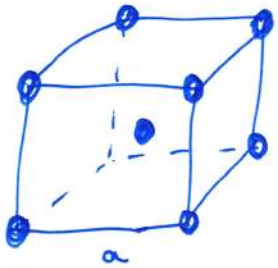
(1)



$\vec{a}_1 = a(1,0,0) \rightarrow \vec{A}_1 = \frac{2\pi}{a}(1,0,0)$

$\vec{a}_2 = a(0,1,0) \qquad \vec{A}_2 = \frac{2\pi}{a}(0,1,0)$

$\vec{a}_3 = a(0,0,1) \qquad \vec{A}_3 = \frac{2\pi}{a}(0,0,1)$

$\vec{r}_1 = 0$

$\vec{r}_2 = a\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$

$S_{\vec{K}} = 1 + e^{-i\vec{K}\cdot\vec{r}_2} \qquad \vec{K} = \frac{2\pi}{a}(m_1, m_2, m_3)$

$S_{\vec{K}} = 1 + e^{-i\pi(m_1+m_2+m_3)} = \begin{cases} 2; & m_1+m_2+m_3 \\ 0; & m_1+m_2+m_3 \end{cases}$

a) $2d\sin\frac{\vartheta}{2} = \lambda$

$d = \frac{2\pi}{|\vec{K}|} = \frac{a}{\sqrt{m_1^2+m_2^2+m_3^2}}$ $\Bigg\}$ $\sin\frac{\vartheta}{2} = \frac{\lambda}{2a}\sqrt{m_1^2+m_2^2+m_3^2}$

$a = 4\,\text{Å},\ \lambda = 3\,\text{Å} \Rightarrow m_1^2+m_2^2+m_3^2 \leq 7$

$\{100\}$ $m_1+m_2+m_3$ lih

$\{110\} \Rightarrow \vartheta_1 = 64.1°$

$\{111\}$ $m_1+m_2+m_3$ lih

$\{200\} \Rightarrow \vartheta_2 = 97.1°$

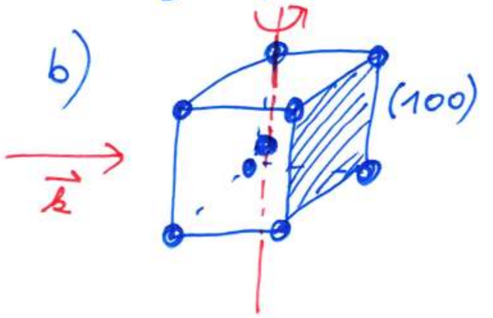
$\{210\}$ $m_1+m_2+m_3$ lih

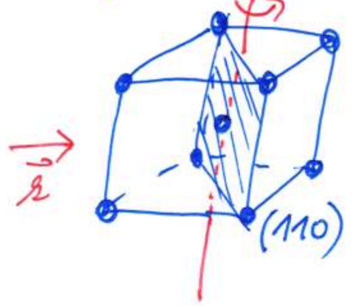
$\{211\} \Rightarrow \vartheta_3 = 133.4°$

$\{220\}$ $m_1^2+m_2^2+m_3^2 = 8 > 7.1$

3/4⁻

b)

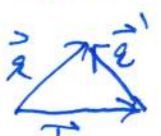




Pri rotaciji monokristala kot med $\vec{k}$ in pravokotnico na ravnini (100) in (110) zavzame vse vrednosti med 0° in 90°, zato bomo Braggova odboja na teh ravninah pri kotih 64.1° in 97.1° opazili.

Poglejmo še odboje na ravninah $\{211\}$:

$\vec{k} - \vec{k}' = \vec{K} \rightarrow \vec{k}\cdot\vec{K} = \frac{K^2}{2}$



$(k_x, k_y, 0)\cdot\frac{2\pi}{a}(m_1, m_2, m_3) = \left(\frac{2\pi}{a}\right)^2(m_1^2+m_2^2+m_3^2)$

$m_1 k_x + m_2 k_y = \frac{1}{2}\frac{2\pi}{a}(m_1^2+m_2^2+m_3^2) \qquad k_x = \frac{2\pi}{\lambda}\cos\alpha$

$k = |\vec{k}| = |\vec{k}'| = \frac{2\pi}{\lambda} \qquad m_1\cos\alpha + m_2\sin\alpha = \frac{\lambda}{2a}(m_1^2+m_2^2+m_3^2) \qquad k_y = \frac{2\pi}{\lambda}\sin\alpha$

Za ravnine $\{211\}$: $m_1 \cos\alpha + m_2 \sin\alpha = \frac{\lambda}{2a} 6 = \frac{9}{4}$

Braggov odboj bomo opazili, če obstaja zasuk kristala α, ki reši to enačbo. Ker je $\frac{9}{4} > 2$, mora biti $m_1 = 2$ in $m_2 = 1$ (ali obratno):

$$2\cos\alpha + 1\sin\alpha = \mathrm{Re}\{2e^{i\alpha} - ie^{i\alpha}\} =$$
$$= \mathrm{Re}\{\sqrt{5}\, e^{i(\alpha+\delta)}\} \leq \sqrt{5} < \frac{9}{4}$$

1/4+

Braggovega odboja pri kotu 133.4° torej ne opazimo.

② a) $\varepsilon_2(\vec{q}) = \frac{\hbar^2 q^2}{2m_2} \rightarrow g_2(\varepsilon) = \frac{\sqrt{2m_2^3}}{\pi^2\hbar^3}\sqrt{\varepsilon}\,\theta(\varepsilon)$ iz vaj

$\varepsilon_1(\vec{q}) = -\frac{\hbar^2 q^2}{2m_1} \rightarrow g_1(\varepsilon) = \frac{\sqrt{2m_1^3}}{\pi^2\hbar^3}\sqrt{-\varepsilon}\,\theta(-\varepsilon)$

$$g(\varepsilon) = g_1(\varepsilon) + g_2(\varepsilon)$$

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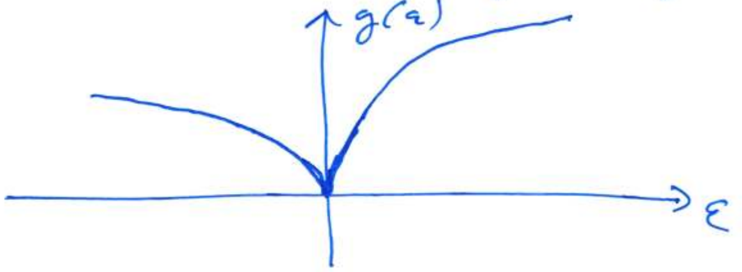
b) $m_2 > m_1 \rightarrow$ za $\varepsilon > 0$: $g(\varepsilon) = g(-\varepsilon)\left(\frac{m_2}{m_1}\right)^{3/2} > g(-\varepsilon)$



$\mu(T=0K) = 0$ Če bi pri $T > 0K$ μ ostal enak 0, bi se št. elektronov v sistemu povečalo, saj bi bilo št. zasedenih stanj v prevodnem pasu večje od št. nezasedenih stanj v valenčnem pasu. Zato je $\mu(T>0K) < 0$.

1/4

c) $\frac{E}{V} = \int_{-\infty}^{\infty} d\varepsilon\, \varepsilon\, g(\varepsilon) f(\varepsilon)$ za $m_1 = m_2$ je $g(-\varepsilon) = g(\varepsilon)$ in $\mu(T) = 0$.

$$\frac{E}{V} = \int_{-\infty}^{0} d\varepsilon\, \varepsilon\, g(\varepsilon) + \int_{-\infty}^{0} d\varepsilon\, \varepsilon\, g(\varepsilon)[f(\varepsilon) - 1] + \int_{0}^{\infty} d\varepsilon\, \varepsilon\, g(\varepsilon) f(\varepsilon) =$$

$$= \frac{E(T=0K)}{V} + \int_{-\infty}^{0} d\varepsilon\, \varepsilon\, g(\varepsilon)[-f(-\varepsilon)] + \int_{0}^{\infty} d\varepsilon\, \varepsilon\, g(\varepsilon) f(\varepsilon)$$

$$\frac{E}{V} - \frac{E(T=0K)}{V} = 2\int_0^{\infty} d\varepsilon\, \varepsilon\, g(\varepsilon) f(\varepsilon) \propto \int_0^{\infty} d\varepsilon\, \varepsilon \sqrt{\varepsilon}\, f(\varepsilon) = \int_0^{\infty} d\varepsilon\, \varepsilon\sqrt{\varepsilon}\, \frac{1}{e^{\beta\varepsilon}+1} \propto$$

$$\propto \beta^{-5/2} \propto T^{5/2} \longrightarrow C \propto T^{3/2}$$

1/4+