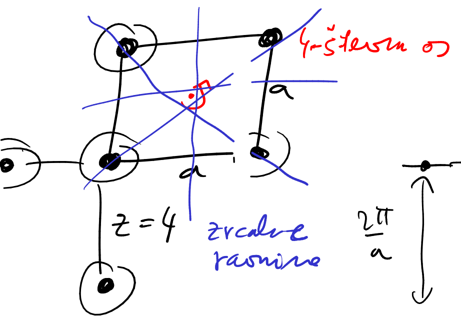
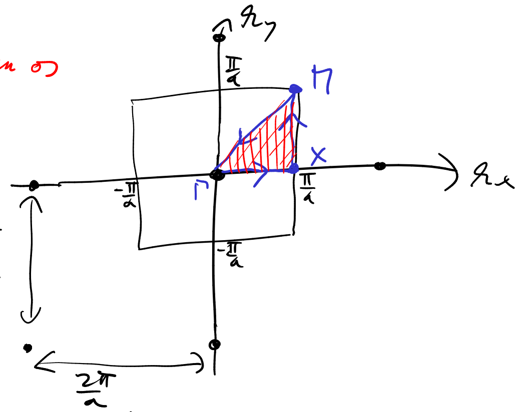
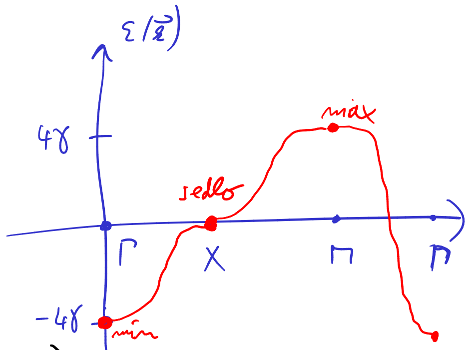
V približku tesne vezi izračunaj gostoto stanj za pas, ki nastane iz s-orbital atomov na kvadratni mreži



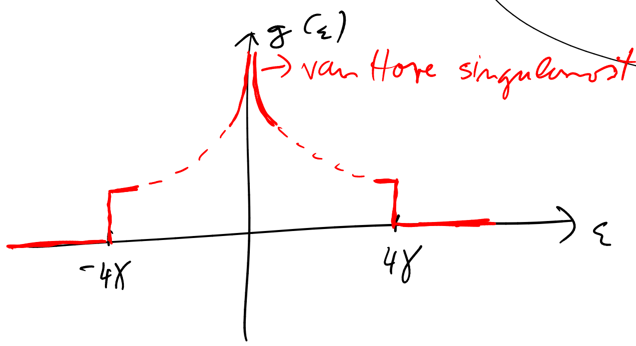




$$\varepsilon(\vec{k}) = \cancel{E-\beta} - \gamma \sum_{\vec{R}=n.s.\neq\vec{0}} e^{i\vec{k}\cdot\vec{R}} = -2\gamma(\cos k_x a + \cos k_y a)$$

$\vec{R} = (a,0)$
$(-a,0)$
$(0,a)$
$(0,-a)$

$\Gamma = (0,0)$ $\quad \varepsilon(\Gamma) = -4\gamma$ min
$X = (\frac{\pi}{a},0)$ $\quad \varepsilon(X) = 0$
$M = (\frac{\pi}{a},\frac{\pi}{a})$ $\quad \varepsilon(M) = 4\gamma$ max



simetrije:

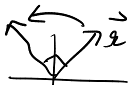
$$\varepsilon(\vec{k}) = -\gamma \sum_{\vec{R}=n.s.\neq\phi} e^{i\vec{k}\cdot\vec{R}}$$

S simetrijske operacije iz simetrijske grupe kvadratne Bravaisove mreže

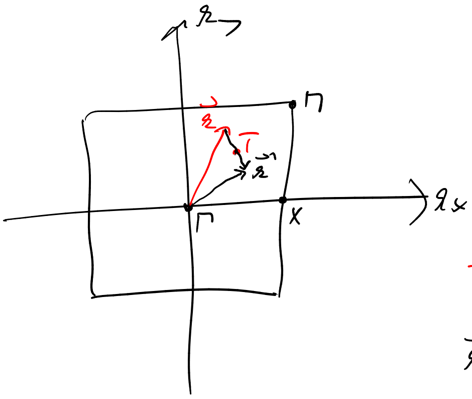
$$\{\vec{R} = n.s.\neq\phi\} = \{S\vec{R} = n.s.\neq\phi\}$$

$$\varepsilon(\vec{k}) = -\gamma \sum_{R=n.s.\neq\phi} e^{i\vec{k}\cdot S\vec{R}} = -\gamma \sum_{R=n.s.\neq\phi} e^{i SS^{-1}\vec{k}\cdot S\vec{R}} = -\gamma \sum_{R=n.s.\neq\phi} e^{i(S^{-1}\vec{k})\cdot\vec{R}} = \varepsilon(S^{-1}\vec{k})$$

rotacija za 90°: $\vec{k} = (k_x, k_y) \to (-k_y, k_x)$



$$\varepsilon(-k_y, k_x) = -2\gamma(\cos(-k_y a) + \cos k_x a) = -2\gamma(\cos k_x a + \cos k_y a) = \varepsilon(k_x, k_y)$$



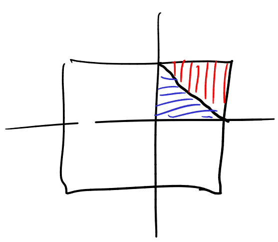
Γ: min
M: max
X: sedlo

$$T = \left(\frac{\pi}{2a}, \frac{\pi}{2a}\right)$$

$$\vec{k}' = T - (\vec{k} - T) = 2T - \vec{k}$$

$$\varepsilon(\vec{k}') = \varepsilon\left(\frac{\pi}{a} - k_x, \frac{\pi}{a} - k_y\right) = -2\gamma(-\cos k_x a - \cos k_y a) = -\varepsilon(\vec{k})$$

$\hookrightarrow g(\varepsilon) = g(-\varepsilon)$ gostota stanj je soda funkcija ε

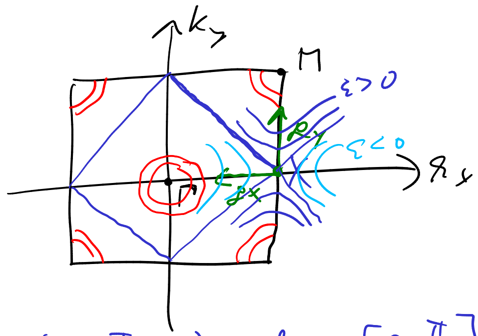


$\varepsilon(\vec{q}) = -2\gamma(\cos q_x a + \cos q_y a)$

$\Gamma = (0,0)$

$\vec{q} = \Gamma + \vec{p} \quad |\vec{p}| \ll \frac{\pi}{a} \qquad \varepsilon(\vec{p}) = -4\gamma + \gamma q^2 a^2 = -4\gamma + \frac{\hbar^2 q^2}{2m^*} \qquad m^* = \frac{\hbar^2}{2\gamma a^2}$

$g(\varepsilon) = \text{konst}\cdot\Theta(\varepsilon + 4\gamma)$



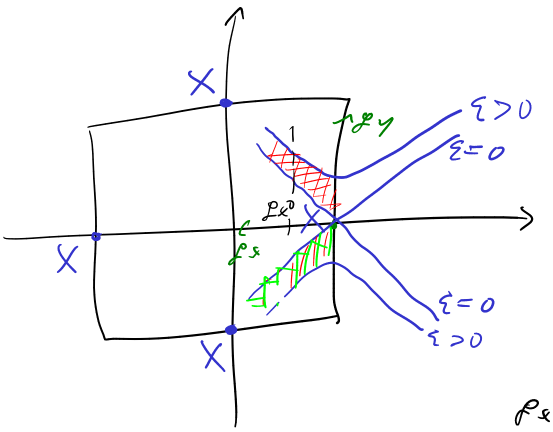
$X:\quad \vec{q} = (q_x, \frac{\pi}{a} - q_x) \quad q_x \in [0, \frac{\pi}{a}]$

$\varepsilon(\vec{q}) = -2\gamma\left(\cos \frac{q_x}{a} + \cos\left(\frac{\pi}{a} - q_x\right)a\right) = 0$

$X = (\frac{\pi}{a}, 0) \quad (q_x, q_y) \to (\frac{\pi}{a} - p_x, p_y)$

$\vec{q} = X + \vec{p} \quad |\vec{p}| \ll \frac{\pi}{a}$

$\varepsilon(\vec{q}) = -2\gamma\left(\cos\left(\frac{\pi}{a} - p_x\right)a + \cos p_y a\right) = -2\gamma(-\cos p_x a + \cos p_y a) \doteq \gamma a^2(-p_x^2 + p_y^2)$



$g(\varepsilon) = \frac{1}{V}\frac{N(\varepsilon, \varepsilon + d\varepsilon)}{d\varepsilon} = \frac{1}{V}\frac{dN(0,\varepsilon)}{d\varepsilon} \qquad V = L^2$

$$N(0,\varepsilon) = \frac{2\cdot 4 \cdot 2 \cdot \int_0^{p_x^0} dp_x \left(\sqrt{\frac{\varepsilon}{\gamma a^2} + p_x^2} - p_x\right)}{\left(\frac{2\pi}{L}\right)^2} =$$

\# = \# $\quad$ 4 sedla $\quad$ spin

$$= \frac{4L^2}{\pi^2}\int_0^{p_x^0}\left(\sqrt{\frac{\varepsilon}{\gamma a^2} + p_x^2} - p_x\right)dp_x$$

$$g(\varepsilon) = \frac{4}{\pi^2}\frac{d}{d\varepsilon}\int_0^{p_x^0}\left(\sqrt{\frac{\varepsilon}{\gamma a^2} + p_x^2} - p_x\right)dp_x =$$

$$= \frac{4}{\pi^2}\int_0^{p_x^0}\frac{1}{2\sqrt{\frac{\varepsilon}{\gamma a^2} + p_x^2}}\cdot\frac{1}{\gamma a^2}\,dp_x =$$

$$= \frac{2}{\pi^2 \gamma a^2}\int_0^{p_x^0}\frac{dp_x}{\sqrt{\frac{\varepsilon}{\gamma a^2} + p_x^2}} = \frac{2}{\pi^2\gamma a^2}\,\mathrm{arsh}\left(p_x^0\sqrt{\frac{\gamma a^2}{\varepsilon}}\right)$$

$$\int\frac{dx}{\sqrt{a^2 + x^2}} = \mathrm{arsh}\frac{x}{a}$$

$g(\varepsilon \to 0)$

$\mathrm{arsh}(u \gg 1) \approx \ln(2u)$

$\mathrm{sh}\, x = y \quad y \gg 1$

$\|$

$\frac{e^x}{2} = y$

$x = \ln(2y)$

$$\to g(\varepsilon) \doteq \frac{2}{\pi\gamma a^2}\ln\left(2p_x^0\sqrt{\frac{\gamma a^2}{\varepsilon}}\right) =$$

$$= \frac{2}{\pi\gamma a^2}\left(\cancel{\ln 2p_x^0\sqrt{\gamma a^2}} - \frac{1}{2}\ln\varepsilon\right)$$

konst $\quad$ divergira

zanemarimo

$$g(\varepsilon) = -\frac{1}{\pi\gamma a^2}\ln\varepsilon \propto -\ln\varepsilon$$