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Naloga: Imamo krogelno simetrični potencial. Delec ima vrtilno količino L = 1, $L_z = 1$;

Definiramo operatorja L^2 in L_z:

$$L^2 \left| LL_z \right\rangle = \hbar^2 L(L+1) \left| LL_z \right\rangle$$

$$L_z \left| LL_z \right\rangle = \hbar L_z \left| LL_z \right\rangle$$

Vnekem trenutku vklopimo zunanje magnetno polje, ki je pod kotom ϑ glede na os z. Uvedemo nov koordinatni sistem `.

(Če se delec vrti ima magnetni moment, zato je energija sorazmerna z L*B)

$$E = -\mu \cdot B$$

$$H = -\frac{\hbar \nabla^2}{2m} + V(r) - \lambda \vec{L}\vec{B}$$

Zaradi mag. polja pa se stanja razcepijo: l = 1;

l_z` = 1, 0, -1;

$\left|11\right\rangle$, $\left|10\right\rangle$, $\left|1-1\right\rangle$;

$$H\left|11\right\rangle` = (E_0 - \hbar B\lambda)\left|11\right\rangle`$$

$$H\left|10\right\rangle` = (E_0 - \hbar B\lambda \cdot 0)\cdot\left|10\right\rangle` = E_0\left|10\right\rangle`$$

$$H\left|1-1\right\rangle` = (E_0 + \hbar B\lambda)\left|1-1\right\rangle`$$

Sedaj zapišemo začetno stanje s temi stanji.

$$\left|11\right\rangle = \sum_{l_z`=-1}^{1} A_{l_z`}\left|1l_z`\right\rangle$$

$$L_z\left|11\right\rangle = \hbar\left|11\right\rangle$$

L_z bi radi zapisali z operatorji novega sistema.

$$L_z = L_z`\cos\vartheta - L_x`\sin\vartheta$$

$$(L_z`\cos\vartheta - L_x`\sin\vartheta)\sum_{l_z`=-1}^{1} A_{l_z`}\left|1l_z`\right\rangle = \hbar\sum_{l_z`=-1}^{1} A_{l_z`}\left|1l_z`\right\rangle$$

Vpeljemo L_+ in L_-

$$L_+ = L_x + iL_y; L_- = L_x - iL_y$$

$$L_x = \frac{L_+ + L_-}{2}; L_y = \frac{L_+ - L_-}{2i}$$

$$L_+\left|ll_z\right\rangle = \hbar\sqrt{l(l+1) - l_z(l_z+1)}\left|ll_z+1\right\rangle; L_-\left|ll_z\right\rangle = \hbar\sqrt{l(l+1) - l_z(l_z-1)}\left|ll_z-1\right\rangle$$

$$\sum_{l_z`=-1}^{1} A_{l_z`}(\cos\vartheta\hbar l_z\left|ll_z`\right\rangle - \sin\vartheta\frac{1}{2}(\hbar\sqrt{l(l+1) - l_z`(l_z`+1)}\left|ll_z`+1\right\rangle + \hbar\sqrt{l(l+1) - l_z`(l_z`-1)}\left|ll_z`-1\right\rangle) = \hbar\sum_{l_z`=-1}^{1} A_{l_z`}\left|ll_z`\right\rangle$$

Z leve pomnožimo z $\left\langle lL_z\right|$

$$\sum_{l_z`=-1}^{1} A_{l_z`}(\cos\vartheta\hbar l_z`\langle lL_z|ll_z`\rangle - \sin\vartheta\frac{1}{2}(\hbar\sqrt{l(l+1)-l_z`(l_z`+1)}\langle lL_z|ll_z`+1\rangle +$$

$$+\hbar\sqrt{l(l+1)-l_z`(l_z`-1)}\langle lL_z|ll_z`-1\rangle) = \hbar\sum_{l_z`=-1}^{1} A_{l_z`}\langle lL_z||ll_z`\rangle\rangle|ll_z`\rangle$$

$$\sum_{l_z`=-1}^{1} A_{l_z`}(\cos\vartheta\hbar l_z`\delta(l_z`,L_z) - \sin\vartheta\frac{1}{2}(\hbar\sqrt{l(l+1)-l_z`(l_z`+1)}\delta(l_z`+1,L_z) +$$

$$+\hbar\sqrt{l(l+1)-l_z`(l_z`-1)}\delta(l_z`-1,L_z)) = \hbar\sum_{l_z`=-1}^{1} A_{l_z`}\delta(l_z`,L_z)$$

$$A_{L_z}(\cos\vartheta L_z) - A_{L_{z-1}}\sin\vartheta\frac{1}{2}\sqrt{l(l+1)-L_z(L_z-1)} - A_{L_{z+1}}\sin\vartheta\frac{1}{2}\sqrt{l(l+1)-L_z(L_z+1)} = A_{L_z}$$

Imamo system treh enačb s tremi neznankami.(L_z = -1, 0, 1)

L=1,L_z=1:

$$A_1\cos\vartheta - A_0\frac{1}{2}\sin\vartheta\sqrt{2} - A_2\frac{1}{2}\sin\vartheta\sqrt{0} = A_1$$

L=1,L_z=-1:

$$-A_{-1}\cos\vartheta - A_0\frac{1}{2}\sin\vartheta\sqrt{2} = A_{-1}$$

L=1,L_z=0:

$$A_{-1}\frac{1}{2}\sin\vartheta\sqrt{2} - A_1\frac{1}{2}\sin\vartheta\sqrt{2} = A_0$$

Zapišemo v matrični obliki:

$$\begin{bmatrix} \cos\vartheta - 1, -\frac{\sqrt{2}}{2}\sin\vartheta, 0 \\ -\frac{\sqrt{2}}{2}\sin\vartheta, -1, -\frac{\sqrt{2}}{2}\sin\vartheta \\ 0, -\frac{\sqrt{2}}{2}\sin\vartheta, -1-\cos\vartheta \end{bmatrix} * \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = 0,$$

Imamo homogen system.Eno neznanko si lahko izberemo.

A_1=1;

$$\cos\vartheta - 1 - A_0\frac{\sqrt{2}}{2}\sin\vartheta = 0 \Rightarrow A_0 = \frac{\cos\vartheta - 1}{\frac{1}{\sqrt{2}}\sin\vartheta} = -\frac{\sqrt{2}\sin(\vartheta/2)}{\cos(\vartheta/2)} = -\sqrt{2}tg(\vartheta/2)$$

$$\frac{\sqrt{2}}{2}\sqrt{2}\sin\vartheta tg(\vartheta/2) = (\cos\vartheta + 1)A_{-1} \Rightarrow A_{-1} = \frac{\sin\vartheta tg(\vartheta/2)}{1+\cos\vartheta} = tg^2(\vartheta/2)$$

Normalizacija:

$$|A_i|^2 = 1$$

$$\lambda^2(2tg^2(\frac{\vartheta}{2})) + 1 + tg^4(\frac{\vartheta}{2}) = 1$$

$$\lambda^2(1+tg^2(\frac{\vartheta}{2}))^2=1 \Rightarrow \lambda=\frac{1}{tg^2(\frac{\vartheta}{2})+1}=(\frac{1}{\cos^2(\vartheta/2)})^{-1}=\cos^2(\vartheta/2)$$

$$A_1=\cos^2(\vartheta/2), A_0=-\sqrt{2}\sin(\vartheta/2)\cos(\vartheta/2)=-\frac{1}{\sqrt{2}}\sin\vartheta, A_{-1}=\sin^2(\vartheta/2)$$

Ob času t=0:

$$|\psi,0\rangle=\cos^2(\vartheta/2)|11\rangle`-\sqrt{2}\sin(\vartheta/2)\cos(\vartheta/2)|10\rangle`+\sin^2(\vartheta/2)|1-1\rangle`$$

Časovni razvoj:

$$|\psi,t\rangle=\cos^2(\vartheta/2)e^{i\lambda Bt}|11\rangle`-\sqrt{2}\sin(\vartheta/2)\cos(\vartheta/2)|10\rangle`+\sin^2(\vartheta/2)e^{-i\lambda Bt}|1-1\rangle`;\omega=\lambda\cdot B$$