$$\hbar\dot{\vec{k}} = \vec{F} = e(\vec{E} + \vec{v}\times\vec{B}), \qquad \vec{v} = \frac{1}{\hbar}\frac{d\varepsilon(\vec{k})}{d\vec{k}} \qquad \Big\}\ \text{kvaziklasični približek}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2m}$$

(1) $\vec{E} = 0,\quad \vec{B}(\vec{r}) = (0,0,B)$

prosti elektroni: $V(\vec{r}) = \emptyset$ — el. kroži s ciklotronsko frekvenco $\frac{|e|B}{m} = \omega_c$

v periodičnem potencialu v okolici min, max ali sedla

$$\varepsilon(\vec{k}) = \frac{\hbar^2}{2}\vec{k}\cdot(m^*)^{-1}\vec{k} + \varepsilon_0$$

↳ tenzor efektivne mase (simetričen)

$$v_\alpha = \frac{1}{\hbar}\frac{\partial\varepsilon}{\partial k_\alpha} = \frac{1}{\hbar}\frac{\partial}{\partial k_\alpha}\left(\frac{\hbar^2}{2}k_\beta (m^*)^{-1}_{\beta\gamma}k_\gamma\right) = \frac{\hbar}{2}\left(\delta_{\alpha\beta}(m^*)^{-1}_{\beta\gamma}k_\gamma + k_\beta(m^*)^{-1}_{\beta\gamma}\delta_{\gamma\alpha}\right) =$$

$$= \frac{\hbar}{2}\left((m^*)^{-1}_{\alpha\gamma}k_\gamma + k_\beta(m^*)^{-1}_{\beta\alpha}\right) =$$

$$= \frac{\hbar}{2}\left((m^*)^{-1}_{\alpha\gamma}k_\gamma + (m^*)^{-1}_{\alpha\beta}k_\beta\right) = \hbar(m^*)^{-1}_{\alpha\gamma}k_\gamma = \left(\hbar(m^*)^{-1}\vec{k}\right)_\alpha$$

$$\vec{v} = \hbar(m^*)^{-1}\vec{k} \;\longrightarrow\; m^*\vec{v} = \hbar\vec{k}$$

$$m^*\dot{\vec{v}} = e\,\vec{v}\times\vec{B}$$

$$m^*\dot{\vec{v}} = e\begin{pmatrix}v_x\\v_y\\v_z\end{pmatrix}\times\begin{pmatrix}0\\0\\B\end{pmatrix} = e\begin{pmatrix}v_y B\\-v_x B\\0\end{pmatrix} \qquad m^* = \begin{pmatrix}m_{xx} & m_{xy} & m_{xz}\\ m_{yx} & m_{yy} & m_{yz}\\ m_{zx} & m_{zy} & m_{zz}\end{pmatrix}$$

$$\vec{v}(t) = \vec{v}_0 e^{-i\omega t} \qquad \vec{r}(t) = \int\vec{v}(t)\,dt$$

$$-i\omega m^*\vec{v}_0 = e\,\vec{v}_0\times\vec{B}$$

$$m^*\vec{v}_0 = \frac{ie}{\omega}\begin{pmatrix}v_{0y}B\\-v_{0x}B\\0\end{pmatrix}$$

$$\begin{pmatrix} m_{xx}, & m_{xy}-\frac{ieB}{\omega}, & m_{xz}\\ m_{yx}+\frac{ieB}{\omega}, & m_{yy}, & m_{yz}\\ m_{zx}, & m_{zy}, & m_{zz}\end{pmatrix}\begin{pmatrix}v_{0x}\\v_{0y}\\v_{0z}\end{pmatrix} = 0$$

$$\det(\ldots) = 0$$

$$m_{xx}m_{yy}m_{zz} + \left(m_{xy}-\frac{ieB}{\omega}\right)m_{yz}m_{zx} + \left(m_{yx}+\frac{ieB}{\omega}\right)m_{zy}m_{xz} -$$

$$- m_{xz}m_{yy}m_{zx} - m_{yz}m_{zy}m_{xx} - \left(m_{xy}-\frac{ieB}{\omega}\right)\left(m_{yx}+\frac{ieB}{\omega}\right)m_{zz} = 0$$

($m_{yz} = m_{zy}$, $m_{zx} = m_{xz}$, $m_{yx} = m_{xy}$)

$$\det m^* + \left(\frac{ieB}{\omega}\right)^2 m_{zz} = 0 \qquad \vec{B} = B\hat{e}$$

$$\det m^* = \frac{e^2B^2}{\omega^2}m_{zz} \qquad m_{zz} = \hat{e}\cdot m^*\hat{e}$$

$$\omega^2 = \frac{e^2B^2\, m_{zz}}{\det m^*} \;\longrightarrow\; \omega = \frac{|e|B}{\sqrt{\frac{\det m^*}{m_{zz}}}}$$

$$\omega = \frac{|e|B}{m_c^*} \qquad m_c^* = \sqrt{\frac{\det m^*}{\hat{e}\cdot m^*\hat{e}}}$$

↳ ciklotronska efektivna masa

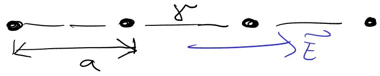
$m_c^* > 0$ v okolici min. pasu

↳ gostota stanj: prosti elektroni: $g(\varepsilon) = \frac{1}{\pi^2\hbar^3}\sqrt{2m}\sqrt{\varepsilon}$ v 3D

v periodičnem potencialu v bližini min.: $g(\varepsilon) = \frac{1}{\pi^2\hbar^3}\sqrt{2m_g^*}\sqrt{\varepsilon-\varepsilon_0}$

$$m_g^* = \sqrt[3]{\det m^*}$$

Blochove oscilacije v 1D verigi v približku tesne vezi



$$\hbar \dot{k} = eE$$

$$k(t) = k(0) + \frac{eE}{\hbar} t$$

$E > 0$, $e < 0$ ⇒ $\frac{eE}{\hbar} < 0$

$$\varepsilon(k) = -\gamma \sum_{\vec{R} = n.s.}^{} e^{i\vec{k}\cdot\vec{R}} = -\gamma\left(e^{ika} + e^{ik(-a)}\right) =$$

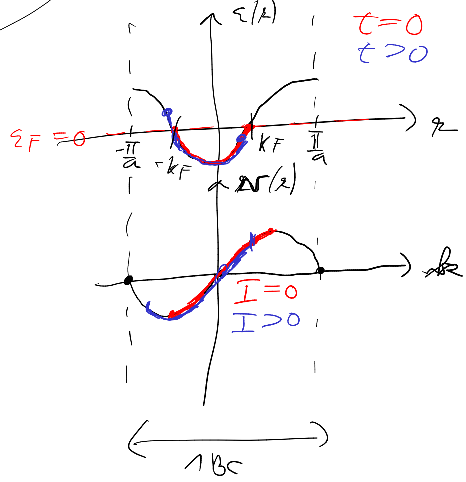
$$\boxed{\varepsilon(k) = -2\gamma \cos ka}$$

$$v = \frac{1}{\hbar}\frac{d\varepsilon}{dk} = \frac{2\gamma a}{\hbar}\sin ka$$

$(j = nev)$

$$I(t) = e\,2 \sum_{|k(0)|<k_F} v(k(t)) \cdot \frac{1}{L}$$ (2 — spin)

$\Delta k = \frac{L}{2\pi}$



$$I(t) = \frac{2e}{L} \sum_{|k(0)|<k_F} v\left(k(0) + \frac{eE}{\hbar}t\right) =$$

$$= \frac{2e}{L}\frac{L}{2\pi}\int_{-k_F}^{k_F} dk(0)\, v\left(k(0) + \frac{eE}{\hbar}t\right) =$$

$$= \frac{e}{\pi}\int_{-k_F}^{k_F} dk(0)\, \frac{2\gamma a}{\hbar}\sin\left(k(0) + \frac{eE}{\hbar}t\right)a =$$

$$= \frac{2e\gamma a}{\pi\hbar}\,\frac{1}{a}\left[\cos\left(k(0) + \frac{eE}{\hbar}t\right)a\right]\Big|_{-k_F}^{k_F} =$$

cc − ss, cc + ss

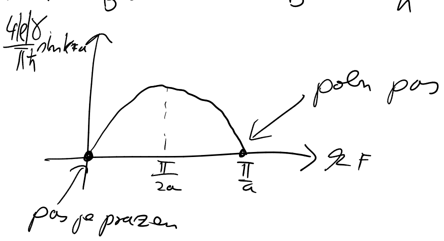
$$= \frac{2e\gamma}{\pi\hbar}\left(\cos\left(k_F + \frac{eE}{\hbar}t\right)a - \cos\left(-k_F + \frac{eEt}{\hbar}\right)a\right) =$$

$$= -\frac{4e\gamma}{\pi\hbar}\sin k_F a \,\sin\frac{eEta}{\hbar} =$$

$$= \frac{4|e|\gamma}{\pi\hbar}\sin k_F a\,\sin\frac{|e|Ea}{\hbar}t =$$

$$= \frac{4|e|\gamma}{\pi\hbar}\sin k_F a\,\sin\omega_B t$$

$\omega_B = \frac{|e|Ea}{\hbar}$ Blochova frekvenca

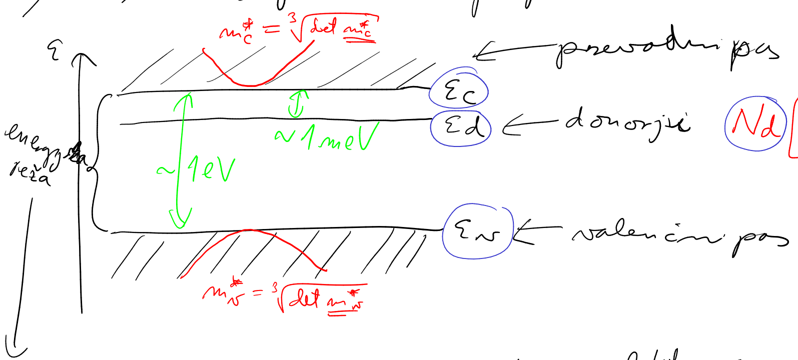


$\tau \ll \frac{1}{\omega_B}$ (realne kovine)

čas med trki

→ Ohmov zakon (Drude)

$\mu(T)$ v dopiranem polprevodniku



$$g_c(\varepsilon) = \frac{\sqrt{2}\, m_c^{*}}{\pi^2 \hbar^3}\sqrt{\varepsilon-\varepsilon_c} \quad \left[\frac{1}{J m^3}\right]$$

$N_d \left[\frac{1}{m^3}\right]$

$$g_v(\varepsilon) = \frac{\sqrt{2}\, m_v^{*}}{\pi^2 \hbar^3}\sqrt{\varepsilon_v-\varepsilon} \quad \left[\frac{1}{J m^3}\right]$$

$\varepsilon_g = \varepsilon_c - \varepsilon_v$

koncentracija elektronov v prevodnem pasu

vrzeli v valenčnem pasu

ohranitev naboja: $n_c = p_v + p_d$ → vrzeli na donorskih nivojih

→ za dovolj nizke T

$$n_c = \int_{\varepsilon_c}^{\infty} d\varepsilon\, g_c(\varepsilon) f(\varepsilon) = \int_{\varepsilon_c}^{\infty} d\varepsilon \frac{\sqrt{2}\, m_c^*}{\pi^2\hbar^3}\sqrt{\varepsilon-\varepsilon_c}\; \frac{1}{e^{\beta(\varepsilon-\mu)}+1} =$$

$\frac{1}{e^{\beta(\varepsilon-\mu)}+1} \approx e^{-\beta(\varepsilon-\mu)}$ → nedegeneriran polprevodnik

$$= \int_{\varepsilon_c}^{\infty} d\varepsilon \frac{\sqrt{2}\, m_c^*}{\pi^2\hbar^3}\sqrt{\varepsilon-\varepsilon_c}\; e^{-\beta(\varepsilon-\mu)} =$$

$\int_0^\infty dx \sqrt{x}\, e^{-x} = \frac{\sqrt{\pi}}{2}$

$$= \frac{\sqrt{2}\, m_c^*}{\pi^2\hbar^3} \int_{\varepsilon_c}^{\infty} d\varepsilon \sqrt{\varepsilon-\varepsilon_c}\; e^{-\beta(\varepsilon-\varepsilon_c)}\; e^{-\beta(\varepsilon_c-\mu)} =$$

(x = $\beta(\varepsilon-\varepsilon_c)$)

$$= \frac{\sqrt{2}\, m_c^*}{\pi^2\hbar^3}\, \frac{\sqrt{\pi}}{2}\, \beta^{-3/2}\, e^{-\beta(\varepsilon_c-\mu)} \longrightarrow$$

$$\boxed{n_c = N_c\, e^{-\beta(\varepsilon_c-\mu)} \qquad N_c = \frac{1}{4}\left(\frac{2 m_c^* k_B T}{\pi\hbar^2}\right)^{3/2}}$$

$$p_v = \int^{\varepsilon_v} d\varepsilon\, g_v(\varepsilon)\left[1-f(\varepsilon)\right] = \dots =$$

$$\boxed{p_v = P_v\, e^{-\beta(\mu-\varepsilon_v)} \qquad P_v = \frac{1}{4}\left(\frac{2 m_v^* k_B T}{\pi\hbar^2}\right)^{3/2}}$$

1 donorski atom:

$$\langle N\rangle = \frac{\sum_i N_i\, e^{-\beta(E_i-\mu N_i)}}{\sum_i e^{-\beta(E_i-\mu N_i)}} = \frac{0\, e^{-\beta(0-\mu 0)} + 2\cdot 1\, e^{-\beta(\varepsilon_d-1\mu)} + \cancel{2 e^{-\beta(2\varepsilon_d+U-2\mu)}}}{e^{-\beta(0-\mu 0)} + 2 e^{-\beta(\varepsilon_d-1\mu)} + \cancel{e^{-\beta(2\varepsilon_d+U-2\mu)}}}$$

zanemarimo

$\sum_i$ →

stanje	E_i
(prazno)	0
↑	ε_d
↓	ε_d
↑↓	$2\varepsilon_d + U$

U velik

$$\langle N\rangle = \frac{2 e^{-\beta(\varepsilon_d-\mu)}}{1+2e^{-\beta(\varepsilon_d-\mu)}}$$

$$p_d = N_d\left(1-\langle N\rangle\right)$$

$$\boxed{p_d = N_d \frac{1}{1+2e^{-\beta(\varepsilon_d-\mu)}}}$$

$$n_c = \cancel{p_v} + p_d$$

$$N_c e^{-\beta(\varepsilon_c-\mu)} = \cancel{P_v e^{-\beta(\mu-\varepsilon_v)}} + \frac{N_d}{1+2e^{-\beta(\varepsilon_d-\mu)}}$$

→ polinom 3 stopnje za neznanko $e^{\beta\mu}$.

zanemarimo pri nizkih temperaturah → kvadratna enačba za $e^{\beta\mu}$