

$S_{\vec{K}} = 0$ če $m_1+m_2+m_3$ lih

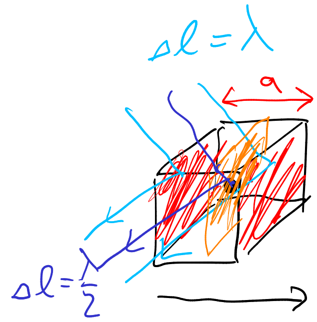
Braggov pogoj: $2d\sin\frac{\vartheta}{2} = N\lambda$

razdalja med mrežnimi ravninami

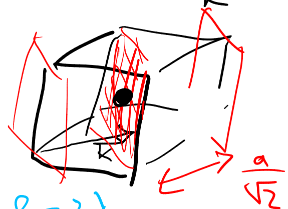


$|\vec{k}| = |\vec{k}'|$

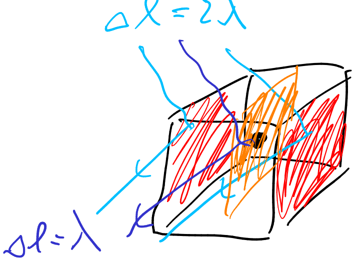
$$d = \frac{2\pi}{|\vec{K}|} = \frac{a}{\sqrt{m_1^2+m_2^2+m_3^2}}$$



$\vec{K} = \frac{2\pi}{a}(1,0,0)$ $\quad d = a$ $\quad \rightarrow 2d\sin\frac{\vartheta}{2} = \lambda$



$\vec{K} = \frac{2\pi}{a}(1,1,0)$ $\quad d = \frac{a}{\sqrt{2}}$



$\vec{K} = \frac{2\pi}{a}(2,0,0)$ $\quad d = \frac{a}{2}$

$$2d \sin\frac{\vartheta}{2} = \lambda$$

$$d = \frac{2\pi}{|\vec{k}|} = \frac{a}{\sqrt{m_1^2+m_2^2+m_3^2}}$$

$$2\frac{a}{\sqrt{m_1^2+m_2^2+m_3^2}} \sin\frac{\vartheta}{2} = \lambda$$

$$a = \frac{\lambda\sqrt{m_1^2+m_2^2+m_3^2}}{2\sin\frac{\vartheta}{2}}$$

$\lambda = 2\,Å$

$\vartheta_1 = 83.3°$

$\vartheta_2 = 141.1°$

~~BCC~~

$m_1\, m_2\, m_3$

$\left.\begin{matrix}1\,0\,0\\0\,1\,0\\0\,0\,1\end{matrix}\right\}$ $\quad a = \dfrac{\lambda}{2\sin\frac{\vartheta_1}{2}} = 1.50\,Å$

$\left.\begin{matrix}1\,1\,0\\1\,-1\,0\\0\,1\,1\\0\,1\,-1\\1\,0\,1\\1\,0\,-1\end{matrix}\right\}$ $\quad a = \dfrac{\lambda\sqrt{2}}{2\sin\frac{\vartheta_2}{2}} = 1.50\,Å$

BCC

$1\,1\,0 : \quad a = \dfrac{\sqrt{2}\,\lambda}{2\sin\frac{\vartheta_1}{2}} = 2.13\,Å$

$2\,0\,0 : \quad a = \dfrac{2\lambda}{2\sin\frac{\vartheta_2}{2}} = 2.13\,Å$

$0\,2\,0$

$0\,0\,2$

$m_1+m_2+m_3 = 2 \qquad I_{200} = I_{020} = I_{002}$

$$I \propto |S_{\vec{k}}|^2 = \left|1 + e^{-i2\pi\frac{x}{a}(m_1+m_2+m_3)}\right|^2$$

$I_{1-10} = I_{10-1} = I_{01-1} \propto 4$

$m_1+m_2+m_3 = 0$

$I_{110} = I_{101} = I_{011} \propto \left|1 + e^{-i4\pi\frac{x}{a}}\right|^2 = 2 + 2\cos\frac{4\pi x}{a} = 4\cos^2 2\pi\frac{x}{a}$

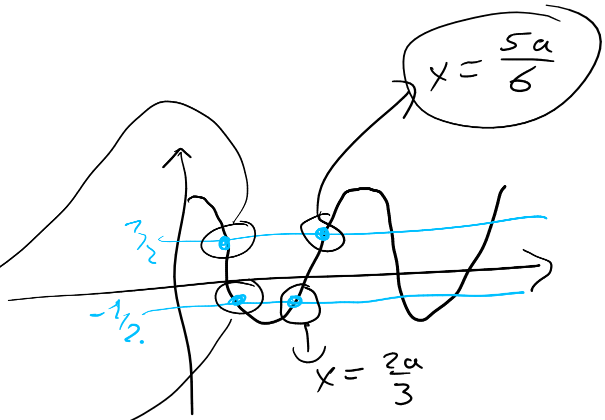
$m_1+m_2+m_3 = 2$

$$\frac{I_{110}}{I_{1-10}} = \frac{1}{4} = \frac{4\cos^2 2\pi\frac{x}{a}}{4} = \cos^2 2\pi\frac{x}{a}$$

$$\cos 2\pi\frac{x}{a} = \pm\frac{1}{2}$$

$2\pi\frac{x}{a} = \frac{\pi}{3} \qquad x = \frac{a}{6}$

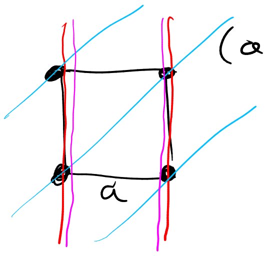
$2\pi\frac{x}{a} = \frac{2\pi}{3} \qquad x = \frac{a}{3}$



1. Atomi dvodimenzionalnega kristala tvorijo kvadratno mrežo z medatomsko razdaljo a.

   (a) Na praškast vzorec, v katerem vsi kristali ležijo v isti ravnini, posvetimo z rentgensko svetlobo z valovno dolžino λ tako, da tudi vpadni žarek leži v tej ravnini. Največ kolikšna je lahko λ, da dobimo Braggove odboje pri vsaj treh različnih sipalnih kotih?

   (b) Na monokristal posvetimo z belo rentgensko svetlobo, ki vsebuje vse valovne dolžine, daljše od $\lambda_{\min}$. Največ kolikšna je lahko $\lambda_{\min}$, da dobimo vsaj tri Braggove odboje, če žarek vpada v ravnini kristala pod kotom 20° glede na zveznico med atomi?



(a) $$2d\sin\frac{\vartheta}{2}=\lambda$$

$$d=\frac{2\pi}{|\vec K|}=\frac{a}{\sqrt{m_1^2+m_2^2}}$$

$$\vec K=\frac{2\pi}{a}(m_1,m_2)$$

$$\sin\frac{\vartheta}{2}=\frac{\lambda}{2a}\sqrt{m_1^2+m_2^2}$$

$d=a$

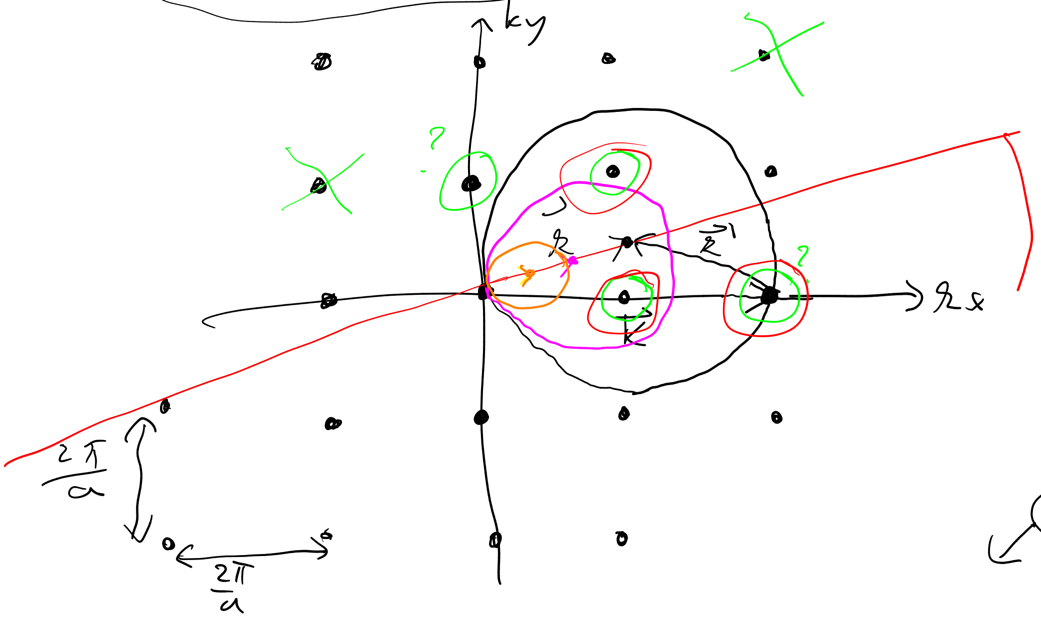
$m_1m_2=10 \longrightarrow \sin\frac{\vartheta}{2}=\frac{\lambda}{2a} \rightarrow \lambda\leq 2a$

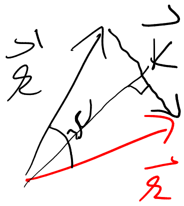
$d=\frac{a}{\sqrt{2}}$

$m_1m_2=11 \longrightarrow \sin\frac{\vartheta}{2}=\frac{\sqrt{2}\lambda}{2a} \rightarrow \lambda\leq\sqrt{2}a$

$m_1m_2=20 \longrightarrow \sin\frac{\vartheta}{2}=\frac{2\lambda}{2a}=\frac{\lambda}{a} \rightarrow \lambda\leq a$

(b) $$\vec K=\frac{2\pi}{a}(m_1,m_2)$$





$$|\vec k'|=|\vec k|=\frac{2\pi}{\lambda}$$

$$\vec k-\vec k'=\vec K$$

$$\vec k\cdot\frac{\vec K}{K}=\frac{K}{2}$$

$$\vec k\cdot\vec K=\frac{K^2}{2}\equiv 2d\sin\frac{\vartheta}{2}=\lambda$$

$$\vec k=\frac{2\pi}{\lambda}(\cos\alpha,\sin\alpha)$$

$$\vec K=\frac{2\pi}{a}(m_1,m_2)$$

$$\frac{4\pi^2}{a\lambda}(m_1\cos\alpha+m_2\sin\alpha)=\frac{4\pi^2}{a^2}\frac{m_1^2+m_2^2}{2}$$

$$\lambda=\frac{m_1\cos\alpha+m_2\sin\alpha}{m_1^2+m_2^2}2a$$

$m_1m_2=10$: $\lambda=2a\frac{\cos\alpha}{1}=1.88a$

$m_1m_2=11$: $\lambda=2a\frac{\cos\alpha+\sin\alpha}{2}=1.28a$

$m_1m_2=20$: $\lambda=2a\frac{2\cos\alpha}{4}=0.94a$

$m_1m_2=01$: $\lambda=2a\frac{\sin\alpha}{1}=0.68a$

$$\lambda_{min}=0.94a$$

Na matematiki imamo Erasmus študentko, za katero bi potrebovali tutorja. Delo: v angleščino bi prevajal besedila nalog, ki jih delamo na vajah pri predmetu Fizika 1, in stare kolokvije. Plačilo: 3KT ali 9.63EUR/h.