2. KOLOKVIJ IZ KVANTNE MEHANIKE I

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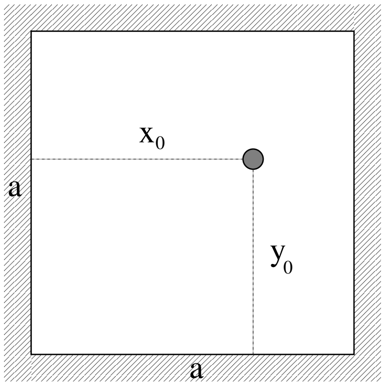

1. We want to couple a square quantum dot to a very thin wire that is connected perpendicularly to the dot. The quantum dot may be represented as a twodimensional infinite potential well with the side length $a = 100\,\mathrm{nm}$. The influence of the wire is described by a delta potential $V(x, y) = W_0\,\delta(x-x_0)\delta(y-y_0)$, where $x_0,\ y_0$ are the coordinates of the wire attachment point and $W_0 = 10^{-40}\,\mathrm{Jm}^2$. As we wish to use the first excited, doubly degenerate state of the electron with the mass $m_e = 9.1 \times 10^{-31}\,\mathrm{kg}$ to store quantum information, the wire is desired to have a minimal influence on this state.
   - Calculate the energy shift of the ground and the first excited (degenerate) state of the electron in the dot due to the influence of the wire as a function of the attachment point.
   - Where must the wire be attached in order for the first excited state to remain degenerate in the first order of the perturbation?

2. Two ions with the spins $S = 1/2$ caught in a trap are prepared such that at the time $t = 0$ each of them is in a state with a well defined component of the angular momentum in the z direction, $S_z^{(1,2)} = +\hbar/2$. From the time $t = 0$ until the time $t = T = 10^{-3}\,\mathrm{s}$ we act on only one of the ions with the magnetic field in the y direction. The total Hamiltonian operator during this time interval is given as $H = bS_y^{(2)}$ where $b = 10^3\,\mathrm{s}^{-1}$.
   - Write down the state of the system after the time T (hint: this is the product of the state of the first spin that remains unaltered, multiplied by the state of the second spin that is appropriately rotated due to the action of H).
   - Calculate the expected value of the square of the total angular momentum $J^2 = (\vec{S}^{(1)} + \vec{S}^{(2)})^2$ of both spins and the expected value of the component of the angular momentum of both spins in the z direction $J_z = S_z^{(1)} + S_z^{(2)}$ at the time T.
   - What is the probability to find the two spins in the state with a well defined quantum number of the total angular momentum $J = 0$ at the time T?

$\hbar = 1.05 \times 10^{-34}\,\mathrm{Js}$