Fizika

Opis gibanja – Kinematika

– Premo enakomerno gibanje

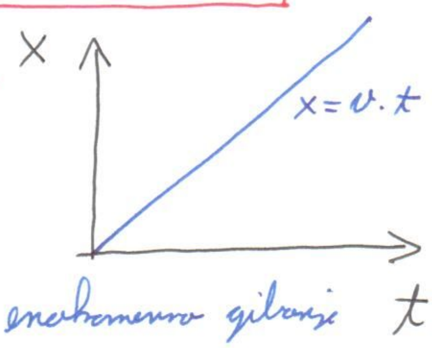
Koordinatni sistem:

Premo gibanje: $x(t)$ odmik v odvisnosti od časa

Enakomerno gibanje: $\boxed{x = v \cdot t}$

v: hitrost $[\frac{m}{s}]$

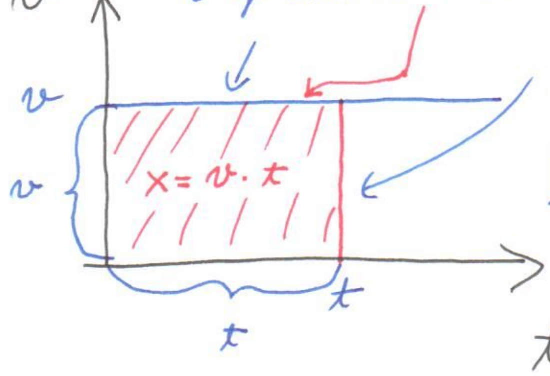
t: čas $[s]$



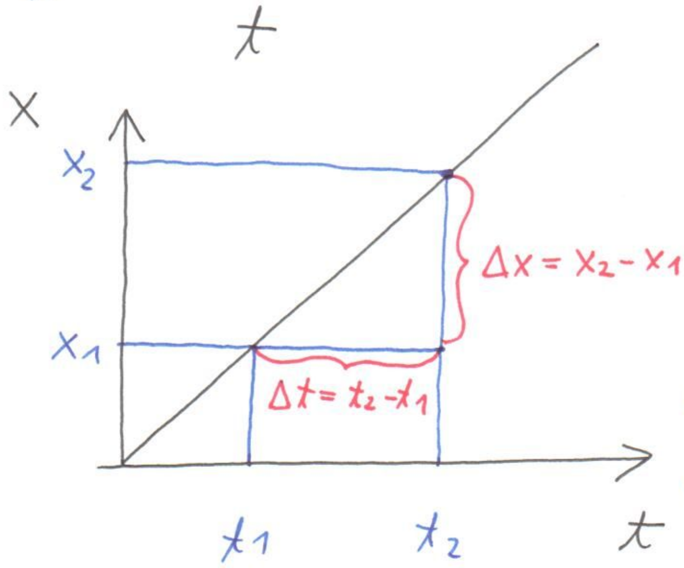
Graf odmika v odvisnosti od časa: premica

v sliki

v je konstantna $\Leftrightarrow$ enakomerno gibanje



$x = v \cdot t$ si lahko predstavljamo kot ploščino pod "krivuljo" $v(t)$



$$\boxed{v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}}$$

Hitrost: sprememba odmika (poti) deljena s časovnim intervalom

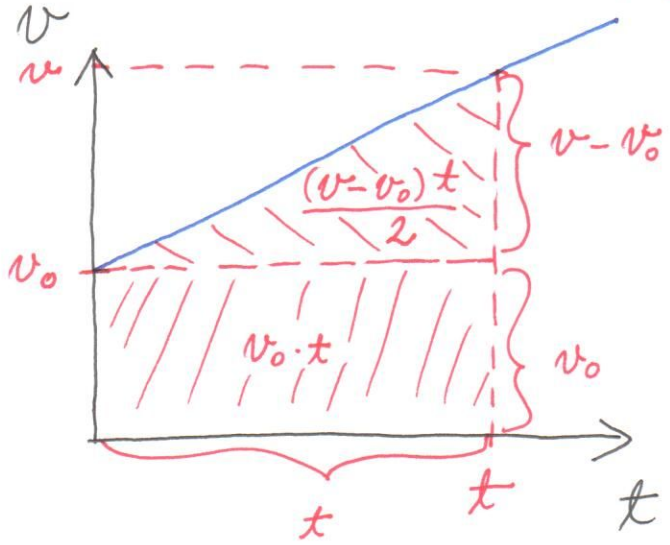
– Enakomerno pospešeno premo gibanje:

Hitrost se linearno spreminja s časom:

$$\boxed{v = v_0 + a t} \quad (1)$$

v_0: začetna hitrost (ob času $t=0$)

a: pospešek $\left[\frac{m}{s^2}\right]$



$$x = v_0 t + \frac{v - v_0}{2} \cdot t$$

$$v - v_0 = a t \Rightarrow$$

$$\boxed{x = v_0 t + \frac{a t^2}{2}} \quad (2)$$

Iz enačb (1) in (2) izpeljimo še eno koristno zvezo:

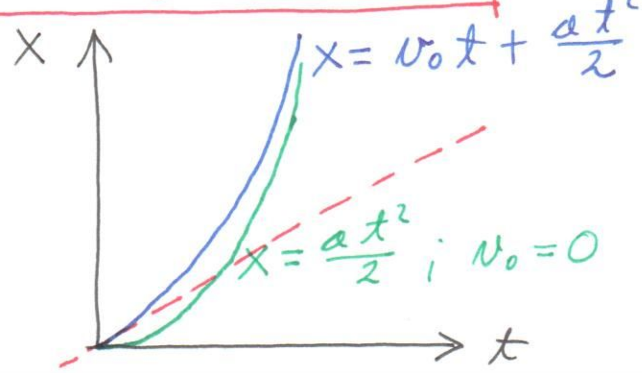
$$x = v_0 t + \frac{a t^2}{2} \; ; \quad t = \frac{v - v_0}{a}$$

$$x = \frac{v_0 (v - v_0)}{a} + \frac{(v - v_0)^2}{2a} =$$

$$= \frac{v - v_0}{2a}\left[\underbrace{2 v_0 + v - v_0}_{v + v_0}\right] = \frac{v^2 - v_0^2}{2a}$$

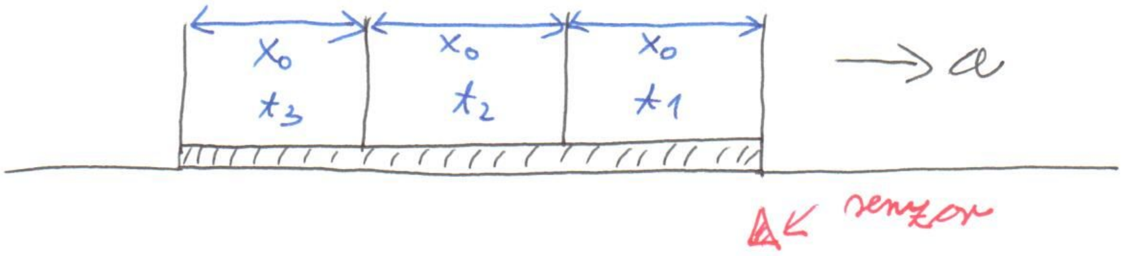
$$\boxed{v^2 = v_0^2 + 2 a x} \quad (3)$$

Graf $x(t)$:



$\boxed{x(t) \text{ je parabola}}$

Primer: meritev enak. pospešenega gibanja:



t_1, t_2, t_3 časi prehodov ~~a~~ prečk preko senzorja

Enačbe:

$$x_0 = v_0 t_1 + \frac{a t_1^2}{2}$$

$$x_0 = v_1 t_2 + \frac{a t_2^2}{2}$$

$$v_1 = v_0 + a t_1$$

$$x_0 = v_0 t_1 + \frac{a t_1^2}{2} \quad | : t_1$$

$$x_0 = (v_0 + a t_1) t_2 + \frac{a t_2^2}{2} \quad | : t_2$$

ter odštejemo

$$\frac{x_0}{t_1} - \frac{x_0}{t_2} = -a t_1 - \frac{a t_2}{2} + \frac{a t_1}{2} = -\frac{a}{2}(t_1 + t_2)$$

$$a = \frac{2 x_0 \left(\frac{1}{t_2} - \frac{1}{t_1} \right)}{t_1 + t_2}$$

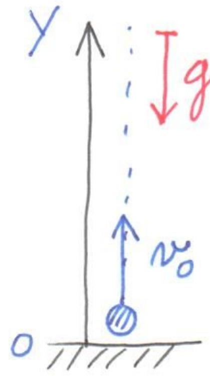
Iz simetrije pa mora veljati tudi:

$$a = \frac{2 x_0 \left(\frac{1}{t_3} - \frac{1}{t_2} \right)}{t_2 + t_3}$$

Pozor: enačbi za pospešek sta rešitvi konkretnega problema in nista del "obvezne snovi".

– Navpični met – prosti pad – primera enakomerno pospešenega gibanja

I



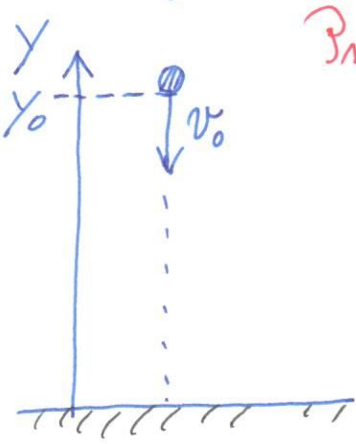
$$v = v_0 - g \cdot t$$

g: težnostni pospešek $g = 9{,}8\ \frac{m}{s^2}$

$$y = v_0 t - \frac{g t^2}{2}$$

Čas dviga: $v = 0 \Rightarrow 0 = v_0 - g\, t_D \Rightarrow \boxed{t_D = \frac{v_0}{g}}$

II



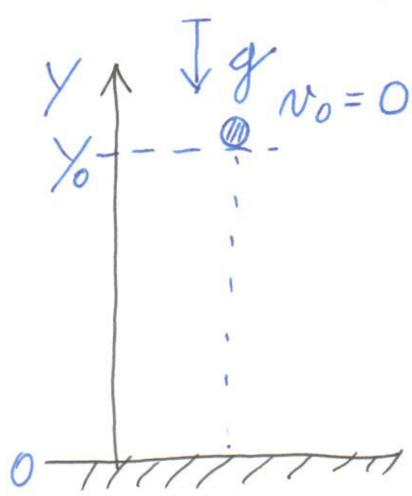
Predmet vržemo navpično navzdol: z višine y_0

$$y = y_0 - v_0 t - \frac{g t^2}{2}$$

$$v = -v_0 - g t$$

III

Predmet prosto pade z višine y_0



$$y = y_0 - \frac{g t^2}{2}$$

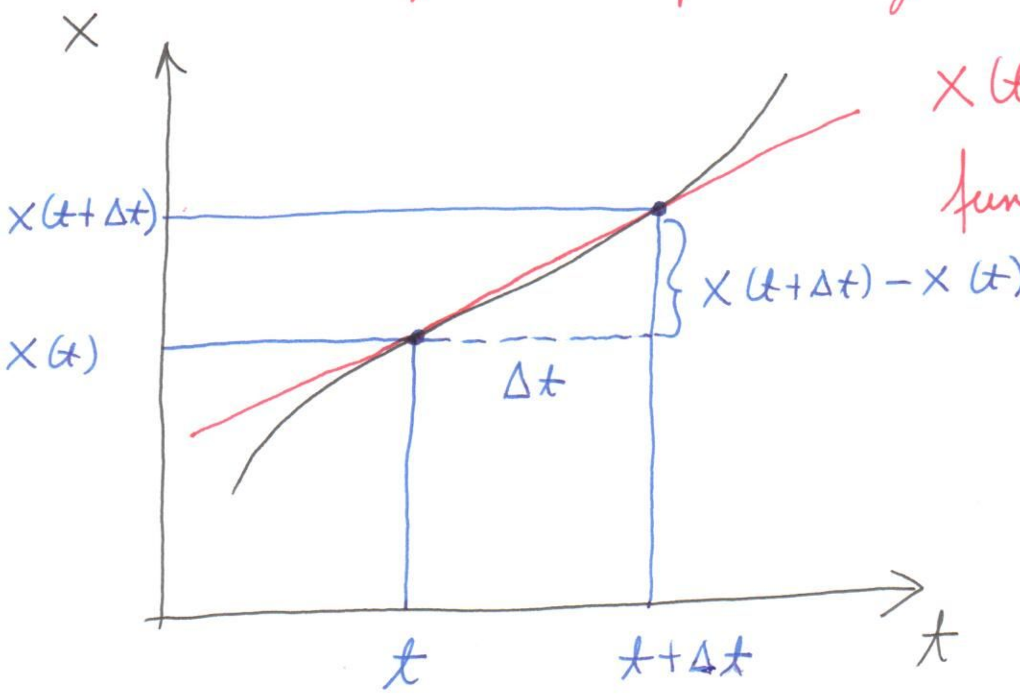
Čas padanja: $0 = y_0 - \frac{g t_0^2}{2} \Rightarrow$

$$t_0 = \sqrt{\frac{2 y_0}{g}}$$

Hitrost: $v = -g t$

Hitrost pri tleh: $v = -\sqrt{\frac{2 y_0}{g}} \cdot g = -\sqrt{2 y_0 g}$

Pospešeno premo gibanje



$x(t)$: poljubna funkcija časa

Povprečna hitrost $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x(t+\Delta t) - x(t)}{\Delta t}$

Predstavlja hitrost, ki bi jo imelo telo, če bi se gibalo med časoma t ter $t+\Delta t$ <u>enakomerno</u>.

Vendar, nas zanima trenutna hitrost v času t!

$$v(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \frac{dx}{dt} = \dot{x}(t)$$

odvod $x(t)$ po času t!

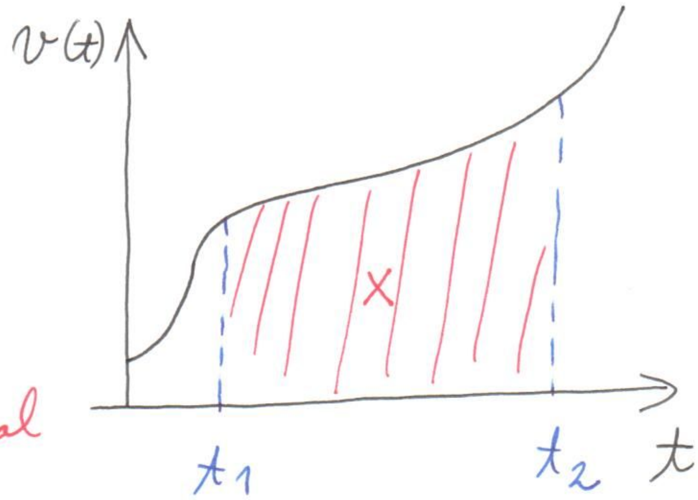
$$v(t) = \frac{dx}{dt}$$

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

pospešek

<u>drugi</u> odvod $x(t)$ po času t

Obratna pot ?



Določeni integral

$$x = \int_{t_1}^{t_2} v(t)\,dt\,;$$

Pot med časoma t_1 in t_2

$$v = \int_{t_1}^{t_2} a(t)\,dt\,;$$

Običajno: $x = \int_0^t v(t)\,dt$

in $v = \int_0^t a(t)\,dt$

Primeri:

$$x = v_0 t + \frac{at^2}{2}\,; \quad v = \frac{dx}{dt} = v_0 + at\,;$$

saj velja $\frac{d(v_0 t)}{dt} = v_0 \frac{dt}{dt} = v_0\,; \quad \frac{dt^2}{dt} = 2t$

$$x = x_0 \sin \omega t$$

$$v = \frac{dx}{dt} = x_0 \omega \cos \omega t$$

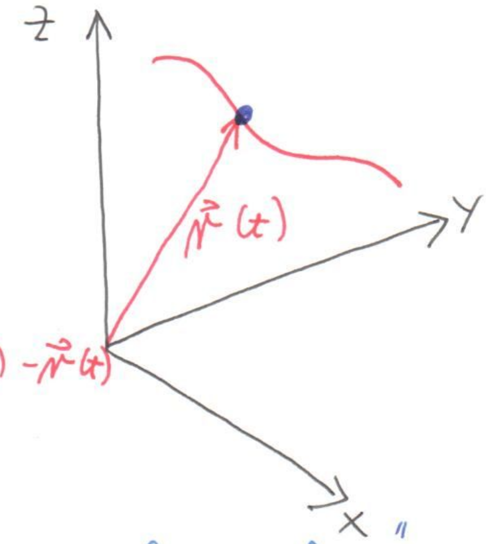
Pozor: odvod posredne funkcije

$$a = \frac{dv}{dt} = -x_0 \omega^2 \sin \omega t$$

Krivo gibanje točkastega telesa (gibanje v prostoru)

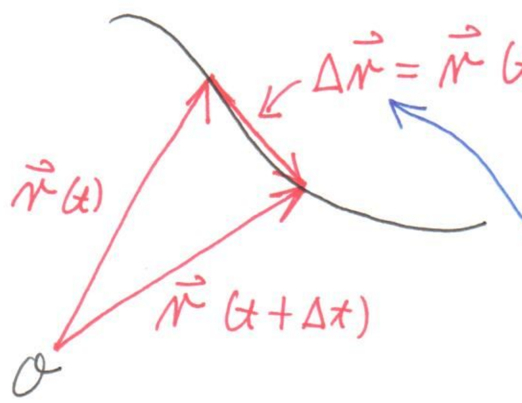
Gibanje opišemo s tirom (krivulja v prostoru) po katerem se giblje telo

$$\vec{r}(t) = (x(t), y(t), z(t))$$



Kako določimo hitrost?

$$\Delta\vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$$



$\Delta\vec{r}$ je vektor, ki "približno" kaže v smeri gibanja telesa

izhodišče koordinat. sistema

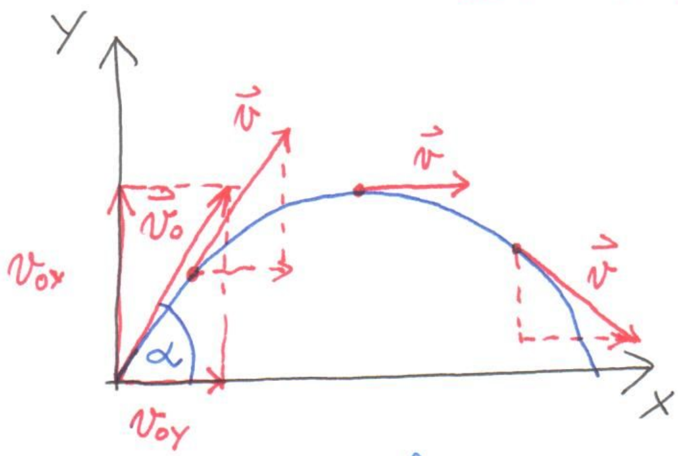
$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v}(t) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = (v_x(t), v_y(t), v_z(t))$$

podobno: $$\vec{a}(t) = \left(\frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\right) = (a_x(t), a_y(t), a_z(t))$$

Velikost hitrosti: $v(t) = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Poševni met



$v_{0x} = v_0 \cos\alpha$

$v_{0y} = v_0 \sin\alpha$

$\vec{a} = (0, -g)$

ali zapis po komponentah

$\vec{v} = (v_0 \cos\alpha,\ v_0 \sin\alpha - g\,t)$

$\vec{r} = \left(v_0 \cos\alpha \cdot t,\ v_0 \sin\alpha \cdot t - \frac{g t^2}{2}\right)$

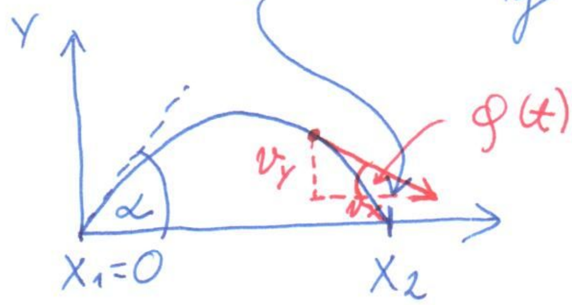
$v_x(t) = v_0 \cos\alpha$

$v_y(t) = v_0 \sin\alpha - g\,t$

$$\left.\begin{aligned} x &= v_0 \cos\alpha\ t \\ y &= v_0 \sin\alpha\ t - \frac{g t^2}{2} \end{aligned}\right\} \begin{matrix} x(t) \\ y(t) \end{matrix} \qquad \Rightarrow t = \frac{x}{v_0 \cos\alpha}$$

$$y = \operatorname{tg}\alpha \cdot x - \frac{g\, x^2}{2 v_0^2 \cos^2\alpha}$$: Parabola

Domet: $y = 0 \Rightarrow \quad x_1 = 0 \qquad x_2 = \frac{2 v_0^2 \sin\alpha \cos\alpha}{g} = \frac{v_0^2 \sin 2\alpha}{g}$



Maksimalni domet pri fiksni v_0 in g če velja $\sin 2\alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$

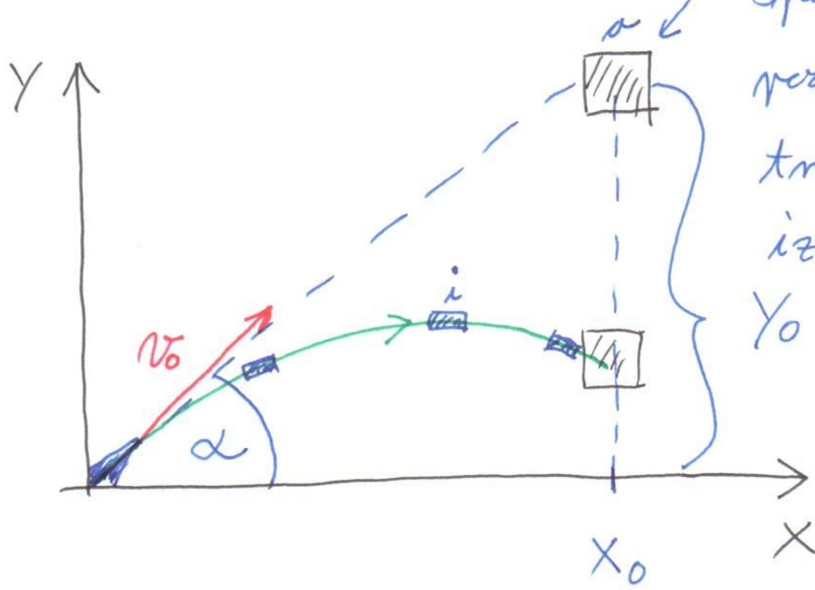
ali $\alpha = 45^0$

$$\operatorname{tg}\varphi(t) = \frac{v_y(t)}{v_x(t)} = \frac{v_0 \sin\alpha - g\,t}{v_0 \cos\alpha} =$$

$$= \operatorname{tg}\alpha - \frac{g}{v_0} \frac{t}{\cos\alpha}$$

Primer:

Spustimo, da prosto pada tar v istem trenutku izstrelimo izstrelek iz izhodišča



poševni met

$$\vec{r}_i = \left(v_0 \cos\alpha \cdot t \,,\; v_0 \sin\alpha \cdot t - g\frac{t^2}{2} \right)$$

$$\vec{r}_o = \left(X_0 \,,\; Y_0 - g\frac{t^2}{2} \right)$$

prosti pad z "višine" Y_0

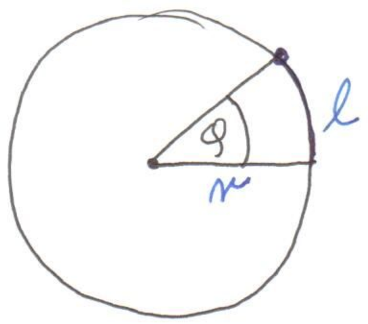
Pogoj za zadetek: $\vec{r}_i = \vec{r}_o$

$$\left.\begin{aligned} v_0 \cos\alpha \cdot t &= X_0 \\ v_0 \sin\alpha \cdot t - \cancel{g\frac{t^2}{2}} &= Y_0 - \cancel{g\frac{t^2}{2}} \end{aligned}\right\}$$ enačbi delimo

$$\boxed{\operatorname{tg}\alpha = \frac{Y_0}{X_0}}$$ (kot kaže slika)

Rezultat je neodvisen od v_0! Pogoj pa je, da do trka pride preden objekt – o pade na tla!

Enakomerno kroženje



$l = \varphi \cdot r$; l: dolžina krožnega loka

l [m] r: radij kroga

φ [rd] Poseben primer:

$o = 2\pi r$

o: obseg kroga

$\varphi = 2\pi$: polni kot

$$l = \varphi \cdot r \;/ : t$$

$$v = \frac{l}{t} = \frac{\varphi}{t} \cdot r$$

$$\boxed{v = \omega \cdot r}$$

$$\boxed{\varphi = \omega \cdot t} :$$

ω: $[\frac{rd}{s} = \frac{1}{s}]$ kotna hitrost

v: $[\frac{m}{s}]$ obodna hitrost

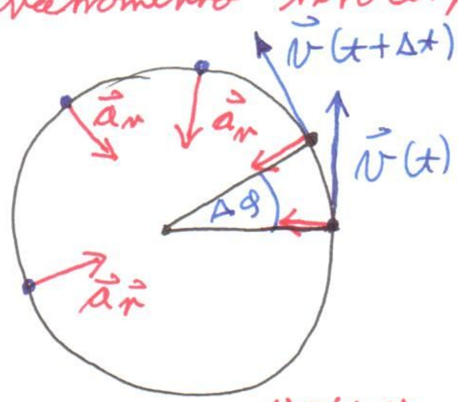
$$2\pi = \omega \cdot t_0$$

t_0: obhodni čas (čas enega obhoda)

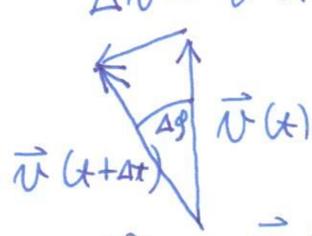
$$\omega = \frac{2\pi}{t_0} = 2\pi\nu$$

ν: frekvenca

Enakomerno kroženje je pospešeno gibanje:



$$\Delta\vec{v} = \vec{v}(t+\Delta t) - \vec{v}(t)$$



$$\vec{a}_r = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t}$$

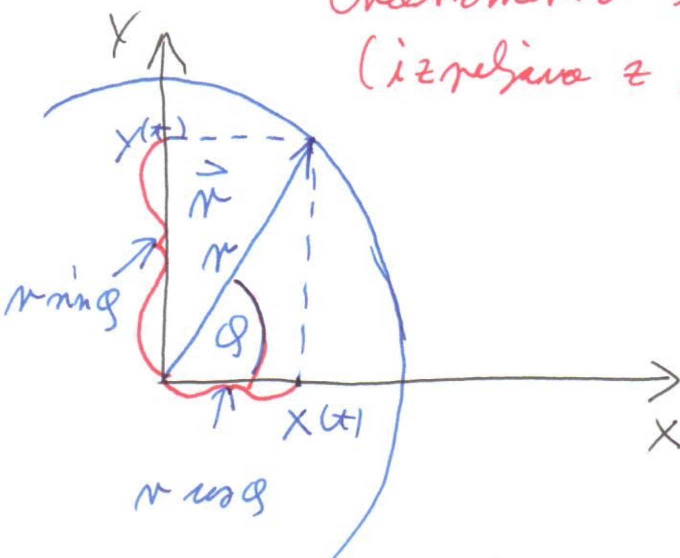
$$v = \omega \cdot r$$

$$a_r = v \cdot \omega = \omega^2 \cdot r = \frac{v^2}{r}$$

Smer: radialno v središče

$$a_r = \frac{\Delta v}{\Delta t} = \frac{v \cdot \Delta\varphi}{\Delta t} = v \cdot \omega$$

Enakomerno kroženje (izpeljava z vektorji)



$$(\ x(t)\ ,\ \ y(t)\ )$$

$$\vec r(t) = (r\cos\varphi,\ r\sin\varphi)$$

$$\varphi = \omega t$$

$$\vec r(t) = (r\cos\omega t,\ r\sin\omega t)$$
$$= r(\cos\omega t,\ \sin\omega t)$$

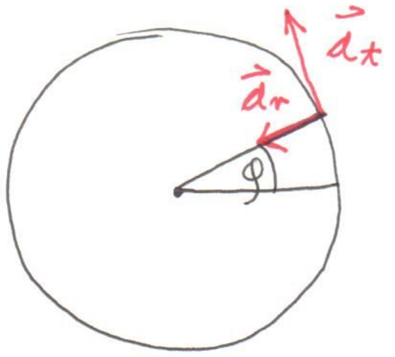
$$\vec v = \frac{d\vec r}{dt} = r\omega(-\sin\omega t,\ \cos\omega t)$$

$$\vec a = \frac{d\vec v}{dt} = -\underbrace{r\omega^2(\cos\omega t,\ \sin\omega t)}_{\vec r(t)}$$

$$\vec a = -\omega^2\cdot\vec r \quad ; \quad a_r = \omega^2\cdot r$$

$\vec a$ kaže v smeri $\vec r$ z ⊖ minusom, torej

Enakomerno pospešeno kroženje



$$\omega = \omega_0 + \alpha t$$

$$\varphi = \omega_0 t + \frac{\alpha t^2}{2}$$

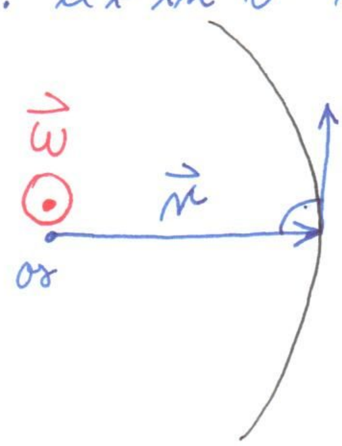
$$v = v_0 + a_t \cdot t$$

$a_r =$

$$a_t = \alpha \cdot r \qquad \alpha\text{: kotni pospešek } \left[\frac{rad}{s^2} = \frac{1}{s^2}\right]$$

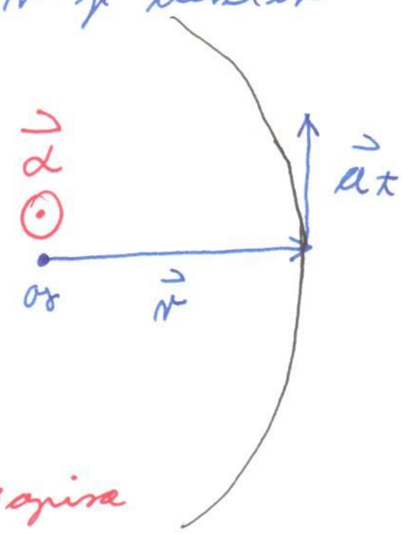
$$v = \omega \cdot r$$

Toda: $\vec a_t$ in $\vec v$ sta vektorja! Tudi $\vec r$ je vektor



$$\boxed{\begin{aligned}\vec v &= \vec \omega \times \vec r \\ \vec a_t &= \vec \alpha \times \vec r\end{aligned}}$$

$$\vec a_r = \vec \omega \times \vec v$$



Primer vektorskega zapisa pospešenega kroženja

$$\vec r(t) = r\,(\cos\varphi(t), \sin\varphi(t))$$

$$\vec v(t) = r\frac{d\varphi}{dt}(-\sin\varphi(t), \cos\varphi(t)) \quad ; \quad \frac{d\varphi}{dt} = \omega$$

$$\vec v(t) = r\,\omega(t)\,(-\sin\varphi(t), \cos\varphi(t))$$

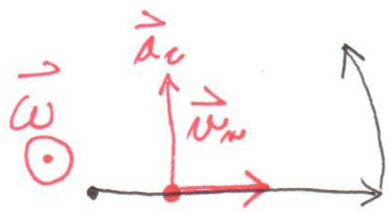
$$\vec a(t) = r\underbrace{\frac{d\omega}{dt}}_{\alpha}(-\sin\varphi(t), \cos\varphi(t)) - r\omega^2(\cos\varphi(t), \sin\varphi(t))$$

$$\vec a(t) = r\alpha\,(-\sin\varphi(t), \cos\varphi(t)) - r\omega^2(\cos\varphi(t), \sin\varphi(t))$$

$$\vec a(t) = a_t\,(-\sin\varphi(t), \cos\varphi(t)) - a_r\,(\cos\varphi(t), \sin\varphi(t))$$

Coriolisov pospešek

(Neobvezno!)



$$r = v_r \cdot t;$$

$$\vec{r} = v_r \cdot t\,(\cos\omega t, \sin\omega t)$$

Telo se giblje iz središča kroženja navzven. Kroženje je enakomerno.

$$\vec{v} = \frac{d\vec{r}}{dt} = v_r\,(\cos\omega t, \sin\omega t) + v_r t\,(-\omega\sin\omega t, \omega\cos\omega t)$$

$$\vec{a} = v_r\,(-\omega\sin\omega t, \omega\cos\omega t) + v_r\,(-\omega\sin\omega t, \omega\cos\omega t)$$

$$-\,\omega^2 v_r t\,(\cos\omega t, \sin\omega t) =$$

$$= 2\,v_r\,\omega\,(-\sin\omega t, \cos\omega t) - \omega^2 \overset{v_r t}{\overset{\|}{r}}\,(\cos\omega t, \sin\omega t)$$

$$\vec{a} = a_c\,(-\sin\omega t, \cos\omega t) \quad - a_r\,(\cos\omega t, \sin\omega t)$$

$$a_c = 2\,v_r \cdot \omega\;; \quad \boxed{\vec{a}_c = 2\,\vec{\omega}\times\vec{v}_r}$$