SCHWABL 6.37:

$$R_{n\ell}(r) = -\left[\frac{(n-\ell-1)!\,\left(\frac{2}{n r_B}\right)^3}{2n\,((n+\ell)!)^3}\right]^{1/2}\left(\frac{2r}{n r_B}\right)^{\ell} e^{-\frac{r}{n r_B}}\, L^{2\ell+1}_{n+1}\left(\frac{2r}{n r_B}\right)$$

$\boxed{\ell = n-1}$

$$R_{n,n-1}(r) = \underbrace{\left(\frac{2}{n r_B}\right)^{n+1/2}\frac{1}{2n\sqrt{(2n-1)!}}}_{A_1}\, r^{n-1} e^{-\frac{r}{n r_B}}$$

; r_B ~ BOROV RADIJ

SCHWABL 5.22:

$$Y_{\ell m}(\vartheta,\varphi) = (-1)^{(m+|m|)/2} P_{\ell |m|}(\cos\vartheta)\, e^{im\varphi}\left(\frac{2\ell+1}{4\pi}\cdot\frac{(\ell-|m|)!}{(\ell+|m|)!}\right)^{1/2}$$

$\boxed{m = \ell = n-1}$

$$Y_{\ell\ell} = \underbrace{(-1)^{\ell}(2\ell-1)!!\left(\frac{2\ell+1}{4\pi(2\ell)!}\right)^{1/2}}_{A_2} \sin^{\ell}\vartheta\, e^{i\ell\varphi}$$

NAŠ NASTAVEK:

$$\psi_{n,n-1,n-1}(r,\vartheta,\varphi) = Y_{n-1,n-1}(\vartheta,\varphi)\cdot R_{n,n-1}(r)$$

NORMALIZACIJA:

$$\int |R_{n\ell} Y_{\ell m}|^2 d^3r = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta\, Y^*_{\ell m} Y_{\ell m}\int_0^{\infty} R^2_{n\ell}\, r^2\, dr = 1$$

$\ell = n-1$

STRNAD 3, STR. 196:

$$\langle n\ell|r|n\ell\rangle = \int_0^{\infty} r^3 R^2_{n\ell}\, dr = \frac{1}{2} r_B\left[3n^2 - \ell(\ell+1)\right]$$

$$\langle n\ell|r^2|n\ell\rangle = \int_0^{\infty} r^4 R^2_{n\ell}\, dr = \frac{1}{2} r_B^2\, n^2\left[5n^2 + 1 - 3\ell(\ell+1)\right]$$

NAŠ PRIMER:

$$\langle n\ n-1|r|n\ n-1\rangle = \frac{1}{2} r_B\left[3n^2 - (n-1)n\right] = \frac{1}{2} r_B\left[2n^2 + n\right] = \underline{n r_B\left[n + \tfrac{1}{2}\right]}$$

$$\langle n\ n-1|r^2|n\ n-1\rangle = \frac{1}{2} r_B^2 n^2\left[5n^2 + 1 - 3(n-1)n\right] = \underline{\underline{\frac{1}{2} r_B^2\, n^2\left[2n^2 + 1 + 3n\right]}}$$

$$\delta r = \sqrt{\langle r^2\rangle - \langle r\rangle^2} = r_B n\sqrt{n^2 + \tfrac{1}{2} + \tfrac{3}{2}n - \cancel{n^2} - n - \tfrac{1}{4}} = r_B n\sqrt{\tfrac{1}{2}n + \tfrac{1}{4}} = \underline{\underline{\frac{1}{2} r_B n\sqrt{2n+1}}}$$

$m = \ell = n-1$

$$\langle \sin\vartheta\rangle = |A_2|^2\int_0^{2\pi} d\varphi\int_0^{\pi}\sin^2\vartheta\,\sin^{2n-2}\vartheta\, d\vartheta = |A_2|^2\, 2\pi\int_0^{\pi}\sin^{2n}\vartheta\, d\vartheta = \text{MATHEMATICA}$$

$$= |A_2|^2\, 2\pi\,\frac{\sqrt{\pi}\,\Gamma\left(\frac{1}{2}+n\right)}{\Gamma(1+n)}$$

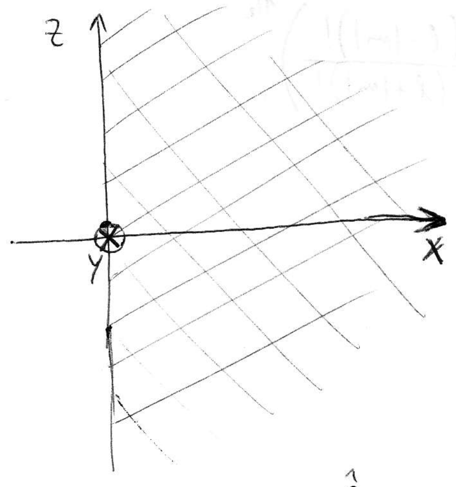
$$\langle \sin^2\vartheta\rangle = |A_2|^2\int_0^{2\pi} d\varphi\int_0^{\pi}\sin^3\vartheta\,\sin^{2n-2}\vartheta\, d\vartheta = |A_2|^2\, 2\pi\int_0^{\pi}\sin^{2n+1}\vartheta\, d\vartheta = \text{MATHEMATICA}$$

$$= |A_2|^2\, 2\pi\,\frac{\sqrt{\pi}\,\Gamma(1+n)}{\Gamma\left(\frac{3}{2}+n\right)}$$

$$\delta(\sin\vartheta)=\sqrt{\langle \sin^2\vartheta\rangle-\langle \sin\vartheta\rangle^2}=$$

$$=\sqrt{|A_2|^2\,2\pi\,\frac{\sqrt{\pi}\,\Gamma(m+1)}{\Gamma\left(\frac{3}{2}+m\right)}-|A_2|^4\,4\pi^2\,\pi\left(\frac{\Gamma\left(\frac{1}{2}+m\right)}{\Gamma(1+m)}\right)^2}=$$

$$=|A_2|^2\,2\pi\sqrt{\frac{\sqrt{\pi}\,\Gamma(m+1)}{\Gamma\left(\frac{3}{2}+m\right)}-|A_2|^2\,2\pi^2\left(\frac{\Gamma\left(\frac{1}{2}+m\right)}{\Gamma(m+1)}\right)^2}$$

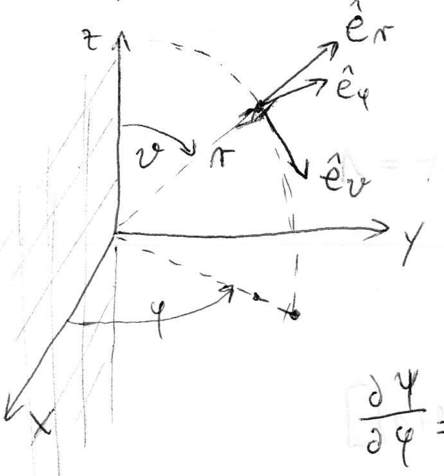


GLEDAMO PRETOK SKOZI ŠRAFIRANO POLOVICO RAVNINE Z-X.

$$\varphi=0$$

$$j=\frac{\hbar}{i2m}\left(\psi^*\nabla\psi-\psi\nabla\psi^*\right)$$

ZANIMA NAS SAMO j PRAVOKOTNO NA RAVNINO Z-X, TOREJ V SMERI $\hat{e}_\varphi$. ZATO VZAMEMO SAMO TA DEL GRADIENTA:



$$\nabla\psi=\frac{1}{r\sin\vartheta}\frac{\partial\psi}{\partial\varphi}\,\hat{e}_\varphi$$

$$A=A_1\cdot A_2$$

$$\psi=A\sin^{m-1}\vartheta\, e^{i(m-1)\varphi}\, r^{m-1}\, e^{-\frac{r}{m r_B}}$$

$$\frac{\partial\psi}{\partial\varphi}=A\sin^{m-1}\vartheta\, r^{m-1}\, e^{-\frac{r}{m r_B}}\, i(m-1)\varphi\, e^{i(m-1)\varphi}$$

$$j=\frac{\hbar}{i2m}|A|^2\sin^{2(m-1)}\vartheta\, r^{2(m-1)}\, e^{-\frac{2r}{m r_B}}\left[i(m-1)+i(m-1)\right]=$$

$$j=\frac{\hbar}{m}|A|^2\sin^{2(m-1)}\vartheta\, r^{2(m-1)}\, e^{-\frac{2r}{m r_B}}(m-1)$$

$$\int j\,\overset{r\,dr\,d\vartheta}{dS}=\frac{\hbar}{m}|A|^2\int_0^\pi \sin^{2(m-1)}\vartheta\, d\vartheta\int_0^\infty r^{2m-1}e^{-\frac{2r}{m r_B}}dr=\frac{\hbar}{m}|A|^2\cdot\frac{\sqrt{\pi}\,\Gamma\left(m-\frac{1}{2}\right)}{\Gamma(m)}\cdot 2^{-2m}\left(\frac{1}{m r_B}\right)^{-2m}\Gamma(2m)$$