Prepustnost in odbojnost na paru δ potencialov

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Naloga: Poišči amplitudi prepustnosti in odbojnosti na potencialu oblike $V(x) = W_0(\delta(x+a) + \delta(x-a))$.

Na potencial $V(x) = W_0(\delta(x+a) + \delta(x-a))$ pošljemo curek delcev z leve strani. Zapišemo valovno funkcijo za tri območja:levo stran δ funkcij (1), znotraj (2) in desno stran (3):

$$\psi_1(x) = Ae^{ik(x+a)} + Be^{-ik(x+a)}$$
$$\psi_2(x) = Ce^{ikx} + De^{-ikx}$$
$$\psi_3(x) = Fe^{ik(x-a)}$$

$$Odbojnost = \left|\frac{B}{A}\right|^2 \qquad Prepustnost:\ T = \left|\frac{F}{A}\right|^2$$

Robni pogoji:

$$\psi_1(-a) = \psi_2(-a) \quad (1)$$
$$\psi_2(a) = \psi_3(a) \quad (2)$$
$$\frac{\partial}{\partial x}\psi_1(-a) - \frac{\partial}{\partial x}\psi_2(-a) = -\frac{2mW_0}{\hbar^2}\psi_1(-a) \quad (3)$$
$$\frac{\partial}{\partial x}\psi_2(a) - \frac{\partial}{\partial x}\psi_3(a) = -\frac{2mW_0}{\hbar^2}\psi_3(-a) \quad (4)$$

$$(1) \rightarrow A + B = Ce^{-ika} + De^{ika} \quad (5)$$
$$(2) \rightarrow Ce^{ika} + De^{-ika} = F \quad (6)$$
$$(3) \rightarrow ikaA - ikaB - ikaCe^{-ika} + ikaDe^{ika} = -\frac{2mW_0a}{\hbar^2}(A+B) \quad (7)$$
$$(4) \rightarrow ikaCe^{ika} - ikaDe^{-ika} - ikaF = -\frac{2mW_0a}{\hbar^2}F \quad (8)$$

$$ka \rightarrow x$$
$$\frac{2mW_0a}{\hbar^2} \rightarrow x_0$$

Enačbe prepišemo v brezdimenzijsko obliko:

$$A + B = Ce^{-ix} + De^{ix} \quad (9)$$
$$Ce^{ix} + De^{-ix} = F \quad (10)$$
$$ixA - ixB - ixCe^{-ix} + ixDe^{ix} = -x_0(A+B)$$
$$ixCe^{ix} - ixDe^{-ix} - ixF = -x_0F$$

Zadnji dve delimo z ix in preuredimo:

$$A(1-\frac{ix_0}{x})+B(-1-\frac{ix_0}{x})=Ce^{-ix}+De^{ix} \tag{11}$$

$$Ce^{ix}-De^{-ix}=F(1+\frac{ix_0}{x}) \tag{12}$$

$$(10)+(12)\rightarrow C=(1+\frac{ix_0}{2x})e^{-ix}F$$

$$(10)-(12)\rightarrow D=-\frac{ix_0}{2x}e^{ix}F$$

$$(5)\rightarrow B=-A+Ce^{-ix}+De^{ix}$$

$$(11)\rightarrow A(1-\frac{ix_0}{2x})+(-A+Ce^{-ix}+De^{ix})(-1-\frac{ix_0}{2x})=Ce^{-ix}+De^{ix}$$

...

$$\frac{A}{F}=e^{-2ix}-\frac{ix_0^2}{2x^2}\sin 2x$$

$$\left|\frac{F}{A}\right|^2=\frac{1}{1+\frac{x_0^2}{2x^2}\sin^2 2x+\frac{x_0^4}{4x^4}\sin^2 2x}$$

$$\frac{B}{A}=\frac{\frac{x_0}{x}\sin 2x-\frac{ix_0^2}{2x^2}\sin 2x}{e^{-2ix}-\frac{ix_0^2}{2x^2}\sin 2x}$$

$$\left|\frac{B}{A}\right|^2=\frac{\frac{x_0^2}{x^2}\sin^2 2x+\frac{x_0^4}{4x^4}\sin^2 2x}{1+\frac{x_0^2}{2x^2}\sin^2 2x+\frac{x_0^4}{4x^4}\sin^2 2x}=\frac{1+\frac{x_0^2}{4x^2}}{\frac{1}{2}+\frac{x_0^2}{4x^2}+\frac{x^2}{x_0^2\sin^2 2x}}$$