$$n_c = p_d$$

$$N_c e^{-\beta(\varepsilon_c - \mu)} = N_d \frac{1}{1+2e^{-\beta(\varepsilon_d - \mu)}} \quad ; \quad N_c = \frac{1}{4}\left(\frac{2 m_c^* k_B T}{\pi \hbar^2}\right)^{3/2}$$

$$e^{\beta\mu} = \frac{1}{4} e^{\beta \varepsilon_d}\left[-1 + \sqrt{1 + \frac{8N_d}{N_c} e^{\beta(\varepsilon_c - \varepsilon_d)}}\right]$$

iz vaj.

a) $T = 2K$

$$\frac{8N_d}{N_c} e^{\beta(\varepsilon_c - \varepsilon_d)} = 2 \times 10^8 \gg 1$$

$$\mu \doteq \frac{\varepsilon_c + \varepsilon_d}{2} + k_B T \ln \sqrt{\frac{N_d}{2N_c}}$$ iz vaj

$$\frac{\varepsilon_c - \mu}{k_B T} = \frac{\varepsilon_c - \varepsilon_d}{2} - \ln \sqrt{\frac{N_d}{2N_c}} = 9$$ nedegeneriran polprevodnik

$$n_c = \sqrt{\frac{N_d N_c}{2}} e^{-\beta \frac{\varepsilon_c - \varepsilon_d}{2}} = 1.4 \times 10^{11}/cm^3$$ $1/2^+$

b) $T = 20K$

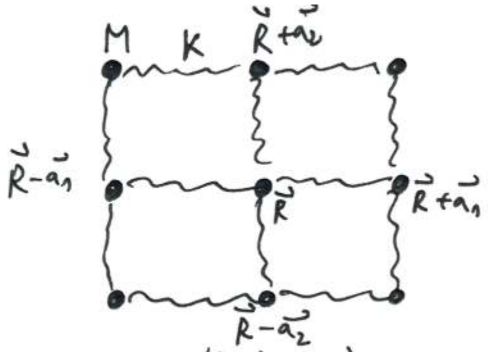
$$\frac{8N_d}{N_c} e^{\beta(\varepsilon_c - \varepsilon_d)} = 1.02$$

$$\frac{\varepsilon_c - \mu}{k_B T} = \frac{\varepsilon_c - \varepsilon_d}{k_B T} - \ln \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c} e^{\beta(\varepsilon_c - \varepsilon_d)}}}{4} = 4$$ nedegeneriran polprevodnik

$$n_c = N_c e^{-\beta(\varepsilon_c - \varepsilon_d)} \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c} e^{\beta(\varepsilon_c - \varepsilon_d)}}}{4} = 8.3 \times 10^{14}/cm^3$$ $1/2^-$

Vrzeli v valenčnem pasu lahko zanemarimo, saj je $\frac{\varepsilon_c - \varepsilon_v}{k_B T} \approx 400$.

② a)



$$M\ddot{u}_{\vec R} = \tilde K\left(u_{\vec R+\vec a_1}+u_{\vec R+\vec a_2}+u_{\vec R-\vec a_1}+u_{\vec R-\vec a_2}-4u_{\vec R}\right)$$

1/4$^-$

$\vec a_1 = (a, 0)$

$\vec a_2 = (0, a)$

$\tilde K = K\,\frac{a-a_0}{a}$ (iz vaj)

b) $u_{\vec R} = u_0\, e^{i(\vec k\cdot\vec R-\omega t)}$

$$-M\omega^2 u_0 = K\left(e^{i\vec k\cdot\vec a_1}+e^{i\vec k\cdot\vec a_2}+e^{-i\vec k\cdot\vec a_1}+e^{-i\vec k\cdot\vec a_2}-4\right)u_0 =$$
$$= -4K\left(\sin^2\frac{\vec k\cdot\vec a_1}{2}+\sin^2\frac{\vec k\cdot\vec a_2}{2}\right)$$
$$\omega = \sqrt{\frac{4K}{M}\left(\sin^2\frac{\vec k\cdot\vec a_1}{2}+\sin^2\frac{\vec k\cdot\vec a_2}{2}\right)}$$

1/4$^-$

c) $|\vec k| \ll \frac{\pi}{a} \Rightarrow \omega = \sqrt{\frac{K}{M}k^2a^2} \Rightarrow c = \sqrt{\frac{K}{M}}\,a$ 1/4$^-$

d)
$$M\ddot{u}_{\vec R} = \tilde K\left(u_{\vec R+\vec a_1}+u_{\vec R+\vec a_2}+u_{\vec R-\vec a_1}+u_{\vec R-\vec a_2}-4u_{\vec R}\right)+Q\left(v_{\vec R}-u_{\vec R}\right)$$
$$M'\ddot{v}_{\vec R} = \tilde K'\left(v_{\vec R+\vec a_1}+v_{\vec R+\vec a_2}+v_{\vec R-\vec a_1}+v_{\vec R-\vec a_2}-4v_{\vec R}\right)+Q\left(u_{\vec R}-v_{\vec R}\right)$$
$$u_{\vec R} = u_0\, e^{i(\vec k\cdot\vec R-\omega t)}$$
$$v_{\vec R} = v_0\, e^{i(\vec k\cdot\vec R-\omega t)}$$
$$-M\omega^2 u_0 = \tilde K\left(-4\sin^2\frac{\vec k\cdot\vec a_1}{2}-4\sin^2\frac{\vec k\cdot\vec a_2}{2}\right)u_0 + Q(v_0-u_0)$$
$$-M'\omega^2 v_0 = \tilde K'\left(-4\sin^2\frac{\vec k\cdot\vec a_1}{2}-4\sin^2\frac{\vec k\cdot\vec a_2}{2}\right)v_0 + Q(u_0-v_0)$$
$$\det\begin{bmatrix} 4\tilde K\left(\sin^2\frac{\vec k\cdot\vec a_1}{2}+\sin^2\frac{\vec k\cdot\vec a_2}{2}\right)+Q-M\omega^2, & -Q \\ -Q, & 4\tilde K'\left(\sin^2\frac{\vec k\cdot\vec a_1}{2}+\sin^2\frac{\vec k\cdot\vec a_2}{2}\right)+Q-M'\omega^2 \end{bmatrix} = 0$$
$$\vec k = 0 \Rightarrow (Q-M\omega^2)(Q-M'\omega^2)-Q^2 = 0$$
$$-(M+M')Q\omega^2 + MM'\omega^4 = 0$$
$$\omega_{opt} = \sqrt{Q\,\frac{M+M'}{MM'}}$$

1/4$^-$

e) $|\vec k| \ll \frac{\pi}{a}$:
$$\det\begin{bmatrix} \tilde K k^2a^2+Q-M\omega^2, & -Q \\ -Q, & \tilde K' k^2a^2+Q-M'\omega^2 \end{bmatrix} = 0$$
$$\omega = ck$$

1/4$^-$

$$\left[Q+(\tilde K a^2-Mc^2)k^2\right]\left[Q+(\tilde K'a^2-M'c^2)k^2\right]-Q^2 = 0$$
$$Q\left[(\tilde K+\tilde K')a^2-(M+M')c^2\right]k^2 + O(k^4) = 0 \Rightarrow c = \sqrt{\frac{\tilde K+\tilde K'}{M+M'}}\,a$$

f) Za $Q=0$ imamo dve akustični veji s $c = \sqrt{\frac{K}{M}}\,a$ in $c' = \sqrt{\frac{K'}{M'}}\,a$.

Izračunajmo najprej prispevek spodnje plasti:

$$c = \frac{1}{N}\frac{dE}{dT} \qquad E = \sum_{\vec k}\hbar\omega_{\vec k}\,\frac{1}{e^{\beta\hbar\omega_{\vec k}}-1} = \frac{Na^2}{(2\pi)^2}\int_0^\infty 2\pi k\,dk\,\hbar ck\,\frac{1}{e^{\beta\hbar ck}-1}$$

② f) $E = \frac{Na^2}{2\pi} \int_0^{\infty} \frac{u^2 du}{e^u - 1} \frac{1}{(\hbar c)^2 \beta^3}$ $\left(\int_0^{\infty} \frac{u^2 du}{e^u - 1} = 2\zeta(3) \right)$

$$c = \frac{a^2}{2\pi} \frac{3k_B^3 T^2}{(\hbar c)^2} \cdot 2\zeta(3)$$

$$c = \frac{3\zeta(3)}{\pi} \left(\frac{k_B T}{\hbar \sqrt{\frac{K}{M}}} \right)^2 k_B$$

Za obe plasti: $C = \frac{3\zeta(3)}{\pi} \left[\left(\frac{k_B T}{\hbar \sqrt{\frac{K}{M}}} \right)^2 + \left(\frac{k_B T}{\hbar \sqrt{\frac{K}{M'}}} \right)^2 \right] k_B$ 1/4 −