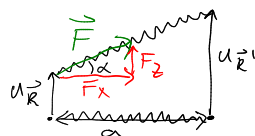
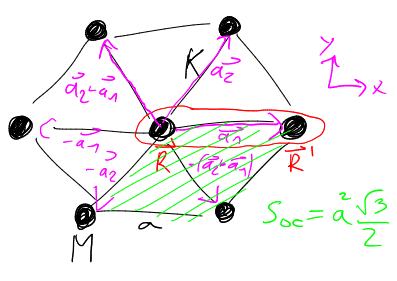


materna mihanya

- trikotna mreža
- odsvaki naj bodo  $\perp$  na ravni kristala
- nameti so prednapete, samo med najbližjimi sosedi

$\vec{a}_1 = a(1, 0)$   
 $\vec{a}_2 = a(\frac{1}{2}, \frac{\sqrt{3}}{2})$



neraztegnjeno namet je dolga  $a_0 < a$

raztezek:

$$\sqrt{a^2 + (u_{\vec{R}'} - u_{\vec{R}})^2} - a_0 = |u_{\vec{R}'} - u_{\vec{R}}| \ll a$$

$$= a \sqrt{1 + \frac{(u_{\vec{R}'} - u_{\vec{R}})^2}{a^2}} - a_0 \approx a + \frac{1}{2} \frac{(u_{\vec{R}'} - u_{\vec{R}})^2}{a} - a_0 = a - a_0 + \frac{1}{2} \frac{(u_{\vec{R}'} - u_{\vec{R}})^2}{a}$$

velikot sile:

$$F = K(a - a_0 + \frac{1}{2} \frac{(u_{\vec{R}'} - u_{\vec{R}})^2}{a})$$

$$F_x = F \cos \alpha = F(1 - \frac{\alpha^2}{2}) = \frac{K}{a} (a - a_0) + \sigma (u_{\vec{R}'} - u_{\vec{R}})^2$$

$$F_z = F \sin \alpha = F \alpha = \frac{K}{a} (a - a_0) (u_{\vec{R}'} - u_{\vec{R}}) + \sigma (u_{\vec{R}'} - u_{\vec{R}})^3$$

$$\alpha = \tan \alpha = \frac{u_{\vec{R}'} - u_{\vec{R}}}{a}$$

$$e^{-i\vec{k}\cdot\vec{r}} \left( -M\omega^2 u_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} = \tilde{K} \sum_{\vec{R}'=n.s.\vec{R}} u_0 (e^{i(\vec{k}\cdot\vec{R}' - \omega t)} - e^{i(\vec{k}\cdot\vec{R} - \omega t)}) \right)$$

$$-M\omega^2 = \tilde{K} \sum_{\vec{R}'=n.s.\vec{R}} [e^{i\vec{k}\cdot(\vec{R}' - \vec{R})} - 1]$$

$$-M\omega^2 = \tilde{K} (e^{i\vec{k}\cdot\vec{a}_1} + e^{i\vec{k}\cdot\vec{a}_2} - 1 + e^{i\vec{k}\cdot(\vec{a}_2 - \vec{a}_1)} - 1 + e^{-i\vec{k}\cdot\vec{a}_1} - 1 + \dots) =$$

$$= \tilde{K} (2 \cos(\vec{k}\cdot\vec{a}_1) - 2 + 2 \cos(\vec{k}\cdot\vec{a}_2) - 2 + 2 \cos[\vec{k}\cdot(\vec{a}_2 - \vec{a}_1)] - 2) = \text{sh}^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

$$= \tilde{K} \left( -4 \sin^2 \frac{\vec{k}\cdot\vec{a}_1}{2} - 4 \sin^2 \frac{\vec{k}\cdot\vec{a}_2}{2} - 4 \sin^2 \frac{\vec{k}\cdot(\vec{a}_2 - \vec{a}_1)}{2} \right)$$

$$\omega^2 = \frac{4\tilde{K}}{M} \left( \sin^2 \frac{\vec{k}\cdot\vec{a}_1}{2} + \sin^2 \frac{\vec{k}\cdot\vec{a}_2}{2} + \sin^2 \frac{\vec{k}\cdot(\vec{a}_2 - \vec{a}_1)}{2} \right)$$

$$\omega^2 = \frac{4\tilde{K}}{M} \left( \sin^2 \frac{2xa}{2} + \sin^2 \frac{(2x + \sqrt{3}2y)a}{4} + \sin^2 \frac{(-2x + \sqrt{3}2y)a}{4} \right) \rightarrow \omega(\vec{k}=0) = 0 \text{ akustična}$$

$$\omega^2 = \frac{4\tilde{K}}{M} \left( \frac{2x^2 a^2}{4} + \frac{(2x^2 + 2\sqrt{3}xy + 3y^2)a^2}{16} + \frac{(2x^2 - 2\sqrt{3}xy + 3y^2)a^2}{16} \right) = \omega(|\vec{k}| \ll \frac{\pi}{a}) = c(\varphi)k$$

$$= \frac{\tilde{K}a^2}{M} \left( \frac{3}{2}2x^2 + \frac{3}{2}2y^2 \right) = \frac{3\tilde{K}a^2}{2M} 2^2 \rightarrow \omega = \sqrt{\frac{3\tilde{K}}{2M}} a k = c + c(\varphi)$$

$\vec{a}_1 = a(1, 0)$   
 $\vec{a}_2 = a(\frac{1}{2}, \frac{\sqrt{3}}{2})$   
 $\vec{a}_2 - \vec{a}_1 = a(-\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\vec{k} = (k \cos \varphi, k \sin \varphi)$

prisperek teh nihanj k specifični topleti kristala:

$$C = \frac{dE}{dT}$$

$$E = \sum_{\vec{R} \in 1BC} \hbar \omega_{\vec{R}} \left( m_{\vec{R}} + \frac{1}{2} \right) = \sum_{\vec{R} \in 1BC} \hbar \omega_{\vec{R}} \left( \frac{1}{e^{\beta \hbar \omega_{\vec{R}}} - 1} + \frac{1}{2} \right) =$$

$$= \left( \frac{L}{2\pi} \right)^2 \int_{1BC} d\vec{k} \hbar \omega_{\vec{k}} \left( \frac{1}{e^{\beta \hbar \omega_{\vec{k}}} - 1} + \frac{1}{2} \right)$$

1.) nizke temperature (velik  $\beta$ )

$\beta \hbar \omega_{\vec{k}} \gg 1$  razen za  $\vec{k}$  v bližini središča 1BC

$\rightarrow m_{\vec{R}} \rightarrow 0$

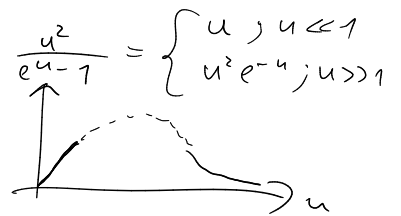
$$E = \left( \frac{L}{2\pi} \right)^2 \int d\vec{k} \hbar c k \frac{1}{e^{\beta \hbar c k} - 1} + \text{konst.}$$

$$E = \left( \frac{L}{2\pi} \right)^2 \int_0^\infty 2\pi k dk \frac{\hbar c k}{e^{\beta \hbar c k} - 1} + \text{konst.}$$

$$u = \beta \hbar c k$$

$$E = \left(\frac{L}{2\pi}\right)^2 \int_0^\infty 2\pi \frac{u}{\beta \hbar c} \frac{du}{\beta \hbar c} \frac{\frac{u}{\beta}}{e^u - 1} + \text{konst} =$$

$$= \frac{L^2}{2\pi \cdot \beta^3 \hbar^2 c^2} \int_0^\infty \frac{u^2 du}{e^u - 1} + \text{konst}$$



specifična toplota na atom

$$\bar{E} = \frac{L^2 (k_B T)^3}{\hbar^2 c^2} \cdot \frac{1}{2\pi} \int_0^\infty \frac{u^2 du}{e^u - 1} + \text{konst}$$

↑  
→ površina kristala

$$C = \frac{1}{N} \frac{d\bar{E}}{dT} = \frac{3}{N \hbar^2 c^2} k_B^3 T^2 \cdot \frac{3}{2\pi} \int_0^\infty \frac{u^2 du}{e^u - 1}$$

↑  
šti atomov

$$c^2 = \frac{3}{2} \frac{\tilde{K}}{M} a^2$$

površina osnovne celice:  $S_{oc} = \frac{a^2 \sqrt{3}}{2}$

$$S = N S_{oc}$$

$$\frac{S}{N} = S_{oc} = \frac{a^2 \sqrt{3}}{2}$$

$$C = \frac{\alpha^2 \sqrt{3}}{2} \frac{k_B^3 T^2}{\hbar^2 \cdot \frac{3}{2} \frac{\tilde{K}}{M} a^2} \cdot \frac{3}{2\pi} \int_0^\infty \frac{u^2 du}{e^u - 1}$$

$$C = k_B \cdot \frac{(k_B T)^2}{(\hbar \sqrt{\frac{\tilde{K}}{M}})^2} \cdot \frac{\sqrt{3}}{2\pi} \int_0^\infty \frac{u^2 du}{e^u - 1}$$

$$\omega_0 = \sqrt{\frac{\tilde{K}}{M}}$$

$$\omega(\text{matruba } 1\text{BC}) \approx \omega_0$$

$$C = k_B \cdot \left(\frac{k_B T}{\hbar \omega_0}\right)^2 \cdot \frac{\sqrt{3}}{2\pi} \int_0^\infty \frac{u^2 du}{e^u - 1}$$

$$C \propto T^2 \rightarrow 2D \text{ kristal}$$

2.) visoka temperatura (majhen  $\beta$ )  $\rightarrow \beta \hbar \omega_0 \ll 1$

$$E = \sum_{\vec{e} \in 1BC} \hbar \omega_{\vec{e}} \left( \frac{1}{e^{\beta \hbar \omega_{\vec{e}}} - 1} + \frac{1}{2} \right) =$$

$$\beta \hbar \omega(\vec{e}) \ll 1 \quad \text{za } \vec{e} \in 1BC$$

$$= \sum_{\vec{e} \in 1BC} \hbar \omega_{\vec{e}} \left( \frac{1}{\beta \hbar \omega_{\vec{e}} - 1} + \frac{1}{2} \right) = \sum_{\vec{e} \in 1BC} \frac{1}{\beta} + \text{konst} =$$

$$= k_B T \left( \sum_{\vec{e} \in 1BC} 1 \right) + \text{konst} = k_B T N + \text{konst}$$

$$C = \frac{1}{N} \frac{dE}{dT} = k_B$$

