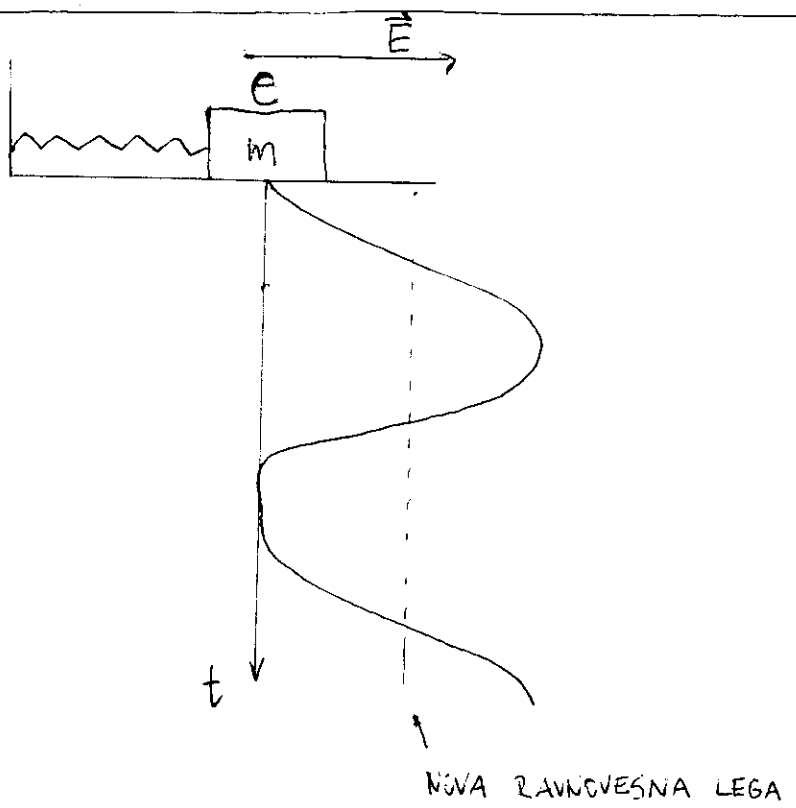
Elektron je v osnovnem stanju harmonskega oscilatorja $H=\frac{p^2}{2m}+\frac{kx^2}{2}$. Ob času $t=0$ v trenutku vključimo električno polje E.

- pokaži, da lahko hamiltonian za $t>0$ zapišemo kot $H=\hbar\omega_0\left(\tilde{a}^+\tilde{a}+\frac{1}{2}\right)$ in poišči povezavo med starimi in novimi anihilacijskimi/kreacijskimi operatorji
- pokaži, da je valovna funkcija ob $t=0$ lastna funkcija novega anihilacijskega operatorja. Razvij funkcijo po novi
- poišči razvoj valovne funkcije



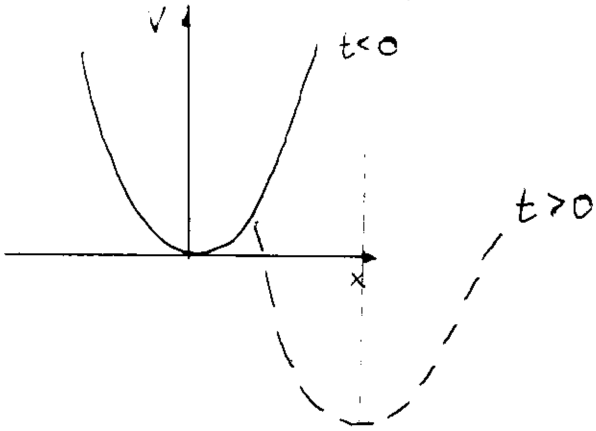
$t<0$: $H_0=\frac{p^2}{2m}+\frac{1}{2}kx^2$

$t>0$: $H=H_0-Eex$

ZAČETNA VALOVNA FUNKCIJA: $|\psi, 0\rangle = |0\rangle$ OSNOVNO STANJE

KAKŠNE SO NOVE LASTNE FUNKCIJE ZA H ?

$$H = \frac{p^2}{2m} + \frac{1}{2} k x^2 - E e x = \frac{p^2}{2m} + \frac{k}{2}\left(x - \frac{1}{k} E e\right)^2 - \frac{e^2 E^2}{2k}$$



DEFINIRAMO: $\tilde{x} = x - \frac{1}{k} E e$

$$\tilde{p} = -i\hbar \frac{\partial}{\partial \tilde{x}} = p$$

$$H = \frac{\tilde{p}^2}{2m} + \frac{1}{2} k \tilde{x}^2 - \cancel{\frac{e^2 E^2}{2k}} \leftarrow$$ konstanten premik energije (lahko črtamo)

$\Rightarrow$ imamo iste oblike lastnih funkcij

$$|n\rangle = \frac{(\tilde{a}^+)^n}{\sqrt{n!}} |0\rangle \qquad a = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} + i \frac{p}{p_0}\right)$$

$$\tilde{a} = a - \underbrace{\frac{1}{\sqrt{2} k} E e \frac{1}{x_0}}_{-z}$$

LASTNE FUNKCIJE ZA $\tilde{a}$?

$a = \tilde{a} - z \qquad (\tilde{a} - z)|0\rangle = 0 \Rightarrow \tilde{a}|0\rangle = z|0\rangle$ $|0\rangle$ je lastno stanje za $\tilde{a}$

$$\tilde{a}|z\rangle = z|z\rangle \qquad \tilde{n} \ldots \text{lastne funkcije } \tilde{a}$$

$$|z\rangle = \sum_{n} c_n |\tilde{n}\rangle$$

$$\tilde{a}|\tilde{n}\rangle = \sqrt{n}\,|\widetilde{n-1}\rangle$$

$$\sum_n c_n \,\tilde{a}|\tilde{n}\rangle = \sum_n z c_n |\tilde{n}\rangle \qquad \Big/ \langle\tilde{m}|$$

$$\sum_n c_n \langle\tilde{m}|\tilde{a}|\tilde{n}\rangle = \sum_n z\langle\tilde{m}|c_n|\tilde{n}\rangle$$

$$\langle\tilde{m}|\tilde{n}\rangle = \delta_{\tilde{m},\tilde{n}}$$

$$\sum_{\tilde{n}} \sqrt{n}\, c_{\tilde{n}} \langle\tilde{m}|\widetilde{n-1}\rangle = z\, c_{\tilde{m}}$$

$$\sqrt{\tilde{m}+1}\, c_{\tilde{m}+1} = c_{\tilde{m}}$$

$$c_{\tilde{n}+1} = \frac{z}{\sqrt{\tilde{n}+1}}\, c_{\tilde{n}}$$

$$\underline{\underline{c_{\tilde{n}} = \frac{(z)^{\tilde{n}}}{\sqrt{\tilde{n}!}}\, c_{\tilde{0}}}} \Rightarrow |z\rangle = c_{\tilde{0}} \sum_{\tilde{n}} \frac{z^{\tilde{n}}}{\sqrt{\tilde{n}!}}|\tilde{n}\rangle$$

$$\langle z|z\rangle = 1 = c_{\tilde{0}}^2 \sum_{\tilde{n}} \frac{|z|^{2\tilde{n}}}{\tilde{n}!} = c_{\tilde{0}}^2\, e^{|z|^2} \Rightarrow c_{\tilde{0}} = e^{-\frac{1}{2}|z|^2}$$

$$|z\rangle = e^{-\frac{1}{2}|z|^2} \sum_{\tilde{n}} \frac{z^{\tilde{n}}}{\sqrt{\tilde{n}!}} \frac{(\tilde{a}^+)^{\tilde{n}}}{\sqrt{\tilde{n}!}}|\tilde{0}\rangle = e^{-\frac{1}{2}|z|^2} e^{z\tilde{a}^+}|\tilde{0}\rangle = |z,0\rangle$$

KAKO SE FUNKCIJA RAZVIJA S ČASOM?

$$|z,t\rangle = e^{-\frac{|z|^2}{2}} \sum_{\tilde{n}} \frac{z^{\tilde{n}}}{\sqrt{\tilde{n}!}} e^{-i\omega(\tilde{n}+\frac{1}{2})t} |\tilde{n}\rangle =$$

$$= e^{-\frac{i\omega t}{2} - \frac{|z|^2}{2}} \sum_{\tilde{n}} \frac{\left(z e^{-i\omega t}\right)^{\tilde{n}}}{\sqrt{\tilde{n}!}} |\tilde{n}\rangle = e^{-\frac{1}{2}(i\omega t + |z|^2)} \sum_{\tilde{n}} \frac{(z e^{-i\omega t})^{\tilde{n}}}{\sqrt{\tilde{n}!}} \frac{(\tilde{a}^+)^{\tilde{n}}}{\sqrt{\tilde{n}!}} |\tilde{0}\rangle$$

$$|z,t\rangle = e^{-\frac{1}{2}(i\omega t + |z|^2)} e^{z e^{-i\omega t} \tilde{a}^+} |\tilde{0}\rangle$$

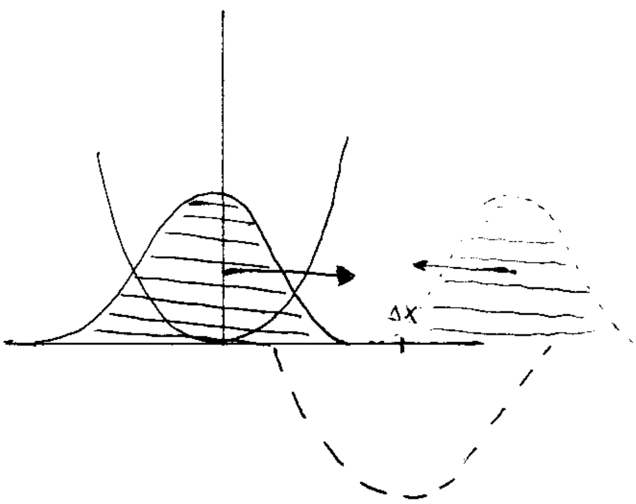
$$\underline{|z,t\rangle = e^{-\frac{i\omega t}{2}} |z e^{-i\omega t}, 0\rangle}$$

NAŠ PROBLEM $\quad \tilde{a}|0\rangle = z|0\rangle$

$$|0\rangle$$

$$|0,t\rangle = e^{-i\frac{\omega t}{2}} |z e^{-i\omega t}, 0\rangle$$

verjetnostna gostota za elektron klasično oscilira



$$\Delta x = \frac{eE}{k}$$