Domača naloga
Sipanje delcev na potencialu

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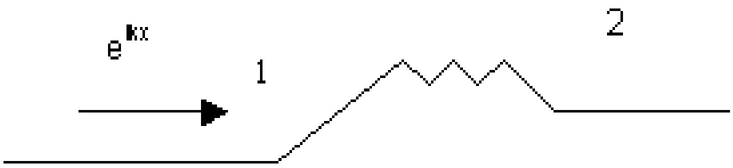
1 Sipanje delcev na potencialu

Imamo nek poljuben potencial na katerem se siplje tok delcev.



Zapišemo valovno funkcijo za prvo območje in drugo območje:

$$\psi_1 = Ae^{ikx} + Be^{-ikx}$$
$$\psi_2 = Ce^{ik'x}$$

Zanima nas prepustnost: $T = \frac{prepuščenitok}{vpadnitok}$

$$j_1 = \frac{i\hbar}{2m}\left(\frac{\partial\psi_1^*}{\partial x}\psi_1 - \psi_1^*\frac{\partial\psi_1}{\partial x}\right) =$$

$$= \frac{i\hbar}{2m}\Big[\left(-ikA^*e^{-ikx} + ikB^*e^{ikx}\right) * \left(Ae^{ikx} + Be^{-ikx}\right) -$$
$$- \left(A^*e^{-ikx} + B^*e^{ikx}\right) * \left(ikAe^{ikx} - ikBe^{-ikx}\right)$$

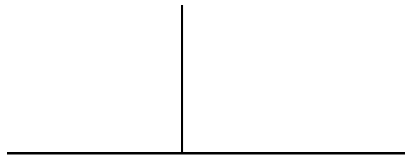
$$= \frac{i\hbar}{2m}\Big[-ikAA^* + ikB^*Ae^{2ikx} -$$
$$-ikA^*Be^{-2ikx} + ikB^*B - ikA^*A + ikBA^*e^{-2ikx} - ikB^*Ae^{2ikx} + ikBB^*$$
$$= \frac{i\hbar}{2m}[-2ikA^*A + 2ikB^*B] = \frac{k\hbar}{m}\left(A^*A - B^*B\right),$$

$$\frac{k\hbar}{m} = hitrost, A^*A - \text{verjetnostna gostota},$$
$$j = \rho v,\ j_2 = \frac{k'\hbar}{m}C^*C.$$

Tako vidimo da je prepustnost : $T = \frac{k'C^*C}{kA^*A}$.

1.1 Primer:

Potencial je delta funkcija $V(x) = V_0\delta(x)$



x=0

$$\psi_1 = Ae^{ikx} + Be^{-ikx}$$
$$\psi_2 = Ce^{ikx}$$

Robni pogoji:
pri x = 0: $\psi_1 = \psi_2 \rightarrow$ A+B=C

$$\frac{\partial\psi_1}{\partial x} - \frac{\partial\psi_2}{\partial x} = -\frac{2V_0 m}{\hbar^2}\psi$$

$$\left[\left(Aike^{ikx} - ikBe^{-ikx}\right) - Cike^{ikx} = -\frac{2V_0 m}{\hbar^2}Ce^{ikx}\right]_{x=0}.$$

$Aik - ikB - ikC = \frac{2V_0}{\hbar^2}C \rightarrow$ če vstavim za B = C – A $2A = 2C\left(-\frac{V_0 m}{ik\hbar^2} + 1\right)$

$$C = \frac{A}{(-\frac{V_0 m}{ik\hbar^2}+1)} \text{ in } C^* = \frac{A}{(\frac{V_0 m}{ik\hbar^2}+1)}.$$

Sedaj lahko izračunamo prepustnost: $T = \frac{CC^*}{AA^*} = \frac{1}{(1+\frac{iV_0 m}{k\hbar^2})(1-\frac{iV_0}{k\hbar^2})} = \frac{1}{(1+(\frac{V_0 m}{k\hbar^2})^2)}$.
Sedaj še vstavimo za $k = (\frac{E2m}{\hbar^2})^{\frac{1}{2}}$ in iz tega sledi, da je

$$T = \frac{1}{1+\frac{V_0^2 m^2\hbar^2}{2mE\hbar^4}} = \frac{1}{1+\frac{V_0^2 m}{2E\hbar^2}} = \frac{E}{E+\frac{mV_0^2}{2\hbar^2}}.$$

Iz tega vidimo da se prepustnost z večanjem energije približuje 1.