Domaca naloga iz kvantne mehanike 1
Spin V
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Naloga:
Elektron v homogenem magnetnem polju je v osnovnem stanju (zanima nas samo spinski del elektronove valovne funkcije). Obravnavaj casovni razvoj sistema, ce na elektron deluje kratek pulz magnetnega polja v smeri pravokotno na stalno magnetno polje, $\vec{B}(t) = (\tilde{B}_x \delta(t), 0, B_z)$.

Resitev:
Hamiltonian je oblike

$$H = g\mu_B \frac{\vec{B}\cdot\vec{S}}{\hbar} = \frac{g\mu_B}{\hbar}(\tilde{B}_x\delta(t)S_x + B_zS_z)$$

Kjer je H_0 hamiltonian katerega lastne funkcije so $|\uparrow>$ in $|\downarrow>$, V(t) pa motnja:

$$H_0 = \frac{g\mu_B}{\hbar}B_zS_z$$
$$V(t) = \frac{g\mu_B}{\hbar}\tilde{B}_x\delta(t)S_x$$

Lastne vrednosti lastnih funkcij operatorja H_0 so:

$$|\uparrow>: \quad E_\uparrow = \frac{1}{2}g\mu_B B_z$$
$$|\downarrow>: \quad E_\downarrow = -\frac{1}{2}g\mu_B B_z$$

Funkcija pred motnjo: $|\Psi >= 1\cdot|\downarrow>$
In po motnji: $|\Psi, t) = c_1|\uparrow, t> + c_2|\downarrow, t>$
Matricni element V_{mn} se v splosnem zapise kot

$$V_{mn}(t) = \int \frac{g\mu_B}{\hbar} e^{\frac{iH_m}{\hbar}t}\Psi^*_m(x,0)V(x,t)e^{-\frac{iH_n}{\hbar}t}\Psi_n(x,0)dx =$$
$$\frac{g\mu_B}{\hbar}e^{\frac{i(E_m-E_n)}{\hbar}t} < m|V(t)|n >= \frac{g\mu_B}{\hbar}e^{i\omega_{mn}t} < m|V(t)|n >$$

Ker je v nasem primeru

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

sta nenicelna samo nediagonalna elementa matrike V_{ij} in ce upostevamo se enacbe za lastne energije:

$$V_{12} = V_{\uparrow\downarrow} = \frac{g\mu_B}{\hbar} e^{\frac{ig\mu_B B_z}{\hbar}t} <\uparrow |S_x| \downarrow> \tilde{B}_x \delta(t) = \frac{g\mu_B}{2} e^{i\omega_z t} \tilde{B}_x \delta(t) = V_{21}^*$$

Resiti je treba matricno enacbo:

$$i\hbar \begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \end{pmatrix} = \begin{pmatrix} 0 & V_{12} \\ V_{21} & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

ce to razpisemo:

$$i\hbar \dot{c}_1 = \frac{g\mu_B}{2} e^{i\omega_z t} \tilde{B}_x \delta(t) c_2$$
$$i\hbar \dot{c}_2 = \frac{g\mu_B}{2} e^{i\omega_z t} \tilde{B}_x \delta(t) c_1$$

Robni pogoji so $c_1(0) = 0$ in $c_2(0) = 1$ in integriramo po $\int_{-\epsilon}^{\epsilon} dt$, $\epsilon \to 0$

$$i\hbar[c_1(0^+) - c_1(0^-)] = \frac{g\mu_B \tilde{B}_x}{2} \left(\frac{c_2(0^+) + c_2(0^-)}{2}\right)$$
$$i\hbar[c_2(0^+) - c_2(0^-)] = \frac{g\mu_B \tilde{B}_x}{2} \left(\frac{c_1(0^+) + c_1(0^-)}{2}\right)$$

pri tem sta $c_1(0^-) = 0$ in $c_2(0^-) = 1$ robna pogoja, $\frac{g\mu_B B_x}{4\hbar} = K$ pa brezdimenzijska kolicina, torej

$$c_1(0^+) = -i\frac{2K}{1+K^2}$$
$$c_2(0^+) = \frac{1-K^2}{1+K^2}$$

Koncna resitev je:

$$|\Psi, t> = -i\frac{2K}{1+K^2} \cdot |\uparrow, t> + \frac{1-K^2}{1+K^2} |\downarrow, t> .$$