$$T = \frac{m}{2} \sum_{\vec{R}} \dot{u}_{\vec{R}}^2$$

$u_{\vec{R}}$ - odmik atoma $\vec{R}$ iz ravnovesne lege

vektorji Bravaisove mreže

$$V = \sum_{\vec{R}} \frac{K'}{2} u_{\vec{R}}^2 + \frac{1}{2} \sum_{\vec{R}, \vec{R}'} \frac{K}{2} \left(\sqrt{a^2 + (u_{\vec{R}} - u_{\vec{R}'})^2} - a_0 \right)^2$$

najbližji sosedi

ker sosede šteješ 2x

$u_{\vec{R}} - u_{\vec{R}'} \ll a$

$$\hookrightarrow V = \sum_{R} \frac{K'}{2} u_{\vec{R}}^2 + \frac{1}{2} \sum_{\vec{R}, \vec{R}'} \frac{K}{2} \left\{ a \left(1 + \left(\frac{u_{\vec{R}} - u_{\vec{R}'}}{a} \right)^2 \right) - a_0 \right\}^2$$

$$= \sum_{R} \frac{K'}{2} u_{\vec{R}}^2 + \frac{1}{2} \sum_{\vec{R}, \vec{R}'} \frac{K}{2} \left\{ \underbrace{(a - a_0)^2}_{\text{konstanta, samo premakne izhodišče, lahko pozabiš}} + \frac{a - a_0}{a} (u_{\vec{R}} - u_{\vec{R}'})^2 \right\}$$

$$\hookrightarrow V = \sum_{\vec{R}} \frac{K'}{2} u_{\vec{R}}^2 + \frac{1}{2} \sum_{\vec{R}, \vec{R}'} \frac{\tilde{K}}{2} (u_{\vec{R}} - u_{\vec{R}'})^2, \qquad \tilde{K} = \frac{a - a_0}{a} K$$

$$L = T - V \qquad \frac{d}{dt} \frac{\partial L}{\partial \dot{u}_{\vec{R}}} = \frac{\partial L}{\partial u_{\vec{R}}}$$

$$\hookrightarrow m \ddot{u}_{\vec{R}} = K' + \frac{1}{\cancel{2}} \cancel{2} \sum_{\vec{R}'} \frac{\tilde{K}}{\cancel{2}} \cancel{2} (u_{\vec{R}} - u_{\vec{R}'})$$

iz odvoda

ker greš 2x čez $\vec{R}$

nastavek: $u_{\vec{R}} = u_0 e^{i\vec{k} \cdot \vec{R} - i\omega t}$

$$\hookrightarrow m\omega^2 = K' + \tilde{K} \sum_{\vec{R}'} \left(1 - e^{i\vec{k} \cdot (\vec{R}' - \vec{R})} \right)$$

$$m\omega^2 = K' + \tilde{K}\left\{(1-e^{ik_xa}) + (1-e^{-ik_xa}) + (1-e^{ik_ya}) + (1-e^{-ik_ya})\right\}$$

$$= K' + 2\tilde{K}\left\{1-\cos(k_xa) + 1-\cos(k_ya)\right\}$$

$$= K' + 4\tilde{K}\left\{\sin^2\left(\frac{k_xa}{2}\right) + \sin^2\left(\frac{k_ya}{2}\right)\right\}$$

nizke $T \Longrightarrow m\omega^2 = K' + 4\tilde{K}\left\{\left(\frac{k_xa}{2}\right)^2 + \left(\frac{k_ya}{2}\right)^2\right\}$

$$= K' + \tilde{K}k^2a^2$$

$\quad k^2 = k_x^2 + k_y^2$

$$\omega = \sqrt{\frac{K'}{m} + \frac{\tilde{K}}{m}a^2k^2} \sim \sqrt{\frac{K'}{m}}\left(1 + \frac{\tilde{K}a^2}{2K'}k^2\right)$$

$$E = \sum_k \hbar\omega(\vec{k})\, n(\vec{k}) \longrightarrow \left(\frac{L}{2\pi}\right)^2 \int_0^\infty \hbar\omega \frac{1}{e^{\beta\hbar\omega}-1}\, 2\pi k\, dk$$

Bose-Einstein

$$\frac{1}{e^{\beta\hbar\omega(\vec{k})}-1}$$

nizke T

↳ Bose-Einstein požre velike k

$$= \frac{L^2}{2\pi}\int_0^\infty \hbar\sqrt{\frac{K'}{m}}\left(1+\frac{\tilde{K}a^2}{2K'}k^2\right) \underbrace{\frac{1}{e^{\beta\hbar\sqrt{\frac{K'}{m}}}\, e^{\beta\hbar\frac{\tilde{K}}{2\sqrt{K'm}}a^2k^2} - 1}} k\, dk$$

nizke T

↳ $\sim e^{-\beta\hbar\sqrt{\frac{K'}{m}}}\, e^{-\underbrace{\beta\hbar\frac{\tilde{K}}{2\sqrt{K'm}}a^2k^2}_{x}}$

$$x = \beta\hbar\frac{\tilde{K}}{2\sqrt{K'm}}a^2k^2$$

$$dx = 2\beta\hbar\frac{\tilde{K}}{2\sqrt{K'm}}a^2 k\, dk$$

↳ $E = \frac{L^2}{2\pi}\int_0^\infty \left(\hbar\sqrt{\frac{K'}{m}} + \frac{x}{\beta}\right) e^{-\beta\hbar\sqrt{\frac{K'}{m}}}\, e^{-x}\, \frac{1}{2}\left(\beta\hbar\frac{\tilde{K}}{2\sqrt{K'm}}a^2\right)^{-1} x^{-\frac{1}{2}}\, dx$

$$E = \frac{L^2}{2\pi}\left(\beta\hbar\frac{\tilde{K}}{2\sqrt{K'm}}a^2\right)^{-1} e^{-\beta\hbar\sqrt{\frac{K'}{m}}}\left\{\hbar\sqrt{\frac{K'}{m}}\underbrace{\int_0^\infty \frac{e^{-x}}{\sqrt{x}}dx}_{} + \frac{1}{\beta}\underbrace{\int_0^\infty \sqrt{x}\,e^{-x}dx}_{}\right\}$$

konstanti

$$= \alpha\, T e^{-\hbar\sqrt{\frac{K'}{m}}/kT} + \gamma\, T^2 e^{-\hbar\sqrt{\frac{K'}{m}}/kT}$$

(pri nizkih temperaturah ta člen prevlada

$$c_v = \frac{1}{L^2}\frac{\partial E}{\partial T} \propto \frac{1}{T} e^{-\hbar\sqrt{\frac{K'}{m}}/kT}$$ ← drugi člen je pri nizkih temperaturah veliko manjši