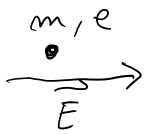
Drudejev model: prevodnost v izmeničnem el. polju

$$\vec{E} = \vec{E}_0 e^{-i\omega t}$$



$$m\ddot{\vec{r}} = e\vec{E} - \frac{m\dot{\vec{r}}}{\tau}$$

$$\vec{r} = \vec{r}_0 e^{-i\omega t}$$

$$-m\omega^2\vec{r}_0 \cancel{e^{-i\omega t}} = e\vec{E}_0 \cancel{e^{-i\omega t}} - \frac{m(-i\omega)\vec{r}_0 \cancel{e^{-i\omega t}}}{\tau}$$

$$\vec{r}_0 = \frac{e\vec{E}_0}{-m\omega^2 - im\omega/\tau}$$

$$\vec{j} = \sigma\vec{E}$$

$$\vec{j}_0 = \sigma(\omega)\vec{E}_0$$

$$\vec{j} = ne\dot{\vec{r}} = ne(-i\omega)\vec{r}_0 e^{-i\omega t} = \underbrace{\left(\frac{ne^2}{m}\,\frac{-i\omega}{-\omega^2 - i\omega/\tau}\,E_0\right)}_{\vec{j}_0} e^{-i\omega t}$$

$$\sigma(\omega) = \frac{ne^2\tau}{m}\,\frac{+i\cancel{\omega}/\tau}{\omega^2 + i\cancel{\omega}/\tau}$$

$$\sigma(\omega) = \frac{ne^2\tau}{m}\,\frac{i}{\omega\tau + i}$$

$$\boxed{\sigma(\omega) = \frac{ne^2\tau}{m}\,\frac{1}{1 - i\omega\tau}}$$

$$\sigma(\omega) = \sigma(0)\,\frac{1}{1 - i\omega\tau}$$

$\omega \ll \frac{1}{\tau}$ $\qquad \sigma(\omega) \approx \sigma(0)$

$\omega \gg \frac{1}{\tau}$ $\qquad \sigma(\omega) \approx i\,\frac{ne^2}{m\omega}$

$\vec{E}, \vec{j}$ sta zamaknjena za 90°

$\vec{j} \cdot \vec{E}$

elektronski plin prostih elektronov

$$V(\vec{r}) = \text{konst.} = 0$$

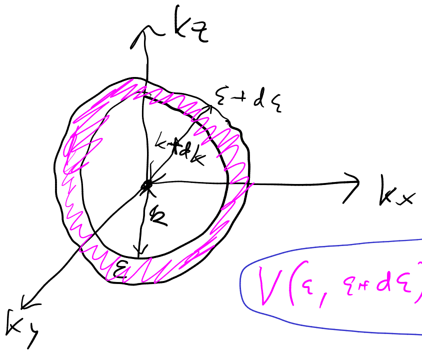
$$\psi(\vec{r}) \propto e^{i\vec{k}\cdot\vec{r}} \qquad \varepsilon(\vec{k}) = \frac{\hbar^2 k^2}{2m} \qquad k = |\vec{k}|$$

1.) izračunaj gostoto stanj $\rightarrow$ 1D, 2D, 3D

2.) tlak

1.) 3D

$$g(\varepsilon) = \frac{1}{V}\frac{dN(\varepsilon, \varepsilon+d\varepsilon)}{d\varepsilon}$$



$$V(\varepsilon, \varepsilon+d\varepsilon) = 4\pi k^2 dk$$

PRP

$$V = L^3 \qquad e^{ik_xL} = 1 \rightarrow k_x = \frac{2\pi}{L}n \qquad n \in \mathbb{Z}$$

$$\Delta k_x = \frac{2\pi}{L}$$

$$V_1 = \left(\frac{2\pi}{L}\right)^3$$

spin

$$dN = 2\frac{4\pi k^2 dk}{\left(\frac{2\pi}{L}\right)^3}$$

$$k = \sqrt{\frac{2m\varepsilon}{\hbar^2}}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2m}$$

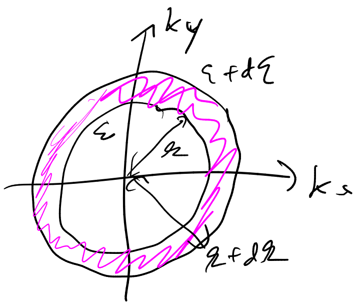
$$\frac{d\varepsilon}{dk} = \frac{\hbar^2}{m}k$$

$$\frac{dk}{d\varepsilon} = \frac{m}{\hbar^2 k}$$

$$g(\varepsilon) = \frac{1}{\cancel{V}}\frac{8\pi k^2 \cancel{L^3}}{8\pi^3}\frac{dk}{d\varepsilon} = \frac{1}{\pi^2}k^2\frac{dk}{d\varepsilon} =$$

$$= \frac{1}{\pi^2}k\frac{m}{\hbar^2} = \frac{m}{\pi^2\hbar^2}\sqrt{\frac{2m\varepsilon}{\hbar^2}}\,\theta(\varepsilon)$$

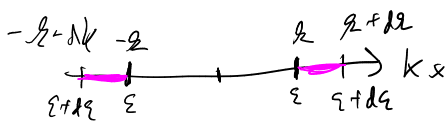
2D



$$dN = \frac{2\pi k\, dk}{\left(\frac{2\pi}{L}\right)^2}\cdot 2 = \frac{L^2}{\pi}k\, dk$$

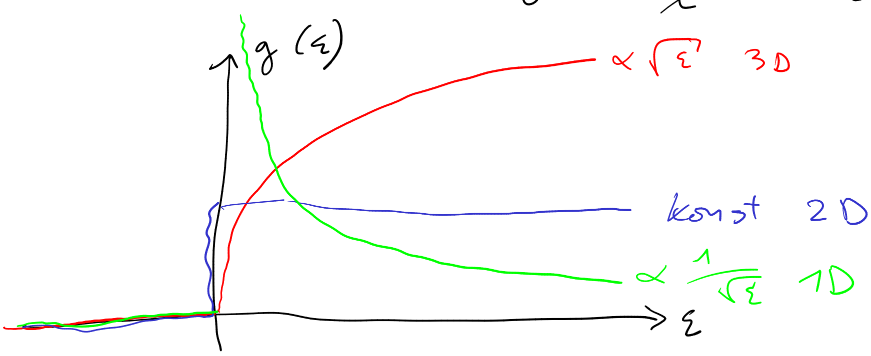
$$g(\varepsilon) = \frac{1}{\cancel{L^2}}\frac{\cancel{L^2}}{\pi}k\frac{dk}{d\varepsilon} = \frac{m}{\pi\hbar^2}\theta(\varepsilon)$$

1D



$$dN = 2\frac{2\,dk}{\frac{2\pi}{L}} = L\frac{2\,dk}{\pi}$$

$$g(\varepsilon) = \frac{1}{\cancel{L}}\cancel{L}\frac{2\,dk}{\pi\, d\varepsilon} = \frac{2}{\pi}\frac{m}{\hbar^2 k} = \frac{2m}{\pi\hbar^2}\sqrt{\frac{\hbar^2}{2m\varepsilon}}\,\theta(\varepsilon)$$

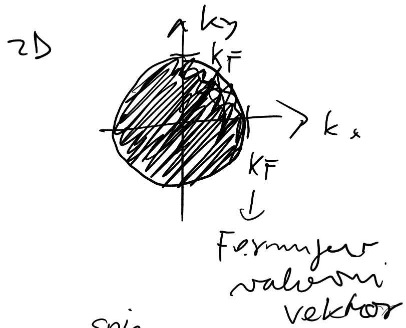


$$g(\varepsilon) = A_d\,\varepsilon^{\frac{d-2}{2}}\,\theta(\varepsilon)$$

2) tlak pri T = 0K

N elektronov

stanja zasedena do k_F



$dE = -p\,dV$

$$p = -\frac{dE}{dV}$$

$$E = 2\sum_{|\vec{k}|<k_F} \frac{\hbar^2 k^2}{2m} =$$

(spin)

3D $\to 2\left(\frac{L}{2\pi}\right)^3 \int d\vec{k}\, \frac{\hbar^2 k^2}{2m} = \frac{L^3}{4\pi^3}\int_0^{k_F} 4\pi k^2 dk\, \frac{\hbar^2 k^2}{2m} =$

$$E = \frac{L^3}{\pi^2}\frac{\hbar^2}{2m}\frac{k_F^5}{5}$$

2D $\to 2\left(\frac{L}{2\pi}\right)^2 \int_0^{k_F} 2\pi k\, dk\, \frac{\hbar^2 k^2}{2m} =$

$$E = \frac{L^2}{\pi}\frac{\hbar^2}{2m}\frac{k_F^4}{4}$$

1D $\to 2\,\frac{L}{2\pi}\int_{-k_F}^{k_F} dk\, \frac{\hbar^2 k^2}{2m} =$

$$E = \frac{L}{\pi}\frac{\hbar^2}{2m}\frac{2k_F^3}{3}$$

$$N = 2\sum_{|\vec{k}|<k_F} 1$$

3D $N = 2\left(\frac{L}{2\pi}\right)^3 \int_0^{k_F} 4\pi k^2 dk = \frac{L^3}{\pi^2}\frac{k_F^3}{3}$

2D $N = 2\left(\frac{L}{2\pi}\right)^2 \int_0^{k_F} 2\pi k\, dk = \frac{L^2}{\pi}\frac{k_F^2}{2}$

1D $N = 2\,\frac{L}{2\pi}\int_{-k_F}^{k_F} dk = \frac{L}{\pi}2k_F$

3D $E = \frac{\hbar^2 k_F^2}{2m}\frac{3}{5}N$ $\qquad E = N\frac{3}{5}\varepsilon_F$

ε_F Fermijeva energija

2D $E = N\cdot\frac{\hbar^2 k_F^2}{2m}\cdot\frac{1}{2}$ $\qquad E = N\frac{1}{2}\varepsilon_F$

1D $E = N\cdot\frac{\hbar^2 k_F^2}{2m}\cdot\frac{1}{3}$ $\qquad E = N\frac{1}{3}\varepsilon_F$

$$E = N\frac{d}{d+2}\varepsilon_F \propto V^{-\frac{2}{d}}$$

$$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m} \propto V^{-\frac{2}{d}}$$

3D $N \propto V k_F^3$ $\quad V = L^3$ $\to k_F \propto V^{-\frac{1}{3}}$

2D $N \propto V k_F^2$ $\quad V = L^2$ $\quad k_F \propto V^{-\frac{1}{2}}$

1D $N \propto V k_F$ $\quad V = L$ $\quad k_F \propto V^{-1}$

$k_F \propto V^{-\frac{1}{d}}$

$$p = -\frac{dE}{dV}$$

$$E = B_d V^{-\frac{2}{d}}$$

$$\frac{dE}{dV} = -\frac{2}{d} B_d V^{-\frac{2}{d}-1} = -\frac{2}{d}\frac{E}{V}$$

$$p = \frac{2}{d}\frac{E}{V} = \frac{2}{\cancel{d}} N\frac{\cancel{d}}{d+2}\frac{\varepsilon_F}{V}$$

$$p = \frac{2}{d+2}\varepsilon_F\frac{N}{V}$$

temperaturna odvisnost kemijskega potenciala

$$N(T=0K) = N(T>0K)$$

$$\varepsilon_F = \mu(T=0K) \neq \mu(T>0K)$$

Sommerfeldov razvoj:

$$\mu(T) = \varepsilon_F - \frac{\pi^2}{6}(k_B T)^2 \frac{g'(\varepsilon_F)}{g(\varepsilon_F)}$$

$$k_B T \ll \varepsilon_F$$

sobna temperatura: $k_B T = \frac{1}{40}\,eV$

$\varepsilon_F \sim 1 - 10\,eV$

elektronski plin: $g(\varepsilon) = A_d\, \varepsilon^{\frac{d-2}{2}}\, \Theta(\varepsilon)$

prosti elektroni

$$g'(\varepsilon) = A_d \frac{d-2}{2} \varepsilon^{\frac{d-2}{2}-1} \Theta(\varepsilon) \qquad \varepsilon > 0$$

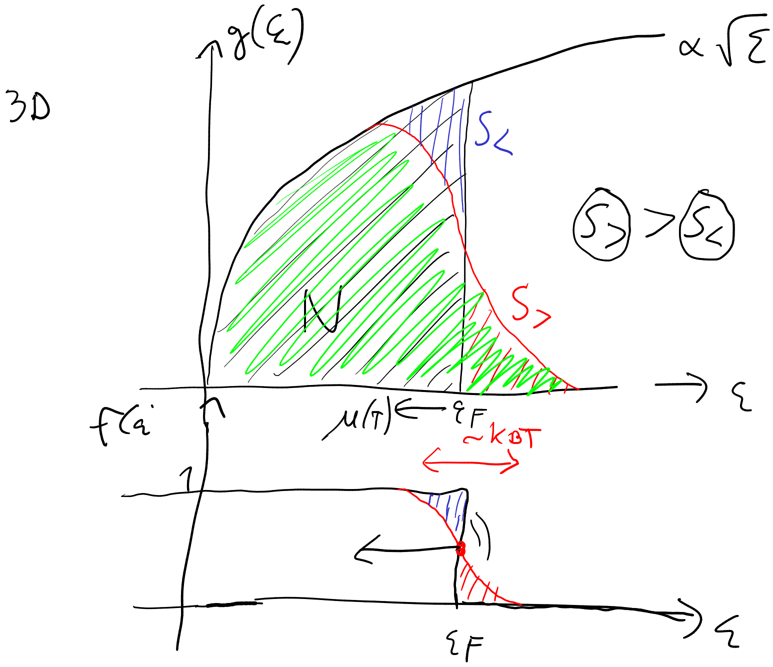
$$g'(\varepsilon) = \frac{d-2}{2} \frac{g(\varepsilon)}{\varepsilon} \qquad \varepsilon > 0$$

$$\mu(T) = \varepsilon_F - \frac{\pi^2}{6}(k_B T)^2 \frac{d-2}{2} \frac{1}{\varepsilon_F}$$

3D: $\mu(T) = \varepsilon_F \left(1 - \frac{\pi^2}{12}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right)$

2D: $\mu(T) = \varepsilon_F$

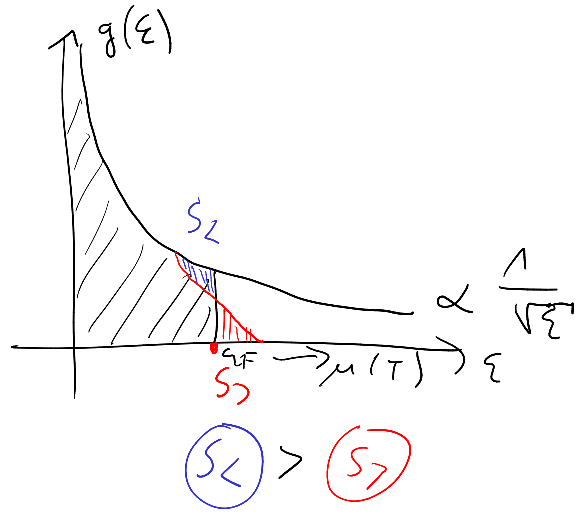
1D: $\mu(T) = \varepsilon_F \left(1 + \frac{\pi^2}{12}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right)$



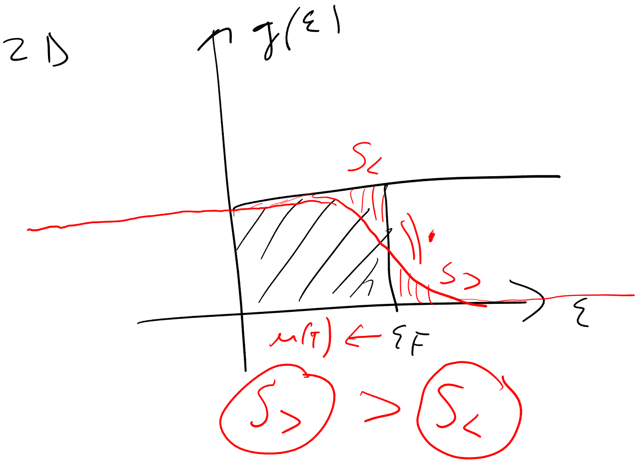
$$N = \int_{-\infty}^{\infty} g(\varepsilon) f(\varepsilon)\, d\varepsilon$$

$$f(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)} + 1}$$

1D



2D



2D

$$N(T=0K) = N(T)$$

$$g(\varepsilon) = A_2\, \theta(\varepsilon)$$

$$\int_0^{\varepsilon_F} g(\varepsilon)\, d\varepsilon = \int_{-\infty}^{\infty} g(\varepsilon) f(\varepsilon)\, d\varepsilon$$

$$\cancel{A_2}\, \varepsilon_F = \cancel{A_2} \int_0^{\infty} \frac{d\varepsilon}{e^{\beta(\varepsilon-\mu)}+1}$$

$$u = e^{\beta(\varepsilon-\mu)}+1$$
$$du = e^{\beta(\varepsilon-\mu)} \beta\, d\varepsilon$$
$$du = \beta(u-1)\, d\varepsilon$$

$$\varepsilon_F = \frac{1}{\beta} \int_{e^{-\beta\mu}+1}^{\infty} \frac{du}{u(u-1)}$$

$$\varepsilon_F = \frac{1}{\beta} \int_{e^{-\beta\mu}+1}^{\infty} \left(-\frac{1}{u} + \frac{1}{u-1}\right) du = \frac{1}{\beta} \ln \frac{u-1}{u} \Big|_{e^{-\beta\mu}+1}^{\infty} =$$

$$= -\frac{1}{\beta} \ln \frac{e^{-\beta\mu}}{e^{-\beta\mu}+1} = -\frac{1}{\beta} \ln \frac{1}{1+e^{\beta\mu}} = \frac{1}{\beta} \ln(1+e^{\beta\mu})$$

$$\beta \varepsilon_F = \ln(1+e^{\beta\mu})$$
$$e^{\beta\varepsilon_F} = 1 + e^{\beta\mu}$$
$$e^{\beta\mu} = e^{\beta\varepsilon_F} - 1$$
$$\beta\mu = \ln(e^{\beta\varepsilon_F}-1) = \ln\left(\underline{e^{\beta\varepsilon_F}}\,\underline{(1-e^{-\beta\varepsilon_F})}\right)$$
$$\beta\mu = \beta\varepsilon_F + \ln(1-e^{-\beta\varepsilon_F}) \qquad \beta\varepsilon_F = \frac{\varepsilon_F}{k_B T} \gg 1$$
$$\beta\mu \doteq \beta\varepsilon_F - e^{-\beta\varepsilon_F}$$

$$\mu(T) = \varepsilon_F - k_B T e^{-\frac{\varepsilon_F}{k_B T}}$$

pri $T=0$ ni analitična