$\hbar \dot{\vec{k}} = \vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$
$\vec{v} = \frac{1}{\hbar} \frac{d\varepsilon(\vec{k})}{d\vec{k}}$
} kvaziklasični približek

① $\vec{E} = 0$, $\vec{B}(\vec{r}) = (0, 0, B)$

prosti elektroni: $V(\vec{r}) = \emptyset$ $\quad \varepsilon = \frac{\hbar^2 k^2}{2m}$ $\quad$ el. kroži s ciklotronsko frekvenco $\frac{|e| B}{m} = \omega_c$

v periodičnem potencialu v okolici min, max ali sedla

$$\varepsilon(\vec{k}) = \frac{\hbar^2}{2} \vec{k} \cdot (m^*)^{-1} \vec{k} + \varepsilon_0$$

↳ tenzor efektivne mase (simetričen)

$$v_\alpha = \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k_\alpha} = \frac{1}{\hbar} \frac{\partial}{\partial k_\alpha} \left(\frac{\hbar^2}{2} k_\beta (m^*)^{-1}_{\beta\gamma} k_\gamma \right) = \frac{\hbar}{2} \left(\delta_{\alpha\beta} (m^*)^{-1}_{\beta\gamma} k_\gamma + k_\beta (m^*)^{-1}_{\beta\gamma} \delta_{\gamma\alpha} \right) =$$

$$= \frac{\hbar}{2} \left((m^*)^{-1}_{\alpha\gamma} k_\gamma + k_\beta (m^*)^{-1}_{\beta\alpha} \right) =$$

$$= \frac{\hbar}{2} \left((m^*)^{-1}_{\alpha\gamma} k_\gamma + (m^*)^{-1}_{\alpha\beta} k_\beta \right) = \hbar (m^*)^{-1}_{\alpha\gamma} k_\gamma = \left(\hbar (m^*)^{-1} \vec{k} \right)_\alpha$$

$$\vec{v} = \hbar (m^*)^{-1} \vec{k} \longrightarrow m^* \vec{v} = \hbar \vec{k}$$

$$m^* \dot{\vec{v}} = e \vec{v} \times \vec{B}$$

$$m^* \dot{\vec{v}} = e \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix} = e \begin{pmatrix} v_y B \\ -v_x B \\ 0 \end{pmatrix} \qquad m^* = \begin{pmatrix} m_{xx} & m_{xy} & m_{xz} \\ m_{yx} & m_{yy} & m_{yz} \\ m_{zx} & m_{zy} & m_{zz} \end{pmatrix}$$

$$\vec{v}(t) = \vec{v}_0 e^{-i\omega t}$$
$$-i\omega m^* \vec{v}_0 = e \vec{v}_0 \times \vec{B}$$

$$\vec{r}(t) = \int \vec{v}(t) dt$$

$$m^* \vec{v}_0 = \frac{ie}{\omega} \begin{pmatrix} v_{0y} B \\ -v_{0x} B \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} m_{xx}, & m_{xy} - \frac{ieB}{\omega}, & m_{xz} \\ m_{yx} + \frac{ieB}{\omega}, & m_{yy}, & m_{yz} \\ m_{zx}, & m_{zy}, & m_{zz} \end{pmatrix} \begin{pmatrix} v_{0x} \\ v_{0y} \\ v_{0z} \end{pmatrix} = 0$$

$$\det(\;) = 0$$

$$m_{xx} m_{yy} m_{zz} + \left(m_{xy} - \cancel{\frac{ieB}{\omega}} \right) \underbrace{m_{yz}}_{= m_{zy}} \underbrace{m_{zx}}_{= m_{xz}} + \left(m_{yx} + \cancel{\frac{ieB}{\omega}} \right) m_{zy} m_{xz} -$$

$$- m_{xz} m_{yy} m_{zx} - m_{yz} m_{zy} m_{xx} - \left(m_{xy} - \frac{ieB}{\omega} \right) \left(\underbrace{m_{yx}}_{= m_{xy}} + \frac{ieB}{\omega} \right) m_{zz} = 0$$

$$\vec{B} = B \hat{e}$$

$$\det m^* + \left(\frac{ieB}{\omega} \right)^2 m_{zz} = 0$$

$$\det m^* = \frac{e^2 B^2}{\omega^2} m_{zz} \qquad \omega = \frac{|e| B}{\sqrt{\frac{\det m^*}{m_{zz}}}} \qquad m_{zz} = \hat{e} \cdot m^* \hat{e}$$

$$\omega^2 = \frac{e^2 B^2 \cdot m_{zz}}{\det m^*}$$

$$\omega = \frac{|e| B}{m_c^*} \qquad m_c^* = \sqrt{\frac{\det m^*}{\hat{e} \cdot m^* \hat{e}}}$$

↳ ciklotronska efektivna masa

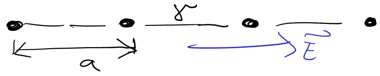
$m_c^* > 0$ v okolici min. pasu

gostota stanj: prosti elektroni: $g(\varepsilon) = \frac{1}{\pi^2 \hbar^3} \sqrt{2m}\, \sqrt{\varepsilon}$ v 3D

v periodičnem potencialu v bližini min: $g(\varepsilon) = \frac{1}{\pi^2 \hbar^3} \sqrt{2 m_g^*}\, \sqrt{\varepsilon - \varepsilon_0}$

$$m_g^* = \sqrt[3]{\det m^*}$$

Blochove oscilacije v 1D verigi
v približku tesne vezi



$\hbar \dot{k} = eE$

$k(t) = k(0) + \frac{eE}{\hbar} t$ $\quad < 0$

$E > 0$
$e < 0$

$\varepsilon(k) = -\gamma \sum_{\vec{R}=n.s.d} e^{i\vec{k}\cdot\vec{R}} =$

$= -\gamma\left(e^{ika} + e^{ik(-a)}\right) =$

$\boxed{\varepsilon(k) = -2\gamma \cos ka}$

$v = \frac{1}{\hbar}\frac{d\varepsilon}{dk} = \frac{2\gamma a}{\hbar} \sin ka$

spin

$I(t) = e\,2 \sum_{|k(0)|<k_F} v(k(t)) \cdot \frac{1}{L}$

$(j = nev)$

$\Delta k = \frac{L}{2\pi}$

$I(t) = \frac{2e}{L} \sum_{|k(0)|<k_F} v\left(k(0) + \frac{eE}{\hbar}t\right) =$

$= \frac{2e}{\cancel{L}} \frac{\cancel{L}}{2\pi} \int_{-k_F}^{k_F} dk(0)\, v\left(k(0) + \frac{eE}{\hbar}t\right) =$

$= \frac{e}{\pi} \int_{-k_F}^{k_F} dk(0)\, \frac{2\gamma a}{\hbar} \sin\left(k(0) + \frac{eE}{\hbar}t\right)a =$

$= \frac{2e\gamma \cancel{a}}{\pi\hbar} \frac{1}{\cancel{a}}\left[\cos\left(k(0) + \frac{eE}{\hbar}t\right)a\right]\Big|_{-k_F}^{k_F} =$

$cc - ss \qquad cc + ss$

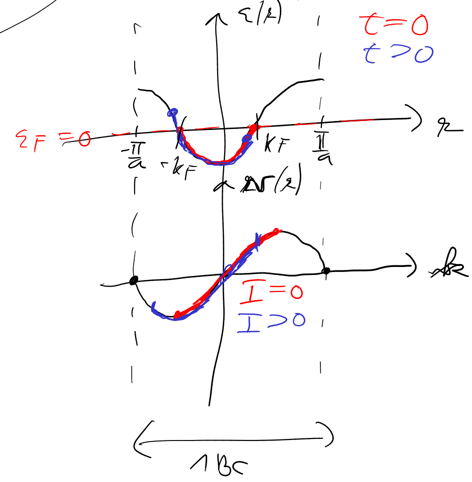
$= \frac{2e\gamma}{\pi\hbar}\left(\cos\left(k_F + \frac{eE}{\hbar}t\right)a - \cos\left(-k_F + \frac{eE}{\hbar}t\right)a\right) =$

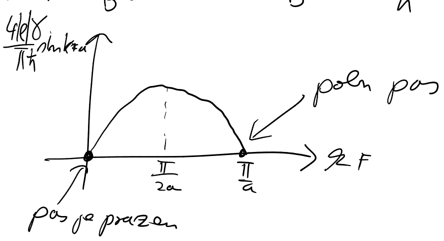
$= -\frac{4e\gamma}{\pi\hbar} \sin k_F a \sin \frac{eEta}{\hbar} =$

$= \frac{4|e|\gamma}{\pi\hbar} \sin k_F a \sin \frac{|e|Ea}{\hbar} t =$

$= \frac{4|e|\gamma}{\pi\hbar} \sin k_F a \sin \omega_B t$

$\omega_B = \frac{|e|Ea}{\hbar}$ Blochova frekvenca



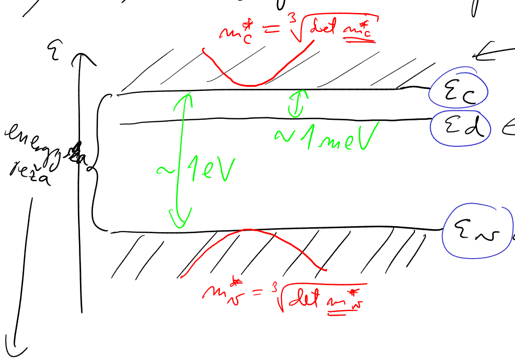


$\tau \ll \frac{1}{\omega_B}$ (realne kovine)

čas med trki

Ohmov zakon (Drude)

$\mu(T)$ v dopiranem polprevodniku



$g_c(\varepsilon) = \frac{\sqrt{2 m_c^*}}{\pi^2\hbar^3}\sqrt{\varepsilon-\varepsilon_c}$ $\left[\frac{1}{J m^3}\right]$

N_d $\left[\frac{1}{m^3}\right]$

$g_v(\varepsilon) = \frac{\sqrt{2 m_v^*}}{\pi^2\hbar^3}\sqrt{\varepsilon_v-\varepsilon}$ $\left[\frac{1}{J m^3}\right]$

$\varepsilon_g = \varepsilon_c - \varepsilon_v$

koncentracija elektronov v prevodnem pasu

vrzeli v valenčnem pasu

ohranitev naboja: $n_c = p_v + p_d$ → vrzeli na donorskih nivojih

→ za dovolj nizke T

$$n_c = \int_{\varepsilon_c}^{\infty} d\varepsilon\, g_c(\varepsilon) f(\varepsilon) = \int_{\varepsilon_c}^{\infty} d\varepsilon \frac{\sqrt{2 m_c^*}}{\pi^2\hbar^3}\sqrt{\varepsilon-\varepsilon_c}\, \underbrace{\frac{1}{e^{\beta(\varepsilon-\mu)}+1}}_{\approx e^{-\beta(\varepsilon-\mu)}} =$$

→ nedegeneriran polprevodnik

$$= \int_{\varepsilon_c}^{\infty} d\varepsilon \frac{\sqrt{2 m_c^*}}{\pi^2\hbar^3}\sqrt{\varepsilon-\varepsilon_c}\; e^{-\beta(\varepsilon-\mu)} =$$

$\int_0^\infty dx\, \sqrt{x}\, e^{-x} = \frac{\sqrt{\pi}}{2}$

$$= \frac{\sqrt{2 m_c^*}}{\pi^2\hbar^3} \int_{\varepsilon_c}^{\infty} d\varepsilon \sqrt{\varepsilon-\varepsilon_c}\; e^{-\underbrace{\beta(\varepsilon-\varepsilon_c)}_{x}}\; e^{-\beta(\varepsilon_c-\mu)} =$$

$$= \frac{\sqrt{2 m_c^*}}{\pi^2\hbar^3}\frac{\sqrt{\pi}}{2}\beta^{-3/2} e^{-\beta(\varepsilon_c-\mu)} \longrightarrow \boxed{n_c = N_c e^{-\beta(\varepsilon_c-\mu)} \quad N_c = \frac{1}{4}\left(\frac{2 m_c^* k_B T}{\pi\hbar^2}\right)^{3/2}}$$

$$p_v = \int^{\varepsilon_v} d\varepsilon\, g_v(\varepsilon)\left[1-f(\varepsilon)\right] = \dots =$$

$$\boxed{p_v = P_v e^{-\beta(\mu-\varepsilon_v)} \qquad P_v = \frac{1}{4}\left(\frac{2 m_v^* k_B T}{\pi\hbar^2}\right)^{3/2}}$$

1 donorski atom:
$$\langle N\rangle = \frac{\sum_i N_i e^{-\beta(E_i-\mu N_i)}}{\sum_i e^{-\beta(E_i-\mu N_i)}} = \frac{0\, e^{-\beta(0-\mu 0)} + 2\cdot 1\, e^{-\beta(\varepsilon_d-1\mu)} + \cancel{2e^{-\beta(2\varepsilon_d+U-2\mu)}}}{e^{-\beta(0-\mu 0)} + 2e^{-\beta(\varepsilon_d-1\mu)} + \cancel{e^{-\beta(2\varepsilon_d+U-2\mu)}}}$$

$\sum_i \to$

	E_i
—	0
↑	ε_d
↓	ε_d
↑↓	$2\varepsilon_d + U$

U velik

zanemarimo

$$\langle N\rangle = \frac{2e^{-\beta(\varepsilon_d-\mu)}}{1+2e^{-\beta(\varepsilon_d-\mu)}}$$

$$p_d = N_d\left(1-\langle N\rangle\right)$$

$$\boxed{p_d = N_d \frac{1}{1+2e^{-\beta(\varepsilon_d-\mu)}}}$$

$$n_c = \cancel{p_v} + p_d$$

$$N_c e^{-\beta(\varepsilon_c-\mu)} = \cancel{P_v e^{-\beta(\mu-\varepsilon_v)}} + \frac{N_d}{1+2e^{-\beta(\varepsilon_d-\mu)}}$$

→ polinom 3 stopnje za neznanko $e^{\beta\mu}$.

zanemarimo pri nizkih temperaturah → kvadratna enačba za $e^{\beta\mu}$

$$N_c e^{-\beta\varepsilon_c} e^{\beta\mu} + 2N_c e^{-\beta(\varepsilon_c+\varepsilon_d)} e^{2\beta\mu} - N_d = 0$$

$$e^{\beta\mu} = \frac{-N_c e^{-\beta\varepsilon_c} \pm \sqrt{(N_c e^{-\beta\varepsilon_c})^2 + 8N_cN_d e^{-\beta(\varepsilon_c+\varepsilon_d)}}}{4N_c e^{-\beta(\varepsilon_c+\varepsilon_d)}}$$

$$e^{\beta\mu} = -\frac{1}{4}e^{\beta\varepsilon_d} \pm \sqrt{\left(\frac{1}{4}e^{\beta\varepsilon_d}\right)^2 + \frac{N_d}{2N_c}e^{\beta(\varepsilon_c+\varepsilon_d)}}$$

$$e^{\beta\mu} = \frac{1}{4}e^{\beta\varepsilon_d}\left[-1 + \sqrt{1+\frac{8N_d}{N_c}e^{\beta(\varepsilon_c-\varepsilon_d)}}\right] \qquad \beta = \frac{1}{k_BT}$$

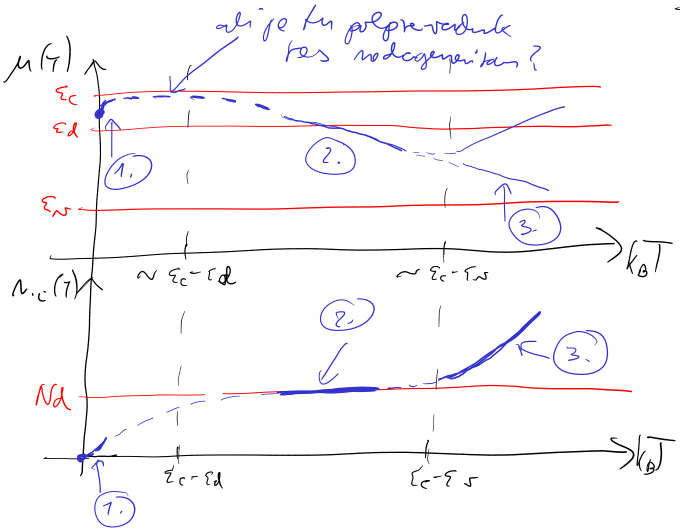
1. $\frac{8N_d}{N_c}e^{\beta(\varepsilon_c-\varepsilon_d)} \gg 1$ nizke temperature $k_BT \ll A(\varepsilon_c-\varepsilon_d)$ $A=\mathcal{O}(1)$ $N_c \propto T^{3/2}$; $\varepsilon_c-\varepsilon_d>0$

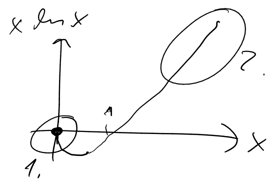
$$e^{\beta\mu} = \frac{1}{4}e^{\beta\varepsilon_d}\sqrt{\frac{8N_d}{N_c}}\,e^{\beta\frac{\varepsilon_c-\varepsilon_d}{2}} = \sqrt{\frac{N_d}{2N_c}}\,e^{\beta\frac{\varepsilon_c+\varepsilon_d}{2}}$$

$$\to \mu = \frac{\varepsilon_c+\varepsilon_d}{2} + \frac{1}{\beta}\ln\sqrt{\frac{N_d}{2N_c}} \doteq \underline{\frac{\varepsilon_c+\varepsilon_d}{2} - \frac{3}{4}k_BT\ln T} + \mathcal{O}(T)$$

$\lim_{T\to 0} = 0$

$$n_c = N_c e^{-\beta(\varepsilon_c-\mu)} = N_c e^{-\beta\varepsilon_c}\cdot\sqrt{\frac{N_d}{2N_c}}\,e^{\beta\frac{\varepsilon_c+\varepsilon_d}{2}} = \underline{\sqrt{\frac{N_dN_c}{2}}\,e^{-\beta\frac{\varepsilon_c-\varepsilon_d}{2}}} \propto T^{3/4}e^{-\frac{\varepsilon_c-\varepsilon_d}{2k_BT}}$$





2. $\frac{8N_d}{N_c}e^{\beta(\varepsilon_c-\varepsilon_d)} \ll 1$ visoke temperature $k_BT \gg A(\varepsilon_c-\varepsilon_v)$ $A=\mathcal{O}(1)$

$$e^{\beta\mu} = \frac{1}{4}e^{\beta\varepsilon_d}\left[-\cancel{1}+\left(\cancel{1}+\frac{1}{2}\frac{8N_d}{N_c}e^{\beta(\varepsilon_c-\varepsilon_d)}\right)\right] = \frac{N_d}{N_c}e^{\beta\varepsilon_c}$$

$$\mu = \varepsilon_c + k_BT\ln\frac{N_d}{N_c} = \varepsilon_c - \frac{3}{2}k_BT\ln T + \mathcal{O}(T)$$

$$n_c = N_c e^{-\beta\varepsilon_c}e^{\beta\mu} = \cancel{N_c}\cancel{e^{-\beta\varepsilon_c}}\frac{N_d}{\cancel{N_c}}\cancel{e^{\beta\varepsilon_c}} = N_d$$

3. $n_c \gg N_d$ $(T \gtrsim \varepsilon_c-\varepsilon_v)$

$$n_c = p_v + \cancel{p_d}$$

$$N_c e^{-\beta(\varepsilon_c-\mu)} = P_v e^{-\beta(\mu-\varepsilon_v)} \qquad N_c = \frac{1}{4}\left(\frac{2m_c^*k_BT}{\pi\hbar^2}\right)^{3/2}$$

$$e^{2\beta\mu} = \frac{P_v}{N_c}e^{+\beta(\varepsilon_c+\varepsilon_v)} \qquad P_v = \frac{1}{4}\left(\frac{2m_v^*k_BT}{\pi\hbar^2}\right)^{3/2}$$

$$\mu = \frac{\varepsilon_c+\varepsilon_v}{2} + \frac{1}{2}k_BT\ln\frac{P_v}{N_c} = \frac{\varepsilon_c+\varepsilon_v}{2} + \frac{3}{4}k_BT\ln\frac{m_v^*}{m_c^*}$$

$$n_c = N_c e^{-\beta\varepsilon_c} e^{\beta\mu} = N_c e^{-\beta\varepsilon_c}\sqrt{\frac{P_v}{N_c}}\, e^{\beta\frac{\varepsilon_c+\varepsilon_v}{2}} =$$

$$= \sqrt{N_c P_v}\, e^{-\beta\left(\frac{\varepsilon_c-\varepsilon_v}{2}\right)} \propto T^{3/2} e^{-\frac{\varepsilon_c-\varepsilon_v}{2k_BT}}$$