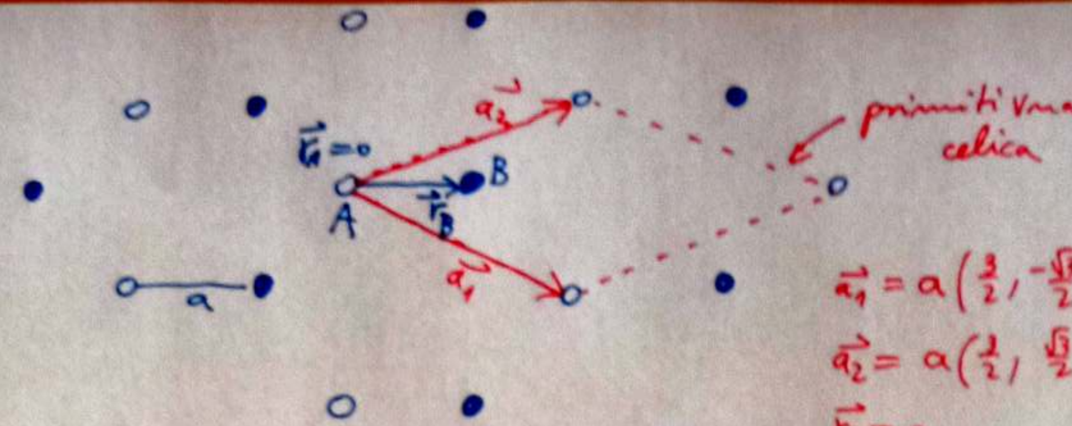
① a)



$\vec{a}_1 = a\left(\frac{3}{2}, -\frac{\sqrt{3}}{2}\right)$

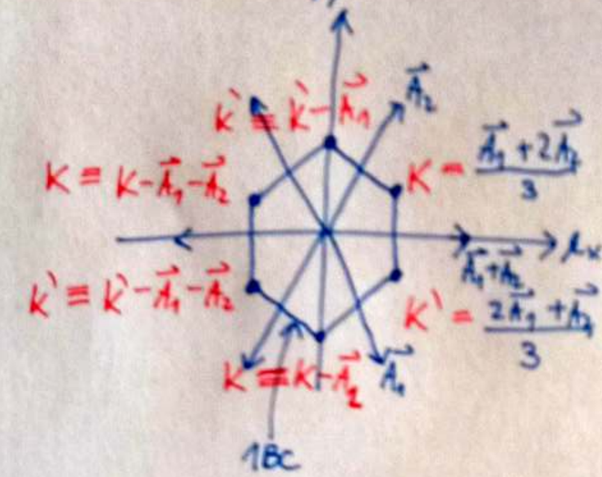
$\vec{a}_2 = a\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

$\vec{r}_A = 0$

$\vec{r}_B = a(1,0)$

$$(\vec{A}_1, \vec{A}_2) = 2\pi \begin{pmatrix} \vec{a}_1 \\ \vec{a}_2 \end{pmatrix}^{-1} \Rightarrow \vec{A}_1 = \frac{4\pi}{3a}\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$\vec{A}_2 = \frac{4\pi}{3a}\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$



1/4 +

b)

$$-\Delta c_{\vec{R}A} - \gamma c_{\vec{R}B} - \gamma c_{\vec{R}-\vec{a}_1 B} - \gamma c_{\vec{R}-\vec{a}_2 B} = E c_{\vec{R}A}$$

$$\Delta c_{\vec{R}B} - \gamma c_{\vec{R}A} - \gamma c_{\vec{R}+\vec{a}_1 A} - \gamma c_{\vec{R}+\vec{a}_2 A} = E c_{\vec{R}B}$$

$$c_{\vec{R}A} = c_A e^{i\vec{k}\cdot\vec{R}}$$

$$c_{\vec{R}B} = c_B e^{i\vec{k}\cdot\vec{R}}$$

$$\begin{bmatrix} -\Delta - E, & -\gamma\left(1 + e^{-i\vec{k}\cdot\vec{a}_1} + e^{-i\vec{k}\cdot\vec{a}_2}\right) \\ -\gamma\left(1 + e^{i\vec{k}\cdot\vec{a}_1} + e^{i\vec{k}\cdot\vec{a}_2}\right), & \Delta - E \end{bmatrix} \begin{bmatrix} c_A \\ c_B \end{bmatrix} = 0$$

$$(-\Delta - E)(\Delta - E) - \gamma^2 \left|1 + e^{i\vec{k}\cdot\vec{a}_1} + e^{i\vec{k}\cdot\vec{a}_2}\right|^2 = 0$$

$$E = \pm\sqrt{\Delta^2 + \gamma^2 \left|1 + e^{i\vec{k}\cdot\vec{a}_1} + e^{i\vec{k}\cdot\vec{a}_2}\right|^2}$$

1/4 +

① c) $E(K+\vec{\kappa}) = \pm\sqrt{\Delta^2+\gamma^2\left|1+e^{i\frac{2\pi}{3}}e^{i\vec{\kappa}\cdot\vec{a}_1}+e^{i\frac{4\pi}{3}}e^{i\vec{\kappa}\cdot\vec{a}_2}\right|^2} =$

$(|\vec{\kappa}|\ll\frac{\pi}{a}) \quad \doteq \pm\sqrt{\Delta^2+\gamma^2\left|ie^{i\frac{2\pi}{3}}\vec{\kappa}\cdot\vec{a}_1+ie^{-i\frac{2\pi}{3}}\vec{\kappa}\cdot\vec{a}_2\right|^2} =$

$= \pm\sqrt{\Delta^2+\gamma^2\cdot\frac{9}{4}a^2\kappa^2}$

$E(K'+\vec{\kappa}) = \pm\sqrt{\Delta^2+\gamma^2\left|1+e^{i\frac{4\pi}{3}}e^{i\vec{\kappa}\cdot\vec{a}_1}+e^{i\frac{2\pi}{3}}e^{i\vec{\kappa}\cdot\vec{a}_2}\right|^2} =$

$(|\vec{\kappa}|\ll\frac{\pi}{a}) \quad \doteq \pm\sqrt{\Delta^2+\gamma^2\left|ie^{-i\frac{2\pi}{3}}\vec{\kappa}\cdot\vec{a}_1+ie^{i\frac{2\pi}{3}}\vec{\kappa}\cdot\vec{a}_2\right|^2} =$

$= \pm\sqrt{\Delta^2+\gamma^2\cdot\frac{9}{4}a^2\kappa^2} = E(K+\vec{\kappa})$

$E(K+\vec{\kappa}) \doteq \pm\Delta\sqrt{1+\frac{9a^2\gamma^2\kappa^2}{4\Delta^2}} \doteq \pm\Delta\left(1+\frac{9}{8}\frac{a^2\gamma^2\kappa^2}{\Delta^2}\right) =$

$= \pm\left(\Delta+\frac{\hbar^2\kappa^2}{2m^*}\right) \qquad m_e^* = \frac{4\hbar^2\Delta}{9\gamma^2a^2} = m_h^*$

1/4

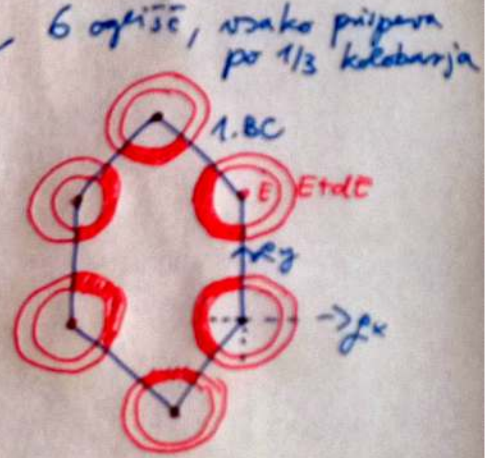
d) za $E>0$:

$g(E) = \frac{1}{S}\frac{dN}{dE} = \frac{1}{S}\,\frac{2\pi\kappa\,d\kappa}{\frac{(2\pi)^2}{S}dE}\cdot 2\cdot 2 =$

spin

6 oglišč, vsako prispeva po 1/3 kolobarja

$= \frac{2}{\pi}\frac{\kappa\,d\kappa}{dE}$

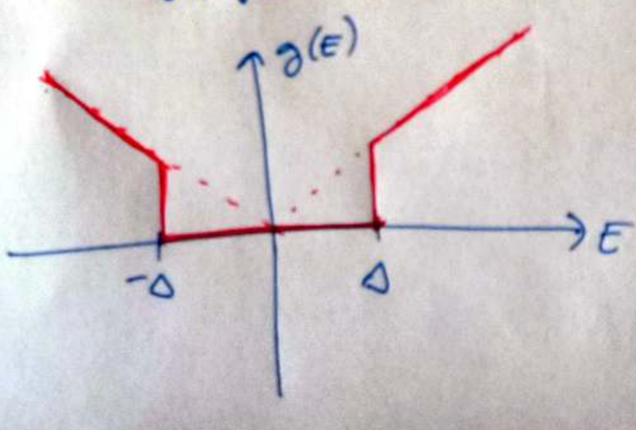


$E^2 = \Delta^2+\gamma^2\frac{9}{4}a^2\kappa^2$

$2E\,dE = \frac{9}{2}\gamma^2a^2\kappa\,d\kappa \rightarrow \frac{\kappa\,d\kappa}{dE} = \frac{4E}{9\gamma^2a^2}$ za $E>\Delta$

$g(-E)=g(E) \Rightarrow$

$g(E) = \frac{8}{9\pi\gamma^2a^2}|E|\,\theta(|E|-\Delta)$



1/4+

(2) a) iz vaj: $\frac{8N_d}{N_c} e^{\beta(\varepsilon_c-\varepsilon_d)} = 1.3\times 10^5 \gg 1$

$$\mu = \frac{\varepsilon_c+\varepsilon_d}{2} + k_BT \ln\sqrt{\frac{N_d}{2N_c}} = \varepsilon_c - 8.8\,\mathrm{meV} = \varepsilon_c - 10.2\, k_BT$$

polprevodnik ni degeneriran

$$n_c = \sqrt{\frac{N_d N_c}{2}}\, e^{-\beta\frac{\varepsilon_c-\varepsilon_d}{2}} = \underline{5.6\times 10^{17}/m^3}$$ 1/2

b) Pri T=0K elektroni iz donorskega nivoja zasedejo vsa stanja akceptorskega nivoja:

$$\underline{p_d = N_a = 10^{19}/m^3}$$ 1/4⁻

c) Ker je donorski nivo delno zaseden, je $\mu = \varepsilon_d$. 1/4⁻

d) $n_c + n_a = p_v + p_d$

Ker je $k_BT \lll (\varepsilon_c, \varepsilon_d) - (\varepsilon_a, \varepsilon_v)$ je $p_v \doteq 0$ in $n_a \doteq N_a$:

$n_c + N_a \doteq p_d$

Ker je $k_BT \ll \varepsilon_c - \varepsilon_d$, je $p_d \doteq N_a$:

$$N_a \doteq p_d = \frac{N_d}{1+2e^{-\beta(\varepsilon_d-\mu)}}$$

$$\mu \doteq \varepsilon_d + k_BT \ln\frac{N_d-N_a}{2N_a} = \varepsilon_c - 11.4\,\mathrm{meV} = \varepsilon_c - 13.2\,k_BT,$$ ni degeneriran

$$n_c = N_c e^{-\beta(\varepsilon_c-\mu)} = \frac{N_c(N_d-N_a)}{2N_a} e^{-\beta(\varepsilon_c-\varepsilon_d)} = \underline{2.8\times 10^{16}/m^3}$$

1/2⁻