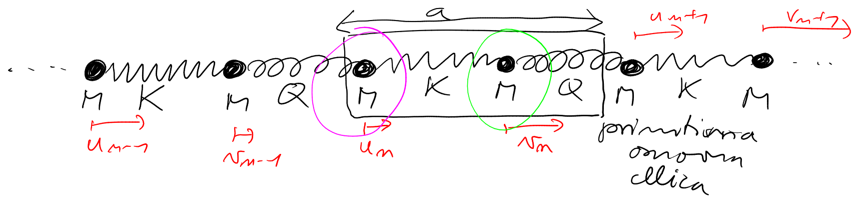
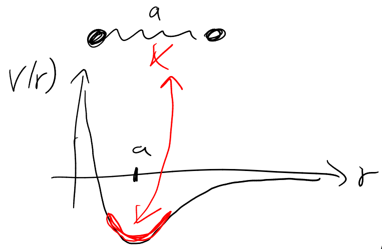


$$R_n = na$$

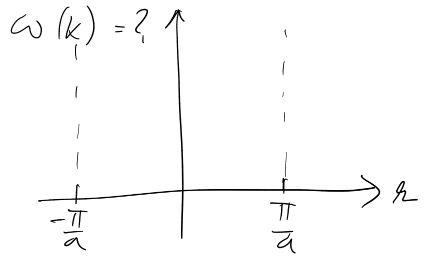


$$M\ddot{u}_n = K(v_n - u_n) - Q(u_n - v_{n-1})$$

$$M\ddot{v}_n = Q(u_{n+1} - v_n) - K(v_n - u_n)$$

$$u_n = u_0 e^{i(kna-\omega t)} \qquad u_n = \mathrm{Re}\{\sim\}$$

$$v_n = v_0 e^{i(kna-\omega t)} \qquad v_n = \mathrm{Re}\{\ \}$$



$$-M\omega^2 u_0 e^{\cancel{i(kna-\omega t)}} = K\left(v_0 e^{\cancel{i(kna-\omega t)}} - u_0 e^{\cancel{i(kna-\omega t)}}\right) - Q\left(u_0 e^{\cancel{i(kna-\omega t)}} - v_0 e^{i(k(\cancel{n}-1)a-\cancel{\omega t})}\right)$$

$$-M\omega^2 u_0 = K(v_0 - u_0) - Q(u_0 - v_0 e^{-ika})$$

$$-M\omega^2 v_0 = Q(u_0 e^{ika} - v_0) - K(v_0 - u_0)$$

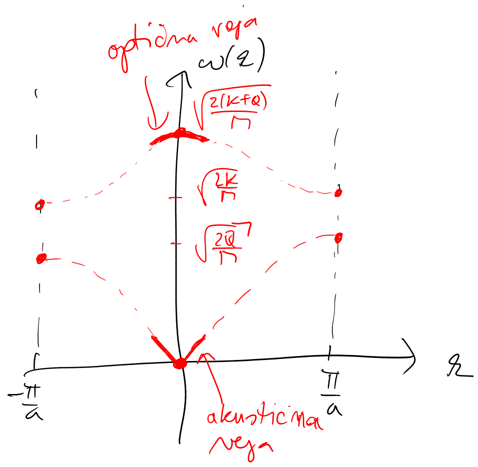
$$\begin{pmatrix} K+Q-M\omega^2, & -K-Qe^{-iqa} \\ -Qe^{iqa}-K, & K+Q-M\omega^2 \end{pmatrix}\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = 0$$

$$\det(\ ) = 0 = (K+Q-M\omega^2)^2 - |K+Qe^{iqa}|^2 = 0$$

$$(M\omega^2)^2 - 2(K+Q)M\omega^2 + \underbrace{(K+Q)^2 - (K^2+Q^2+2KQ\cos qa)}_{2KQ(1-\cos qa) = 4KQ\sin^2\frac{qa}{2}} = 0$$

$$M\omega^2 = K+Q \pm \sqrt{(K+Q)^2 - 4KQ\sin^2\frac{qa}{2}}$$

$$\omega^2 = \frac{K+Q}{M} \pm \sqrt{\left(\frac{K+Q}{M}\right)^2 - \frac{4KQ}{M^2}\sin^2\frac{qa}{2}} \longrightarrow \omega = + \sqrt{\cdots}$$



$$q = 0: \quad \omega^2 = \frac{K+Q}{M} \pm \frac{K+Q}{M} = \begin{cases} 0 & (-) \\ \frac{2(K+Q)}{M} & (+) \end{cases}$$

$$q = \pm\frac{\pi}{a}: \quad \omega^2 = \frac{K+Q}{M} \pm \sqrt{\left(\frac{K-Q}{M}\right)^2} =$$

$$= \frac{K+Q}{M} \pm \left|\frac{K-Q}{M}\right| = \begin{cases} \frac{2Q}{M} & (-) \\ \frac{2K}{M} & (+) \end{cases}$$

Omejimo se na $K > Q$

$$|q| \ll \frac{\pi}{a} \qquad \omega^2 = \frac{K+Q}{M} \pm \frac{K+Q}{M}\sqrt{1 - \frac{\cancel{4}KQ}{(K+Q)^2}\left(\frac{qa}{\cancel{2}}\right)^2} = \frac{K+Q}{M} \pm \frac{K+Q}{M}\left(1 - \frac{KQ}{2(K+Q)^2}q^2a^2\right) =$$

$$= \begin{cases} (-): \ \dfrac{KQ\,q^2a^2}{2M(K+Q)} \longrightarrow \omega = c|q| \qquad c = \sqrt{\dfrac{KQa^2}{2M(K+Q)}} \\ (+): \ \dfrac{2(K+Q)}{M} - \dfrac{KQ\,q^2a^2}{2M(K+Q)} \longrightarrow \omega = \sqrt{\dfrac{2(K+Q)}{M}} - \mathcal{O}(q^2) \end{cases}$$

$$\frac{2(K+Q)}{M}\left(1 - \frac{KQ\,q^2a^2}{4(K+Q)^2}\right) \to \omega = \sqrt{} \doteq \sqrt{\frac{2(K+Q)}{M}}\left(1 - \frac{KQ\,q^2a^2}{8(K+Q)^2}\right)$$

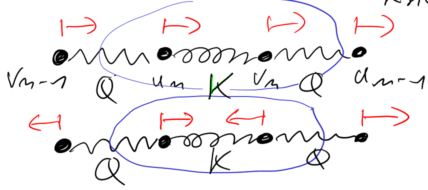
$$\begin{pmatrix} K+Q-M\omega^2, & -K-Qe^{-ika} \\ -K-Qe^{ika}, & K+Q-M\omega^2 \end{pmatrix}\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = 0$$

$$k=0 \qquad \begin{pmatrix} K+Q-M\omega^2, & -K-Q \end{pmatrix}\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = 0$$

akustična: $\omega = 0$ $\qquad \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

optična veja: $M\omega^2 = 2(K+Q)$ $\qquad \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} \propto \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

translacija celotnega kristala



$k=0 \rightarrow u_n = u_0$
$v_n = v_0$

$$k = \pm\frac{\pi}{a} \qquad \begin{pmatrix} K+Q-M\omega^2, & -K+Q \end{pmatrix}\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = 0$$

akustična: $\omega^2 = \frac{2Q}{M}$ $\qquad \begin{pmatrix} K-Q, & -K+Q \end{pmatrix}\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = 0 \qquad \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

optična: $\omega^2 = \frac{2K}{M}$ $\qquad \begin{pmatrix} -K+Q, & -K+Q \end{pmatrix}\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = 0 \qquad \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} \propto \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$u_n = u_0 e^{i(kna-\omega t)} = u_0 (-1)^n e^{-i\omega t}$$
$$v_n = v_0 e^{i(kna-\omega t)} = v_0 (-1)^n e^{-i\omega t}$$